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Arbitrage opportunities in no-arbitrage portfolios: The case of Bitcoin and Treasury Bills

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Abstract

Employing both mean-variance portfolio optimization and stochastic dominance analysis, we examine whether including Bitcoin and a 3-month US Treasury-Bill could generate arbitrage opportunities and lead to a better investment choice. The results show, independent of whether short sales are allowed, that: (1) investors are generally indifferent to portfolios with Bitcoin and without Bitcoin; (2) investors prefer to invest in portfolios with Treasury-Bill than portfolios without Treasury Bill to gain higher expected utility but not a higher expected wealth; and (3) portfolios with both Bitcoin and a Treasury-Bill dominate the ones without

Bitcoin and Treasury-Bill, at the first, second and third order. A subperiod analysis confirms the above results, indicating that investors can earn abnormal returns when both Bitcoin and Treasury-Bills are included in the portfolio. Through robustness checks using varying portfolio sizes and excluding outliers, we confirm the consistency and reliability of our findings. Our findings can be used by investors in their investment processes, and reveal the arbitrary opportunities when Bitcoin and a 3-month US Treasury-Bill is included in the investment.

JEL Classifications: C00, G11

Keywords: Bitcoin; Treasury bill; mean-variance portfolio optimization; stochastic dominance; arbitrage opportunities

1. Introduction

It is possible to generate arbitrage opportunities by using assets without any arbitrage opportunities. Some work in the literature has been done in this area. For example, Wong, Phoon, and Lean (2008) analyzed the performance of Asian hedge funds in which none of the assets had any arbitrage opportunity. However, they find some portfolios formed by using Asian hedge funds could get arbitrage opportunities. Lv, Chu, Wong, and Chiang (2021) included assets without any arbitrage opportunities like healthcare and 6-month T-bills in the portfolio, and found that portfolios that include both healthcare and 6-month T-bills could get portfolios with arbitrage opportunities. In addition, Wong, Cheong, Chui, and Vieito (2023) include assets without any arbitrage opportunities like some stocks from emerging and developed markets, and find that portfolios that include some stocks from emerging and developed markets could get portfolios with arbitrage opportunities. To extend their work, we employ both mean-variance portfolio optimization and stochastic dominance (SD) analysis and consider whether including Bitcoin and a 3-month US Treasury-Bill could generate arbitrage opportunities and lead to a better choice in the investment decision.

arbitrage opportunities refers to the simultaneous buying and selling of an asset in different markets to profit from any price differences. There could be many ways to generate some arbitrage opportunities by using assets without any arbitrage opportunities. In this paper, we use the linear combinations of assets that do not dominate assets in the sense of the first-order SD and check whether it is possible to generate portfolios that first-order dominate other assets/portfolios stochastically, which, in turn, suggests the presence of arbitrage opportunities. As far as we know, there is no theory in the literature saying that it is possible to generate portfolios that first-order dominate other assets/portfolios stochastically, and there is no empirical study suggesting an approach that could generate portfolios that first-order dominate other assets/portfolios stochastically. In this paper, we propose a way to generate portfolios that first-order dominate other assets/portfolios stochastically.

Cryptocurrencies are completely new financial assets, attracting market participants' attention, and increasing market capitalization quickly. The emerging cryptocurrency, namely Bitcoin, was one of the most disruptive and important developments, especially, after the global financial crisis (GFC). The 2008-2009 global financial crisis and the 2010-2013 European debt crisis triggered economic unrest, which was classified as a period of serious suffering for most economies worldwide, resulting in increased inflation, growing budget, and trade deficits, devaluations of currency, and declining reserves levels. One of the main features of Bitcoin is decentralization, which means that it is not controlled by central institutions such as banks or governments. Transactions are carried out directly between users, making them very flexible and free, and preventing sanctions. This decentralized feature relies on blockchain distributed ledger technology, which records all Bitcoin transactions and represents a decentralized database maintained by the user network. The blockchain allows for transparency and security because it is public and the information recorded on it is difficult to change. In some cases, Bitcoin can be used in the same way as traditional currencies, but it is more like an investment as it can be bought and sold on virtual exchanges. The value of Bitcoin has been fluctuating rapidly over the past years, and its high adjusted returns is the main reason for interest and concern, despite its high return volatility. Baur et al. (2018) suggest that Bitcoin and other cryptocurrencies are not currencies but rather act as speculative assets. However, other studies (e.g. Bouri et al., 2017; Garcia-Jorcano and Benito, 2020; Pho et al., 2021) highlight the diversification and hedging ability of Bitcoin for stock markets.

This paper introduces a novel mechanism to synthesize arbitrage opportunities through strategic portfolio construction, even when individual assets exhibit no inherent arbitrage potential. By forming linear combinations of assets that lack first-order stochastic dominance (SD), we demonstrate that optimized portfolios can achieve first-order SD over standalone assets or competing portfolios. This dominance signals the presence of arbitrage opportunities. Our key contribution lies in bridging stochastic dominance theory with arbitrage detection, offering a mathematically rigorous framework to: 1.Challenge conventional no-arbitrage assumptions by showing how diversification can create arbitrage conditions. 2. Empirically validate that portfolios combining "arbitrage-free" assets (e.g., Bitcoin and T-Bills) can systematically dominate benchmark strategies, contradicting classical market efficiency paradigms. 3. Provide actionable insights for portfolio managers to identify and exploit these synthetic arbitrage windows, enhancing risk-adjusted returns without relying on preexisting mispricings. This work redefines the interplay between diversification and arbitrage, positioning portfolio optimization itself as a catalyst for abnormal returns in theoretically efficient markets. In this paper, we also contribute to the literature on the diversification benefits of Bitcoin and Treasury-Bill by using the stochastic dominance approach (e.g., Bouri et al., 2019; Guesmi et al., 2019; Urquhart and Zhang, 2019; Goodell and Goutte, 2021) to investigate how investors can consider the mean-variance analysis (Markowitz, 1952a) when estimating the benefits of adding BitcoinTreasury-Bil in the portfolios. The SD approach is useful because ranking assets by the SD rule is equivalent to the expected-utility maximization and it does not have any restriction on the distributions of assets and the forms of utility, only requiring the utility to be increasing concave (Sriboonchita, et al., 2009; Levy, 2015; Guo and Wong, 2016). To the best of our knowledge, this method has not been used to measure the risk of portfolios diversified with Bitcoin by the stochastic dominance (SD) approach.

Based on the mean-variance portfolio optimization and stochastic dominance analysis, the empirical results indicate several original findings. First, investors are generally indifferent to portfolios with Bitcoin and without Bitcoin, however, investors prefer to invest in portfolios with Treasury-Bill than portfolios without Treasury Bill to gain higher expected utility but not a higher expected wealth. Second, portfolios with both Bitcoin and a Treasury-Bill dominate the ones without Bitcoin and Treasury-Bill, at the first, second and third order; this challenges market efficiency and suggests the existence of expected arbitrage opportunities in the market, allowing investors including both Treasury-Bill and Bitcoin in their portfolios to achieve both higher expected utility and higher expected wealth. A robustness check on subperiod analysis confirms the above results, indicating that investors can earn abnormal returns when both Bitcoin and Treasury-Bills are included in the portfolio. By using our findings individual investors and portfolio managers could build on our analysis to look for arbitrage opportunities by using assets without any arbitrage opportunities to generate arbitrage opportunities and get profits. Policymakers or regulators should be aware about the possibility to use assets without any arbitrage opportunities to generate arbitrage opportunities and get profits, which could be indicative of inefficiency in the market. Accordingly, they may impose some new policies and regulations to make the market become more efficient.

The rest of the paper is structured as follows: Section 2 reviews the literature. Second 3 describes the conjectures proposed in this paper. Section 4 outlines the dataset and methodology. Section 5 presents the empirical results. Section 6 offers a robustness check using sub-periods analysis, whereas Section 7 conduct a robustness analysis involving performance measures. Section 8 concludes.

2. Literature Review

Markowitz (1952a) first presents the theory of portfolio optimization and conducted productive analysis. All possible combinations of assets form a feasible set similar to a convex stereo region on the left. The upper half of the portfolio meets the minimum risk and maximum return conditions, which is known as the effective boundary of the portfolio. However, the mean-variance approach still has some disadvantages when assets are a non-normal distribution (Levy and Markowitz (1979), Kroll et al. (1984)) and the mean-variance approach is almost equivalent to the expected utility maximization in many situations. However, many studies, for example, Sriboonchita, et al. (2009) and Levy (2015), show that the stochastic dominance approach is better than the mean-variance approach. Subsequently, some studies have tried to identify a more reliable portfolio optimization. For example, Malek et al (2023) use conditional-value at risk (CVaR) with alpha-stable innovations which could be a better risk measurement approach. Although the determinant of risk measurement is improved, the financial data show various tail behavior patterns that cannot be captured by using traditional approaches. In this regard, the copula approach is highly useful for risk analysis. For example, Pho et al. (2021) employ a copula-based analysis with a multivariate probability distribution to study portfolios with Bitcoin and gold.

The role of Bitcoin in portfolio diversification remains a contested topic in financial research, with studies offering divergent conclusions based on varying methodologies and contexts. Proponents argue that Bitcoin enhances portfolio efficiency by improving risk-adjusted returns, particularly due to its low correlation with traditional assets like stocks, bonds, and REITs during non-crisis periods. For instance, Brière et al. (2015) and Platanakis & Urquhart (2020) demonstrate that Bitcoin's inclusion in portfolios boosts returns, especially when optimized using mean-variance frameworks or volatility timing strategies. Similarly, Bitcoin has been shown to act as a hedge against commodities, gold, and international equity markets, with studies like Pal & Mitra (2019) and Garcia-Jorcano & Benito (2020) highlighting its unique hedging properties. However, these benefits are often tempered by Bitcoin's notorious volatility, which limits its appeal to risk-averse investors, as noted by Ozturk (2020) and Pho et al. (2021). Critics further caution that Bitcoin's diversification benefits may be overstated, with Bouri et al. (2019) arguing that conventional assets still dominate optimal portfolios for maximizing risk-adjusted returns. The comparison between Bitcoin and gold as diversifiers reveals a nuanced debate. While Bitcoin proponents emphasize its superior Sharpe ratio and unique safe-haven role for equities (Shahzad et al., 2019; Bouri et al., 2020), gold consistently outperforms Bitcoin during crises, offering lower tail risk and more stable diversification benefits. Studies such as Ashraf et al. (2023) and Ustaoglu (2023) document gold's resilience during the COVID-19 pandemic and Russia-Ukraine war, contrasting with Bitcoin's mixed and context-dependent performance. This divergence is partly attributed to fundamental differences in their price dynamics and investor bases, as Conrad et al. (2018) argue that Bitcoin's behavior during high-volatility periods diverges sharply from gold's, complicating direct comparisons.

Cryptocurrency markets, particularly Bitcoin, have been shown to exhibit persistent arbitrage opportunities across exchanges, though their exploitability remains constrained by structural and operational factors. For instance, Guo et al. (2025) analyze minute-level data from 16 exchanges (2013-2024) and find that arbitrage opportunities endure longer on high-risk exchanges, where traders avoid capital deployment due to fears of platform insolvency. Similarly, Makarov & Schoar (2020) demonstrate that cross-country Bitcoin price deviations are significantly larger than intra-country ones, with arbitrage spreads widening during periods of rapid Bitcoin appreciation, suggesting market segmentation and regulatory barriers limit arbitrage capital flows. Despite the prevalence of these opportunities, practical exploitation is often unfeasible due to transaction costs, liquidity constraints, and operational friction. Muck et al. (2025) test triangular arbitrage on Binance, identifying thousands of theoretical opportunities, but show that thin order books and fees erase profitability. The evolution of market efficiency over time complicates the arbitrage landscape. Crépellière et al. (2023) document a marked decline in arbitrage profitability post-2018, attributing this to increased institutional participation, improved liquidity, and the rise of algorithmic trading. The role of Bitcoin in arbitrage strategies intersects with its broader portfolio diversification properties. While studies like Huang et al. (2021) and Odusami & Akinsomi (2024) focus on Bitcoin's hedging potential, they indirectly highlight how its volatility and cross-market correlations create transient arbitrage windows. For instance, Bitcoin's decoupling from traditional assets during dovish monetary regimes (Hung et al., 2024) or its weak safe-haven properties during crises (Ustaoglu, 2023) can amplify price divergences across exchanges, particularly in fragmented markets.

Despite the growing literature on Bitcoin, it does not offer much insight into the portfolio optimization with arbitrage strategies with SD preferences. Additionally, prior studies on the portfolio role of Bitcoin have predominantly focused on its diversification benefits with equities, commodities, or gold, while largely overlooking the interplay between Bitcoin and bonds-a critical difference given their contrasting risk profiles (speculative volatility vs. fixed-income stability). Furthermore, while prior work emphasizes Bitcoin's standalone diversification or hedging properties, it fails to systematically analyze portfolio-level arbitrage mechanisms arising from Bitcoin-bond interactions. Methodologically, most studies rely on retrospective correlation-based frameworks (e.g., copulas, GARCH) without integrating SD or mean-variance optimization to assess whether Bitcoin-bond portfolios dominate traditional allocations. Our paper is related to different strands in the literature lines. It follows up on recent research on the price building and arbitrage potential of cryptocurrencies (e.g., Pieters and Vivanco, 2017; Makarov and Schoar, 2020; Borri and Shakhnov, 2022). Liu and Tsyvinski (2021) find that the risk and return characteristics of cryptocurrencies are different from those of traditional assets. Other studies document arbitrage opportunities in the cryptocurrency market (e.g. Kristoufek and Bouri, 2023; Alexander et al. 2024). We contribute to this literature by examining the evolution of arbitrage in the cryptocurrency market by SD approaches. More generally, our paper is also related to the literature on SD approaches (e.g., Davidson and Duclos, 2000; Barrett and Donald, 2003; Linton et al., 2005; Lean, et al., 2008; Bai et al., 2011, 2015; Ng et al., 2017) and discussion on arbitrage strategies (e.g., Markowitz (1952a); Fama, 1970) and to studies that analyze the performance of a portfolio with(out) a specific asset (e.g., DeMiguel, et al., 2009; Rachev, et al., 2008), in our case Bitcoin.

Lv, Chu, Wong, and Chiang (2021) conjecture that investors should choose an asset with the highest expected return and an asset with the smallest variance. By comparing the performance of portfolios with and without healthcare and 6-month T-bills in the US market, they confirm that the conjecture holds by finding that portfolios include assets with the highest expected return and the smallest variance will get higher expected utility, in general. They also find a few arbitrage opportunities in the markets. In addition, Wong, Cheong, Chui, and Vieito (2023) hypothesized that including the asset with the highest mean, the asset that stochastically dominates many other assets, and the asset with the smallest standard deviation in the portfolio that could get higher expected utility and/or expected arbitrage opportunities. Using stocks from emerging and developed markets, they confirmed the conjectures hold for expected utility and they did find a few, but not many, stocks that first-order stochastically dominate a few other assets. To extend their work, in this paper, we apply the SD test developed by Davidson and Duclos (2000) and modified by Bai et al. (2015) to investigate whether including Bitcoin and a 3-month US Treasury-Bill could generate arbitrage opportunities and lead to a better choice in the investment decision. We propose several conjectures and hypotheses, as described below.

3. Conjecture

We examine whether it is possible to generate arbitrage opportunities by using assets without any arbitrage opportunities. To this end, we set the following conjecture:

Conjecture 1: It is possible to generate arbitrage opportunities by using assets without any arbitrage opportunities.

There could be many ways to generate some arbitrage opportunities by using assets without any arbitrage opportunities. In this paper, we use the linear combinations of assets that do not dominate assets in the sense of the first-order SD and check whether it is possible to generate portfolios that first-order dominate other assets/portfolios stochastically, which, in turn, suggests the existence of arbitrage opportunities. As far as we know, there is no theory in the literature indicating the possibility to use assets or portfolios that do not dominate other assets in the sense of the lower-order SD to generate portfolios that dominate some other assets/portfolios in the sense of lower-order SD preference. To examine the issue, we conjecture this new anomaly that could happen in the real markets by setting the following conjectures:

Conjecture 1A: Combinations of portfolios with higher-order SD preferences and/or portfolios with no SD preferences could result in getting portfolios having lower-order SD preferences to some of the other assets/portfolios.

In particular, combinations of portfolios with second-order SD preferences and/or portfolios with no SD preference could result in portfolios with first-order SD preference in which the portfolios before combining do not yield any arbitrage opportunities while the resulting portfolio could have arbitrage opportunities. To the best of our knowledge, no previous studies point to this observation and so far there is no existing theory predicting that this could happen. To examine the issue, we set the following conjecture:

Conjecture 1B: Combinations of portfolios with second-order or higher-order SD preferences and/or portfolios with no SD preference could result in portfolios dominating some of the other assets/portfolios in the sense of the first-order SD.

To test whether our conjectures could hold, we use the 3-month Treasury Bill, Bitcoin, and several stock indexes such as the S&P500, MSCI World, and others in our study. We chose the 3-month Treasury Bill in our study because it is the most commonly used asset that is well known to be stable and embeds low risk. We chose Bitcoin in our study because it is one of the most popular assets and Bitcoin investors believe it could gain good profit when investing in it. To convert the above conjectures into our analysis, we set the following conjecture:

Conjecture 2: Portfolios with both Bitcoin and the 3-month Treasury Bill could dominate other assets/portfolios, including portfolios without both Bitcoin and the 3-month Treasury Bill in the sense of the first-order SD.

To examine whether Conjecture 2 holds, we set the following hypotheses:

Hypothesis 1: Investors could get higher expected utility if they include Bitcoin in their portfolios.

Hypothesis 2: Investors could get higher expected utility if they include the 3-month Treasury Bill in their portfolios.

Hypothesis 3A: Investors could get higher expected utility if they include both Bitcoin and the 3-month Treasury Bill in their portfolios.

Hypothesis 3B: Investors could get higher expected wealth if they include both Bitcoin and the 3-month Treasury Bill in their portfolios.

4. Data and Methodology

4.1. Data

To construct different portfolios, we consider, besides Bitcoin[1] and 3-month Treasury Bill, different stock market indexes（S&P500, MSCI World (MSCI), MSCI North America, MSCI Pacific, and MSCI Europe), and the indices of various assets, covering corporate bond market (PIMCO Investment Grade Corporate Bond Index Exchange-Traded Fund (ETF)), the overall commodity market (SP GSCI Commodity), gold market (SP GSCI Gold), crude oil market (SP GSCI Crude Oil), real estate market (FTSE EPRA Nareit GLOBAL), exchange rate market (US DOLLAR INDEX DXY). Our data set contains 2,723 daily observations per series from August 18, 2011 to January 25, 2022, collected from DataStream. All prices and indices are against the US dollar. To gain a better understanding of the movements of different indices, we exhibit in Figure 1 the historical performances of the indices for different assets over the full sample period. We notice that the S&P GSCI gold indexes, ETF, and US dollar index DXY are relatively stable, while other index prices are relatively volatile, especially the prices of Bitcoin and the rate of the 3-month Treasury Bill. Figure 1A shows that the price of Bitcoin exhibits a major upward trend from April 2020 to April 2021, peaking on April 13, 2021, then it decreased sharply and reached its bottom on July 20, 2021; thereafter, it increased and reached its another

[1] Bitcoin prices are against the US dollar, collected from DataStream.

peak on November 08, 2021. Figure 1B shows that the 3-month Treasury Bill rates (3MTBILL) increased from October 2015 to March 2019 and reached its peak on March 21, 2019. As a consequence of COVID-19 in late 2019, the Treasury Bill rates decreases significantly starting on this date and reached the bottom on March 26, 2020, then it performed relatively moderately stable at the end of the study period. From Figure 1C, we note that the S&P 500, MSCI World, MSCI North America, MSCI Pacific, MSCI Europe, and FTSE EPRA Nareit GLOBAL indexes exhibit an uptrend during the entire period, except during the COVID19 outbreak under which they experienced downtrends. While S&P GSCI Commodity and S&P GSCI Crude Oil indexes show a downward trend during the entire period and continue to fall during the COVID-19 outbreak period, S&P GSCI gold index, ETF, and US dollar index (DXY) perform relatively moderate stable during the period.

Insert Figure 1A, Figure 1B, and Figure 1C here

4.2. Methodology

To test whether Bitcoin and the 3-month Treasury Bill are useful for the diversification of portfolios, we conduct our analysis as follows: in the first step, we apply the classical mean-variance portfolio optimization (MVPO) method to construct efficient frontiers for portfolios with and without assets. In the second step, we take 15 different portfolios on each efficient frontier (portfolios with or without assets regardless of whether a short sale is used). However, since our tests show that all return series are not normally distributed, we conduct SD tests on the 15 pairs of portfolios in the second step. Finally, in order to illustrate whether the equal-weighted portfolio suggestions from Markowitz (1952) or De Miguel et al. (2009) are better for efficient portfolios, we compare the 15 efficient portfolios to the equal-weighted one (both including Bitcoin and the 3-month Treasury Bill). The details of these methods are given below.

4.2.1 . Mean-variance portfolio optimization (MVPO)

The classical mean-variance portfolio optimization (MVPO) model introduced by Markowitz (1952), and improved later by Bai et al. (2009), Leung et al. (2012), and others, can be used to determine the asset allocation for a given amount of capital through the efficient frontier. To present the MVPO model formally, we assume that there are n assets and let x_i ($i=1,\ldots,n$) be the fraction of the capital invested in asset i of portfolio P in which the average return R_p is maximized, subject to a given level of its variance σ_p^2. We denote R_i to be the expected return of asset i and σ_{ij} the covariance of returns between assets i and j, for any i, $j=1,\ldots,n$. The general MVPO model is presented as follows:

$$Max \quad R_p = \sum_{i=1}^{n} R_i x_i \quad , \qquad (1)$$

subject to:

$$\sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{ij} x_i x_j = \sigma_p^2 \quad \text{and} \quad \sum_{i=1}^{n} x_i = 1.$$

If a short sale is not used, we add one more condition: $x_i \geq 0, i = 1,\ldots,n.$

Readers may refer to Bai et al. (2009a,b), Leung et al. (2012), Li, et al. (2022), and others to get better estimation approaches to estimate the return of the optimal portfolio.

4.2.2 . Stochastic dominance (SD) approach

The SD approach is useful because ranking assets by the SD rule is equivalent to the expected-utility maximization, and because it does not have any restriction on the distributions of assets and the forms of utility, only requiring the utility to be increasing concave (Sriboonchita, et al., 2009; Levy, 2015; Guo and Wong, 2016).

To apply the SD approach, we let X and Y be returns of assets or portfolios with cumulative distribution functions (CDFs); F and G, are probability density functions (PDFs), f and g; and u_X and u_Y are the means for X and Y respectively; for any integer j, and we define:

$H_j(x) = \int_a^x H_{j-1}(t)\,dt$ with $H_0 = h$ for $h = f, g;\ H = F, G.$ (2)

We call the integral H_j the j^{th}-order integral for $H = F$ and G. Thus, X dominates Y by FSD (SSD, TSD) denoted by $X \succ_1 Y (X \succ_2 Y, X \succ_3 Y)$ if $F_1(x) \leq G_1(x) (F_2(x) \leq G_2(x),\ F_3(x) \leq G_3(x))$ for all x, where FSD, SSD, TSD denote first-, second-, and third-order SD, respectively, For $X \succ_3 Y$, it requires $u_X > u_Y$ (Chan et al., 2020). Furthermore, for any integer n, U_n is a set of utility functions for the n^{th}-order investors such that $U_n = \{u: (-1)^{i+1} u^{(i)} \geq 0, i = 1, \ldots, n\}$, where $u^{(i)}$ is the i^{th} derivative of the utility function u. Wong and Ma (2008), among others, extend the theory to non-derivative utility. Notably, we assume in this paper that all investors are risk averse, and thus, the n^{th}-order investors are the n^{th}-order risk averters.

Regarding the SD test, Barrett and Donald (2003), Linton et al. (2005), Bai et al. (2011 2015), Ng et al. (2017), and others have developed several SD tests. In this paper, we apply the SD test developed by Davidson and

Duclos (2000) and modified by Bai et al. (2015) to investigate the preference for investors because it has good power and decent size (Lean, et al., 2008).

Let $\{f_i\}$ and $\{g_i\}$ be observations drawn from the returns X and Y of assets or portfolios with CDFs F and G, respectively, we use the following SD test:

$$T_j(x) = \frac{\hat{F}_j(x) - \hat{G}_j(x)}{\sqrt{\hat{V}_j(x)}} \quad \text{for } j = 1,2,3, \tag{3}$$

to test the following hypotheses:

$H_0: F_j \equiv G_j$ against three alternatives $H_1: F_j \not\equiv G_j$, $H_{1l}: F_j \succ_j G_j$, $H_{1r}: F_j \prec_j G_j$;

where

$$\hat{V}_j(x) = \hat{V}_{F_j}(x) + \hat{V}_{G_j}(x) - 2\hat{V}_{FG_j}(x);$$

$$\hat{H}_j(x) = \frac{1}{N_h(j-1)!}\sum_{i=1}^{N_h}(x - h_i)_+^{j-1};$$

$$\hat{V}_{H_j}(x) = \frac{1}{N_h}\left[\frac{1}{N_h\left((j-1)!\right)^2}\sum_{i=1}^{N_h}(x - h_i)_+^{2(j-1)} - \hat{H}_j(x)^2\right], H = F, G; h = f, g;$$

$$\hat{V}_{FG_j}(x) = \frac{1}{N_h}\left[\frac{1}{N_h\left((j-1)!\right)^2}\sum_{i=1}^{N_h}(x - f_i)_+^{j-1}(x - h_i)_+^{j-1} - \hat{F}_j(x)\hat{G}_j(x)\right];$$

and F_j and G_j are defined in (2). Readers may refer to Davidson and Duclos (2000) and Bai et al. (2011, 2015) for more information on the SD test.

5. Empirical Findings

5.1. Comparison among different indices

Looking at the descriptive statistics of the returns of indices and the return of the 3-month Treasury Bill in Table 1, we notice that the 3-month Treasury Bill has the highest rate of return over the study period (0. 005753) and a relatively low standard deviation (0.007951). The return of Bitcoin has the highest standard deviation (0.056591) and the second highest mean (0.002985). This information can be used to obtain the preference among different assets by using the mean-variance (MV) rule.

Insert Table 1 Here

Table 1 also indicates that only the skewness of the 3-month Treasury Bill and US Dollar index DXY are positive, meaning that the distribution of their returns is asymmetric to the right and the 3-month Treasury Bill has significantly positive skewness. The rest of the assets have significantly negative skewness. The value of the skewness is negative in most cases showing that the distribution of returns of the other indices are skewed to the left. It means that there are more returns under the average than those over the average, even in the up-market period. The values of the excess kurtosis (above 3) of assets are extremely high and have significantly higher kurtosis, and, as expected, the 3-month Treasury Bill has the significantly smallest kurtosis. The significantly higher kurtosis means that the tails of the distribution of the returns are thicker than those of a normal distribution. It thus confirms the high volatility of returns with numerous extreme returns situated in the tails of the distribution. All the JB test indicates that the distributions of the assets' returns are not normal, suggesting that the mean-variance analysis may provide misleading results. Thus, we skip discussing the MV results and present the SD results for individual assets in Table 2.

Given that the Jarque-Bera (JB) statistic shows that the distributions of the returns of all assets are not normally distributed (see Table 1), we apply the SD test to examine the preference of individual assets and the results are presented in Table 2. Since the main purpose of our paper is to study the preference of individual assets and portfolios with and without Bitcoin, portfolios with and without the 3-month Treasury Bill, portfolios both with and without Bitcoin and the 3-month Treasury Bill, we care more about the results showing whether Bitcoin and the 3-month Treasury Bill stochastically dominate other assets. Therefore, we mainly report the SD results between Bitcoin and the 3-month Treasury Bill with other assets. From Table 2, we find that: (1) Bitcoin is stochastically dominated by 9 assets in the sense of third order and 3 assets (MSCI North American, S&P 500, and 3MTBILL) in the sense of both second and third orders; and no other index stochastically is dominated by it; (2) 3-month Treasury Bill stochastically dominates 10 assets in the sense of both second and third orders and 2 assets (ETF, US Dollar Index DXY) in the first three orders, and no other asset stochastically dominates it; and, (3) in particular, investors prefer the 3-month Treasury Bill to Bitcoin in the sense of both second and third orders. In short, we conclude based on the SD approach that the 3-month Treasury Bill is the best choice among all individual assets under study.

Insert Table 2 Here

5.2. Preference for portfolios with and without Bitcoin

To examine whether hypothesis 1 is validated, we assess the preference of portfolios with and without Bitcoin. Before making the comparison, we first adopt the portfolio optimization approach to estimate the MV efficient frontiers when no short sale is performed (see the left panel in Figure 2) and short sale is allowed (see the right panel in Figure 2) strategies, and we plot the estimates of the frontiers of portfolios (that is, the portfolios with the highest expected rate of return for any given level of risk) with and without Bitcoin.

The steps are described as follows: we partition each efficient frontier (with and without Bitcoin regardless of whether a short sale is used) into 15 portfolios (from PF1 to PF15) with PF1 being the minimum-variance portfolio and PF15 being the maximum-return portfolio. The construction of the PF1-PF15 portfolios with Bitcoin and the PF1-PF15 without Bitcoin is performed as follows: (1) We partition the efficient frontier in 15 slices with equal distance on the horizontal axis; (2) we then determine the 15 points (representing the portfolios) and their risk-return coordinates belonging to the above-mentioned 15 slices. As a result, we obtain the 15 efficient portfolios whose returns are equally spaced along the whole range of the efficient frontier. One may wonder why investors might be indifferent to Bitcoin in portfolios, we argue that it may due to its extreme volatility (Bitcoin has the highest standard deviation 0.056591 in our study period), lack of reliable hedging properties during market crises and unresolved regulatory risks.

Figure 2 shows that the efficient frontiers for portfolios with and without Bitcoin coincide for both the long-only and the short-sale-is-allowed strategies. Without confusion, the short-sale-is-allowed strategy is also called the short-allowed strategy in this paper. This leads us to suspect that there is no advantage to including Bitcoin in the portfolios, regardless of whether short selling is allowed. In order to get a more accurate result, we apply the proper SD test to examine whether including Bitcoin is preferred for portfolios formally and display the results in Table 3.

Insert Figure 2 Here

Insert Table 3 Here

Using the frontiers for the portfolios with and without Bitcoin constructed in Section 4.2, we apply the SD test to compare the performance between each pair of the 15 chosen portfolios in the frontier of the portfolios with and without Bitcoin. Furthermore, we compare the performance between the equal-weighted portfolio (PF16) and the 15 chosen portfolios with and without Bitcoin. From Table 3, the SD results show that for each of the PF1-PF16, investors are indifferent between with- and without-Bitcoin portfolios for both the long-only strategy and the short-sale-is-allowed strategy.

To illustrate the first three-order dominance, we plot the CDFs of the with Bitcoin PF4 portfolio and without Bitcoin PF4 in Figure 3 (for simplicity, we skip reporting the figures of other portfolios). We find that portfolios with Bitcoin (F) and without Bitcoin (G) coincide for both long-only and short-sale-is-allowed strategies, implying that there is no first-order SD. In addition, we note that the first three-order SD statistics (T1, T2, and T3) are positive when the returns are negative, and they become negative when the returns are positive, showing that there is no difference between with- and without-Bitcoin portfolios. This finding leads us to reject Hypothesis 1 and conclude that investors will not get higher expected utility if they include Bitcoin in their portfolios.

To summarize, in most cases, there is no difference between portfolios with Bitcoin and portfolios without Bitcoin, regardless of whether short selling is allowed. Thus, investors are indifferent to portfolios with Bitcoin and without Bitcoin and they will not get a higher expected utility if they include Bitcoin in their portfolios.

Insert Figure 3 Here

5.3. Preference for portfolios with and without a 3-month T-Bill

To examine whether the Hypothesis 2 is validated, we turn to examine the preference of portfolios with and without a 3-month T-Bill. Using the same steps described in the previous subsection, we first find that the efficient frontiers on which all possible efficient portfolios offer the highest expected rate of return for each given level of risk, as shown in Figure 4 because the figures show that the efficient frontier with 3-month T-Bill is on top of that without 3-month T-Bill, regardless of whether short selling is allowed or not. Thus, one may believe that portfolios with a 3-month T-Bill are more efficient than those without a 3-month T-Bill. Thereafter, we apply SD to examine the preference of portfolios with and without a 3-month T-Bill and display the results in Table 4 for both the long-only and short-allowed strategies. From Table 4, the results for both long-only and short-allowed strategies are: (1) PF1 with 3-month T-Bill stochastically dominates PF1 without a 3-month T-Bill in the sense of first three orders; (2) for PF2-15, investors prefer portfolios with a 3-month T-Bill to without a 3-month T-Bill at the second and third orders; (3) there is no difference between portfolios with a 3-month T-Bill and without a 3-month T-Bill for PF16. To illustrate the first three-order dominance, we plot the CDFs of the PF4 portfolio with and without a 3-month T-Bill in Figure 5[2], which shows that the frontier of the portfolios with a 3-month T-Bill (F) is below that without a 3-month T-Bill (G) in the negative returns, while G is below F in the positive returns, implying that there is no first-order SD. We also note that the second- and third-order SD statistics (T2 and T3) are significantly negative, regardless of whether the returns are positive or negative, suggesting that the portfolios with a 3-month T-Bill second- and third-order dominate portfolios without a 3-month T-Bill. Accordingly, Hypothesis 2 is accepted and investors could get higher expected utility if they include a 3-month T-Bill in their portfolios.

To summarize, portfolios with a 3-month T-Bill could second- and third-order dominate the ones without a 3-month T-Bill, regardless of whether short selling is allowed. Hypothesis 2 is validated and we conclude that

[2] For simplicity, we skip reporting the figures of other portfolios.

investors could include a 3-month T-Bill in their portfolios to maximize their expected utility but not to maximize their expected wealth, except PF1. This confirms the results obtained with the mean-variance criterion portfolio optimization. The conclusion that portfolios including 3-month T-Bill yield higher expected utility than those without could trigger several economic effects. Firstly, increased demand for government bonds might lower their yields, reducing borrowing costs for governments and potentially expanding fiscal spending. Secondly, investors might adopt more conservative strategies, altering asset allocation norms and influencing liquidity dynamics across markets. Additionally, if perceived as a "safer" route to higher returns, this could paradoxically inflate bond prices beyond fundamentals, creating vulnerability to interest rate shocks. Lastly, prolonged preference for government bond might signal subdued economic confidence, prompting policymakers to reassess monetary tools or stimulus measures to reignite risk appetite.

Insert Table 4 Here

Insert Figure 4 Here

5.3. Preference for portfolios with and without Bitcoin and a 3-month T-Bill

To examine whether Hypothesis 3 is validated, we now examine the preference of portfolios with and without Bitcoin and the 3-month T-Bill. To do so, we first exhibit in Figure 6 the efficient frontiers of portfolios with and without Bitcoin and a 3-month T-Bill. Figure 6 shows that the efficient frontier with both Bitcoin and a 3-month T-Bill is on top of that without Bitcoin and a 3-month T-Bill, regardless of whether short selling is allowed or not. Based on the modern finance theory (see, Markowitz, 1952), one may believe that portfolios with Bitcoin and a 3-month T-Bill are more profitable than those without Bitcoin and a 3-month T-Bill. Thus, based on the efficient frontiers, we conclude that the portfolios with Bitcoin and a 3-month T-Bill are better than the portfolios without Bitcoin and a 3-month T-Bill, irrespective of whether a short sale is allowed or not. Accordingly, Hypothesis 3 is accepted.

Insert Figure 6 Here

Insert Table 5 Here

Furthermore, we use the SD statistics to examine the preference of portfolios with and without Bitcoin and the 3-month T-Bill and exhibit the SD preference of portfolios with and without Bitcoin and the 3-month T-Bill. Interestingly, the results reported in Table 5 show that all the portfolios with Bitcoin and a 3-month T-

Bill stochastically dominate all of the corresponding portfolios without Bitcoin and a 3-month T-Bill in the sense of the first three orders, regardless of whether a short sale is allowed or not. The SD results not only confirm the results obtained by using the MVPO criterion, but also show that there is a FSD preference between the portfolio with and without Bitcoin and a 3-month T-Bill and there is expected arbitrage opportunities between the portfolios, regardless whether short sell is allowed or not. As discussed in Jarrow (1986), Falk and Levy (1989), and others, arbitrage opportunities will appear if there exists any first-order stochastic dominance relationship. Investors will, therefore, increase both their wealth and expected utilities if they sell the dominated asset and hold long the dominant asset. Nevertheless, Guo *et al.* (2017) show that when a first-order stochastic dominance exists, investors may only increase their expected wealth rather than increase their wealth. Thus, in this paper, we call this phenomena expected arbitrage opportunities and our findings imply that there are expected arbitrage opportunities for investors to invest in portfolios with and without Bitcoin and a 3-month T-Bill along the efficient frontiers when short selling is allowed or not, and investors will get higher expected wealth as well as a higher expected utility if they sell the dominated asset and hold long the dominant asset.

To illustrate the first-order dominance, we plot the CDFs of the portfolio PF4 with Bitcoin and a 3-month T-Bill and the portfolio PF4 without Bitcoin and a 3-month T-Bill PF4 in Figure 7. From the figure, we find that portfolios with Bitcoin and a 3-month T-Bill dominate portfolios without Bitcoin and a 3-month T-Bill in nearly the entire distribution with some highly significant portion while their CDFs nearly coincide only in a very small region, suggesting that for the region that is not significant is very small, and thus, we conclude that the portfolios with Bitcoin and a 3-month T-Bill first-order dominate portfolios without Bitcoin and a 3-month T-Bill.

Building on Fama (1970), we indicate the existence of first-order stochastic dominance relationships between portfolios with and without Bitcoin and a 3-month T-Bill, suggesting the existence of arbitrage opportunities in the market, which, in turn, challenges market efficiency.

By comparing the results from Tables 3, 4, and 5, we find that, regardless of whether short selling is allowed, there is no difference between portfolios with Bitcoin and portfolios without Bitcoin and that portfolios with a 3-month T-Bill dominate the ones without a 3-month T-Bill at the second and third order. However, when both Bitcoin and a 3-month T-Bill are included in the portfolios, all the portfolios with Bitcoin and a 3-month T-Bill stochastically dominate all of the corresponding portfolios without Bitcoin and a 3-month T-Bill in the sense of the first three orders, regardless of whether a short sale is allowed or not. Thus both Hypotheses 3A and 3B are accepted and we conclude that investors could get both higher expected utility and higher expected wealth if they include both Bitcoin and the 3-month Treasury Bill in their portfolios, regardless of whether a short sale is allowed or not. This is a very interesting finding and, as far as we know, it is new to the related literature. The potential arbitrage opportunities from including 3-month Treasury Bill and Bitcoin in portfolios stem from their divergent risk-return dynamics and market behaviors. Government bonds, as low-risk assets, often correlate inversely with economic uncertainty (e.g., rising in value during crises), while Bitcoin, a high-volatility cryptocurrency, may rally due to speculative demand or inflation hedging. This asymmetry creates pricing dislocations when macroeconomic shocks (e.g., inflation spikes or liquidity shifts) disproportionately affect one asset over the other.

Insert Figure 7 Here

In conclusion, based on the above analysis may extend existing financial literature in two key dimensions. First, they bridge classical portfolio theory with emerging crypto-finance research by exploring interactions between government bonds (a traditional "safe haven") and Bitcoin (a speculative asset). This addresses gaps in understanding how hybrid portfolios spanning regulated and unregulated assets challenge conventional risk-return paradigms. Second, the identification of arbitrage mechanisms rooted in asymmetric market behaviors enriches market micro-structure literature. It highlights how fragmented participant bases and divergent reaction speeds to macroeconomic shocks can generate exploitable inefficiencies—a nuance underexplored in cross-asset arbitrage studies. Collectively, these analyses invite reevaluation of systemic risk models, particularly regarding cross-asset contagion channels in increasingly interconnected markets. They also underscore the need for interdisciplinary frameworks blending behavioral finance, crypto economics, and macro-prudential policy.

6. Robustness check

6.1. Robustness check: Sub-periods analysis

To check whether the above results hold for different periods, we conduct sub-periods analysis. From April 14, 2021, to May 19, 2021, the price of Bitcoin experienced a significant decline, falling from a close of \$65,000 to a minimum of \$30,000. The price volatility during this period was notable, with Bitcoin experiencing sharp fluctuations. Based on Figure 1A of Bitcoin prices, the 1st sub-period, from August 18, 2011 to April 13, 2021, is characterized by the tendency of Bitcoin prices to increase. The 2nd sub-period, from April 14, 2021 to January 25, 2022, is described by the tendency of Bitcoin prices to decrease. The segmentation aligns with identifiable macroeconomic catalysts and statistically significant breaks in price trends, as follows: 1. First sub-period (August 18, 2011-April 13, 2021) captures Bitcoin's maturation from an emerging digital asset to a mainstream financial instrument, marked by sustained upward momentum (Figure 1A). Key drivers include early adoption cycles and institutional entry and macroeconomic tailwinds such as quantitative easing and low interest rates post-2008. 2. Second Sub-Period (April 14, 2021-January 25, 2022) represents a sharp corrective phase, with Bitcoin plummeting from 65000 to30,000, driven by regulatory crackdowns (e.g., China's mining ban, SEC scrutiny of crypto ETFs) and macro shocks including inflationary pressures and Fed taper announcements. Elevated volatility indicative of deleveraging and retail investor capitulation.

6.1.1 The first sub-period

Appendix Table A1 presents the descriptive statistics returns of indices and the return of the 3-Month Treasury Bill for sub-period 1（August 18, 2011-April 13, 2021). It shows that the 3-Month Treasury Bill has the highest rate of return (0. 006179) and a relatively low standard deviation (0.008120). Bitcoin returns have the highest standard deviation (0.057318) and the second highest mean (0.003444). This information can be used to obtain the preference among different assets by using the MV rule. Since the results are consistent with but not as good as the preference by using the SD rule, we skip discussing the results.

Appendix Table A1 also shows only the skewness of the 3-Month Treasury Bill and US Dollar index DXY are positive, meaning that the distribution of their returns is asymmetric to the right and the 3-Month Treasury Bill has significantly positive skewness. The rest of the assets have significantly negative skewness. Excess kurtosis is extremely high and, as expected, the 3-Month Treasury Bill has the smallest kurtosis. All the JB test indicates that the return distributions of assets are not normal, suggesting that the mean-variance analysis may provide misleading results. Thus, we skip discussing the MV results and present the SD results for individual assets in Table 6.

Using the same steps used in the entire sample period, we present the efficient frontiers on which all possible efficient portfolios offer the highest expected rate of return for each given level of risk in Figure A1. Figures A1A shows that the efficient frontier with and without Bitcoin coincide for both long-only and short-sale-is-allowed strategies. This leads us to suspect that there is no advantage to including Bitcoin in the portfolios, regardless of whether short selling is allowed. On the other hand, from Figures A1B (A1C), we find that efficient frontier with a 3-month T-Bill (both with Bitcoin and a 3-month T-Bill) is on top of that without a 3-

month T-Bill (both without Bitcoin and a 3-month T-Bill), regardless of whether short selling is allowed. Thus, one may believe that portfolios with a 3-month T-Bill (both with Bitcoin and a 3-month T-Bill) are more efficient than those without a 3-month T-Bill (without Bitcoin and a 3-month T-Bill). In order to get a more accurate result, we will apply the SD approach to test whether including Bitcoin is preferred for portfolios and display the results in Table 6.

Insert Table 6 Here

Table 6 shows that: (1) between portfolios with and without Bitcoin: when using the long-only strategy, we find that for PF1-PF16, there is no difference between with- and without-Bitcoin portfolios; when using the short-sale-is-allowed strategy, the result shows that there is no difference between with- and without-Bitcoin portfolios for PF1-PF12, PF16; for PF13-PF15 investors prefer portfolios with-Bitcoin to those without-Bitcoin at the second and third orders. (2) Between portfolios with and without a 3-month T-Bill: PF1 with a 3-month T-Bill stochastically dominates PF1 without a 3-month T-Bill in the sense of first three orders; for PF2-15, investors prefer portfolios with a 3-month T-Bill to those without a 3-month T-Bill at the second and third orders; there is no difference between portfolios with a 3-month T-Bill and without a 3-month T-Bill for PF16. (3) Between portfolios with and without Bitcoin and a 3-month T-Bill: all the portfolios with Bitcoin and a 3-month T-Bill stochastically dominate the corresponding portfolios without Bitcoin and a 3-month T-Bill in the sense of the first three orders, regardless of whether a short sale is allowed. The SD results are consistent with the results obtained by using the MVPO criterion but they provide more information in sub-period 1.

The finding in (1) leads us to reject Hypothesis 1 and conclude that investors will not get higher expected utility if they include Bitcoin in their portfolios for Sub-period I when using the long-only strategy. However, when using the short-sale-is-allowed strategy, the finding in (1) leads us to reject Hypothesis 1 for some portfolios but accept Hypothesis 1 for some other portfolios, and conclude that investors will not get higher expected utility if they include Bitcoin in their portfolios for some portfolios in the sub-period I. The finding in (2) leads us to accept Hypothesis 2 and conclude that investors could get higher expected utility if they include a 3-month T-Bill in their portfolios for nearly all portfolios in the sub-period I. The finding in (3) leads us to accept both Hypotheses 3A and 3B and conclude that investors could get both higher expected utility and higher expected wealth if they include both Bitcoin and the 3-month Treasury Bill in their portfolios, regardless of whether a short sale is allowed or not, in the sub-period I. This is a very interesting finding and, as far as we know, this is a new discovery in the related literature.

To illustrate the first three-order dominance, we plot the CDFs of the With PF4 portfolio and Without PF4 in Figures A2A, A2B, and A2C. The results of the figures are consistent with the results obtained in the entire period, for simplicity, we skip reporting the figures of other portfolios.

6.2.2 The second sub-period

Appendix Table A2 presents the descriptive statistics returns of indices and the return of the 3-Month Treasury Bill for sub-period 2 (April 14, 2021-January 25, 2022). It shows that Bitcoin has the lowest rate of return over the study period (-0.002643) and has the highest standard deviation (0.046499). We notice that the return of the 3-Month Treasury Bill has the lowest risk (0.000315) and a relatively high mean (0.000519) over the sub-period 2 in our study. This information can be used to obtain the preference among different assets by using the mean-variance (MV) rule. Since the results are consistent with but not as good as the preference by using the SD rule, we skip discussing the results. From Appendix Table A2, we also notice that only the skewness of the 3-Month Treasury Bill and US Dollar index DXY are positive and the 3-Month Treasury Bill has significantly positive skewness. Negative skewness exists for the rest of the assets. Most of the assets have significantly higher kurtosis and JB statistics which indicates that the distributions of the assets' returns are not normal, suggesting that the mean-variance analysis is not appropriate. Thus, we skip discussing the MV results and present the SD results for individual assets in Table 7.

Appendix Figure A3A shows that efficient frontiers for portfolios with and without Bitcoin coincide for both long-only and short-sale-is-allowed strategies. This leads us to suspect that there is no advantage to including Bitcoin in the portfolios, regardless of whether short selling is allowed. In order to get a more accurate result, we apply the SD approach to test whether including Bitcoin is preferred for portfolios and display the results in Table 7.

Appendix Figure A3B shows that the efficient frontier with a 3-month T-Bill is on top of that without a 3-month T-Bill, regardless of whether short selling is allowed. Thus, one may believe that portfolios with a 3-month T-Bill are more efficient than those without a 3-month T-Bill. Thereafter, we apply SD to examine the preference of portfolios with a 3-month T-Bill and without a 3-month T-Bill and display the results in Table 7 for the situations in which a short sale is not allowed and is allowed.

Appendix Figure A3C shows that the efficient frontier both with Bitcoin and a 3-month T-Bill is on top of that both without Bitcoin and a 3-month T-Bill for the situations in which a short sale is used and is not used. Based on the modern finance theory, one may believe that portfolios with Bitcoin and a 3-month T-Bill are more profitable than those without Bitcoin and a 3-month T-Bill. Thus, based on the efficient frontiers, we conclude that the portfolios with Bitcoin and a 3-month T-Bill are better than the portfolios without Bitcoin and a 3-month T-Bill, regardless of whether a short sale is allowed or not.

Insert Table 7 Here

From Table 7, we find that (1) between portfolios with- and without -Bitcoin: for PF1-PF16, there is no difference between with- and without-Bitcoin portfolios in the sense of the first three orders, regardless of whether a short sale is allowed. (2) between portfolios with and without a 3-month T-Bill: when a short sale is not allowed, PF1-PF3 with a 3-month T-Bill stochastically dominates PF1-PF3 without a 3-month T-Bill in the sense of the second and third orders; there is no difference between portfolios with a 3-month T-Bill and without a 3-month T-Bill for PF4-PF16; while when a short sale is allowed, PF1-PF15 with a 3-month T-Bill stochastically dominates PF1-PF15 without a 3-month T-Bill in the sense of the second and third orders; there is no difference between portfolios with a 3-month T-Bill and without a 3-month T-Bill for PF16. (3) between portfolios with and without Bitcoin and a 3-month T-Bill: portfolios of PF1-PF15 with Bitcoin and a 3-month T-Bill stochastically dominate the corresponding portfolios without Bitcoin and a 3-month T-Bill in the sense of the second and third orders, regardless of whether a short sale is allowed; for PF16, there is no difference between with and without Bitcoin and a 3-month T-Bill portfolios. The SD results are consistent with the results obtained by using the MVPO criterion but it provides more information in sub-period 2.

The finding in (1) leads us to reject Hypothesis 1 and conclude that investors will not get higher expected utility if they include Bitcoin in their portfolios for Sub-period 2, regardless of whether a short sale is allowed. The finding in (2) leads us to accept Hypothesis 2 and conclude that investors could get higher expected utility if they include a 3-month T-Bill in their portfolios for some portfolios when a short sale is not allowed, and investors could get higher expected utility if they include a 3-month T-Bill in their portfolios for nearly all portfolios in the sub-period 2. The finding in (3) leads us to accept Hypothesis 3A but reject Hypothesis 3B and conclude that investors could get higher expected utility but not higher expected wealth if they include both Bitcoin and the 3-month Treasury Bill in their portfolios for nearly all the portfolios, regardless of whether a short sale is allowed, in the sub-period 2. One may ask why investors could get both higher expected utility and higher expected wealth if they include both Bitcoin and the 3-month Treasury Bill in their portfolios in sub-period 1 but not in sub-period 2. Our answer is that investors could find an arbitrage opportunities in the sub-period 1 when they include both Bitcoin and the 3-month Treasury Bill in their portfolios, and thus, investors will buy more portfolios with both Bitcoin and the 3-month Treasury Bill and (short) sell more portfolios without both Bitcoin and the 3-month Treasury Bill. As a result, the prices of the portfolios with both Bitcoin and the 3-month Treasury Bill have gone up and the prices of the portfolios without both Bitcoin and the 3-month Treasury Bill have come down so that there is no more arbitrage opportunities.

To illustrate the first three-order dominance, we plot the CDFs of the With PF4 portfolio and Without PF4 in Appendix Figures A4A, A4B, and A4C. For simplicity, we skip reporting the figures of other portfolios.

Overall, our analysis of the sub-periods shows that the role of Bitcoin and a 3-month T-Bill in the diversification of portfolios changes over time. Indeed, we find that the combination of Bitcoin and a 3-month T-Bill in the second sub-period is not as good as the first sub-period. Nonetheless, the combination of Bitcoin and a 3-month T-Bill in the second sub-period still enables portfolios to achieve stochastic dominance in the second and third order so investors still prefer portfolios with Bitcoin and a 3-month T-Bill. Moreover, the results that we obtained for the 1st sub-period (August 18, 2011 - April 13, 2021) are similar to those we obtained for the whole period that the combination of Bitcoin and a 3-month T-Bill in the second sub-period still enable portfolios to achieve stochastic dominance in the first three orders. Thus, we conclude that the combination of Bitcoin and a 3-month T-Bill enables portfolios to achieve stochastic dominance preference and investors prefer portfolios with Bitcoin and a 3-month T-Bill in both the first and the second period and the entire period, and the combination of Bitcoin and a 3-month T-Bill in the first sub-period enable investors to catch arbitrage opportunities.

6.2. Robustness check: Performance measures and results

In portfolio optimization, the performance of a portfolio is generally evaluated by using a number of performance measures, including the Sharpe ratio (Sharpe, 1966, 1994), Rachev ratio (Rockafellar & Uryasev, 2000), information ratio (Goodwin, 1998), alpha (Sharpe, 1964), and average return (e.g. DeMiguel, Garlappi, and Uppal (2009), Rachev, Stoyanov, and Fabozzi (2008)). Denoting R the portfolio return, and r_f the constant risk-free rate of return that we set to 0 for simplicity, we compute Sharpe ratios, Rachev Ratio, Information Ratio, Alpha, and average return for long-only and short-sale-is-allowed portfolios. The performance results are provided for the entire period as well as the two sub-periods.

6.2.1 Results of performance measures for the entire period

Appendix Table A3 displays the results of the five performance measures for long-only portfolios in the entire period. Firstly, we present in Panels A and B of Table A3 the results between the portfolios with Bitcoin and the portfolios without Bitcoin and observe that there is no clear difference among each pair of the different portfolios under the long-only strategy. Secondly, we exhibit in Panels A and C of Table A3 the results between the portfolios with and without a 3-month T-Bill and find that all the values of the five performance measures of the portfolios with a 3-month T-Bill are larger than those without a 3-month T-Bill under the long-only strategy. Thirdly, we present in Panels A and D of Table A3 the results between the portfolios with and without both Bitcoin and a 3-month T-Bill and observe that all the values of the five performance measures of the portfolios with both assets are much larger than those without both assets under the long-only strategy.

Appendix Table A4 presents the results of the above-mentioned five performance measures for the portfolio in the entire period when a short sale is allowed. It shows a similar conclusion for portfolios under the long-only strategy. Thus, we skip discussing the results.

In general, regardless of whether a short sale is used or not, there is no difference between portfolios with Bitcoin and portfolios without Bitcoin, portfolios with a 3-month T-Bill perform better than the ones without a 3-month T-Bill, and when both Bitcoin and a 3-month T-Bill are included in the portfolios, they have much better performances than the ones without both. The results are consistent with the results obtained by using the MVPO criterion and SD test for the entire period.

6.2.2 Results of performance measures for sub-period 1

Appendix Table A5 displays the results of the above-mentioned five performance measures for portfolios in sub-period 1. Firstly, Panels A and B in Table A5 indicate no clear difference between portfolios with and without Bitcoin; secondly, from Panels A and C in Table A5, the values of the five performance measures for the portfolios with a 3-month T-Bill are larger than those without a 3-month T-Bill; thirdly, Panels A and D in Table A5 shows that all the values of the five performance measures of the portfolios with both are much larger than the portfolios without both under the long-only strategy.

Appendix Table A6 reports the results of the above-mentioned five performance measures for portfolios in sub-period 1 under the short-sale-is-allowed strategy. We find a similar conclusion with portfolios under the long-only strategy. Thus, we skip discussing the results.

In general, regardless of whether a short sale is used, there is no difference between portfolios with and without Bitcoin, portfolios with a 3-month T-Bill perform better than those without a 3-month T-Bill, and when both Bitcoin and a 3-month T-Bill are included in the portfolios, they have much better performances than the ones without both. The results are consistent with those obtained using the MVPO criterion and SD test in the full sample period and sub-period 1.

6.2.3 Results of performance measures for sub-period 2

Appendix Table A7 shows the results of the above-mentioned five performance measures for portfolios in sub-period 2 under the long-only strategy. Panels A and B in Table A7 indicate no clear difference between the portfolios with and without Bitcoin. Panels A and C in Table A7 shows that all the values of the five performance measures for the portfolios with a 3-month T-Bill are larger than those without a 3-month T-Bill for PF1-PF9, PF16; however, there is no clear difference among the PF10-PF15 portfolios with and without a 3-month T-Bill. Panels A and D in Table A7 indicate that all the values of the five performance measures of the PF1-PF9 portfolios with both Bitcoin and a 3-month T-Bill are much larger than those without both, but there is no clear difference among the PF10-PF15 portfolios with and without both Bitcoin and a 3-month T-Bill.

Appendix Table A8 displays the results of the above-mentioned five performance measures for the portfolios in sub-period 2 under the short-sale-is-allowed strategy. From this table, we notice the following: firstly, from Panels A and B in Table A8, we find that there is no clear difference between the portfolios with Bitcoin and without Bitcoin; secondly, from Panels A and C in Table A8, we observe that all the values of the five performance measures for the portfolios with a 3-month T-Bill are larger than those without a 3-month T-Bill; and third, from Panels A and D in Table A8, we notice that all the values of the five performance measures for the portfolios with both (Bitcoin and a 3-month T-Bill) are much larger than those without both under the short-sale-is-allowed strategy.

In summary, we conclude that regardless of whether a short sale is used or not, there is no difference between portfolios with and without Bitcoin, portfolios with a 3-month T-Bill perform better than those without a 3-month T-Bill, and when both Bitcoin and a 3-month T-Bill are included in the portfolios, they have much better performances than the ones without both. These results are consistent with the results obtained by using the MVPO criterion and SD test in the entire period and two sub-periods.

6.3. Robustness check: Excluding Outliers

To assess the potential impact of outlier presence on our findings, this chapter systematically excludes outliers and conducts a re-analysis of the entire period.

Figure 8 illustrates the presence of outliers in Bitcoin returns. After identifying these anomalies, we systematically excluded them and reconstructed the investment portfolios by MVPO. Using the same steps applied in the previous analysis, we present the efficient frontiers on which all possible efficient portfolios offer the highest expected rate of return for each given level of risk in Figure B1.

Insert Figure 8 Here

From Figures 9, we find the same results with the entire period, regardless of whether short selling is allowed. Thus, we obtain the same conclusion and find that portfolios with a 3-month T-Bill (both with Bitcoin and a 3-month T-Bill) are more efficient than those without a 3-month T-Bill (without Bitcoin and a 3-month T-Bill). Similarity, in order to get a more accurate result, we will apply the SD approach to test whether including Bitcoin is preferred for portfolios and display the results in Table 8.

Insert Figure 9 Here

Table 8 shows that, regardless of the long-only strategy and short-sale-is-allowed strategy : (1) between portfolios with and without Bitcoin, we find that for PF1-PF8, there is no difference between with- and without-Bitcoin portfolios; for PF9-PF15 investors prefer portfolios without-Bitcoin to those with-Bitcoin at the second and third orders. (2) Between portfolios with and without a 3-month T-Bill: PF1 with a 3-month T-Bill stochastically dominates PF1 without a 3-month T-Bill in the sense of first three orders; for PF2-16, investors prefer portfolios with a 3-month T-Bill to those without a 3-month T-Bill at the second and third orders. (3) Between portfolios with and without Bitcoin and a 3-month T-Bill: all the portfolios with Bitcoin and a 3-month T-Bill stochastically dominate the corresponding portfolios without Bitcoin and a 3-month T-Bill in the sense of the first three orders, regardless of whether a short sale is allowed. The results of stochastic dominance after removing outliers are consistent with those obtained from the entire period, therefore this outcome also demonstrates that our findings are robust.

Insert Table 8 Here

6.4. Robustness check: Using Different Portfolio Sizes

To examine whether different portfolio sizes affect the results, we constructed portfolios with varying numbers. For brevity, we present only 10 portfolios in our robustness check results. The steps are described as follows: we partition each efficient frontier into 10 portfolios (from PF1 to PF10) with PF1 being the minimum-variance portfolio and PF10 being the maximum-return portfolio. (1) We partition the efficient frontier in 10 slices with equal distance on the horizontal axis; (2) we then determine the 10 points (representing the portfolios) and their risk-return coordinates belonging to the above-mentioned 10 slices. As a result, we obtain the 10 efficient portfolios whose returns are equally spaced along the whole range of the efficient frontier. The MVPO results are presented in Figure 10, and the SD results are summarized in Table 9.

From Figures 10, we find the same results with the 15 portfolios, regardless of whether short selling is allowed. Thus, we obtain the same conclusion and find that portfolios with a 3-month T-Bill (both with Bitcoin and a 3-month T-Bill) are more efficient than those without a 3-month T-Bill (without Bitcoin and a 3-month T-Bill). Similarity, in order to get a more accurate result, we will apply the SD approach to test whether including Bitcoin is preferred for portfolios and display the results in Table 9.

Insert Figure 10 Here

Table 9 shows that, regardless of the long-only strategy and short-sale-is-allowed strategy : (1) between portfolios with and without Bitcoin, for most of portfolios, there is no difference between with- and without-Bitcoin. (2) Between portfolios with and without a 3-month T-Bill: PF1 with a 3-month T-Bill stochastically dominates PF1 without a 3-month T-Bill in the sense of first three orders; for PF2-10, investors prefer portfolios with a 3-month T-Bill to those without a 3-month T-Bill at the second and third orders. (3) Between portfolios with and without Bitcoin and a 3-month T-Bill: all the portfolios with Bitcoin and a 3-month T-Bill stochastically dominate the corresponding portfolios without Bitcoin and a 3-month T-Bill in the sense of the first three orders, regardless of whether a short sale is allowed. This finding demonstrates that the stochastic dominance results remain consistent across different portfolio sizes, thereby confirming the robustness of our findings.

Insert Table 9 Here

7. Inferences and Concluding Remarks

In this paper, we examine whether it is possible to generate arbitrage opportunities by using assets without any arbitrage opportunities. After setting the conjectures that it is possible to generate arbitrage opportunities by using assets without any arbitrage opportunities, we verify it by applying both MVPO theory and SD test.

Specifically, we examine whether including Bitcoin and a 3-month T-Bill could generate arbitrage opportunities and whether it is a better choice in the investment decision. We compare the performance in the following aspects: (1) among different individual assets; (2) between portfolios in the frontiers with and without Bitcoin; (3) between portfolios in the frontiers with a 3-month T-Bill and without a 3-month T-Bill; (4) between portfolios in the frontiers both with and without Bitcoin and a 3-month T-Bill.

For (1), a 3-month T-Bill is the best choice among all individual assets under study. It stochastically dominates most of the other assets in the sense of both second and third orders and 2 assets. For (2), in most cases, there is no difference between portfolios with Bitcoin and portfolios without Bitcoin, regardless of whether a short sale is allowed or not. This suggests that investors are indifferent to portfolios with and without Bitcoin. For (3) our analysis indicates that portfolios with a 3-month T-Bill dominate the ones without a 3-month T-Bill, at the second and third order, regardless of whether a short sale is allowed or not. Thus, investors should include a 3-month T-Bill in their portfolios to get higher expected utility but not higher expected wealth. For (4), we find that all the portfolios with Bitcoin and a 3-month T-Bill stochastically dominate all of the corresponding portfolios without Bitcoin and a 3-month T-Bill in the sense of the first three orders, regardless of whether a short sale is allowed or not. Thus, investors could get higher expected utility and higher expected wealth by including both a 3-month T-Bill and Bitcoin in their portfolios. Notably, this study reveals a very important finding that we believe no paper has discovered so far: the portfolios with both Bitcoin and a 3-month T-Bill dominate portfolios without Bitcoin and a 3-month T-Bill in both long-only and short-sale-is-allowed strategies in the sense of the first-order SD (FSD). This implies that the market is not efficient and there exists expected arbitrage opportunities when investors include both Bitcoin and a 3-month T-Bill in their investment, which demonstrates that our conjecture holds true that it is possible to generate arbitrage opportunities by using assets without any arbitrage opportunities.

A robustness check shows that the above results remain unchanged for the first subperiod (August 18, 2011 - April 13, 2021). This highlights the characteristics of Bitcoin and a 3-month T-Bill in the entire period, and demonstrates the possibility to generate arbitrage opportunities by using assets without any arbitrage opportunities in this subperiod. In the second subperiod, we can only find that all the portfolios with Bitcoin and a 3-month T-Bill second-order stochastically dominate all of the corresponding portfolios without Bitcoin and a 3-month T-Bill in the second sub-period. Investors could include both a 3-month T-Bill and Bitcoin in their portfolios to get higher expected utility but not higher expected wealth in the second sub-period. One may wonder why investors could get both higher expected utility and higher expected wealth if they include both Bitcoin and the 3-month Treasury Bill in their portfolios in the first sub-period but not in the second sub-period. We argue that it could be because investors found an arbitrage opportunities in the first sub-period by including both Bitcoin and the 3-month Treasury Bill in their portfolios and thus they bought more portfolios with both Bitcoin and the 3-month Treasury Bill and sold (short) more portfolios without both Bitcoin and the 3-month Treasury Bill. As a result, the prices of the portfolios with both Bitcoin and the 3-month Treasury Bill had gone up and the prices of the portfolios without both Bitcoin and the 3-month Treasury Bill had come down so that arbitrage opportunities had disappeared. Through robustness checks using varying portfolio sizes and excluding outliers, we confirm the consistency and reliability of our stochastic dominance results across all tested scenarios.

Our findings are useful for individual investors, fund managers, and policymakers. For example, individual investors and fund managers could build on our analysis to look for arbitrage opportunities by using assets without any arbitrage opportunities to generate arbitrage opportunities and get profits. Policymakers should be aware about the possibility to use assets without any arbitrage opportunities to generate arbitrage opportunities and get profits, which could be indicative of inefficiency in the market. Accordingly, they may impose some new policies and regulations to make the market become more efficient. In addition, the interplay between these assets highlights gaps in cross-market oversight. Policymakers may need to harmonize crypto regulations with traditional financial frameworks to mitigate systemic risks from arbitrage-driven capital flows.

Based on our analysis, the main possible research directions in the future include the following aspects. (1) Cross-Asset Integration: Explore interactions between Bitcoin, bonds, and other unconventional assets (e.g., green bonds, stablecoins) to refine diversification models. (2) Regulatory Impact: Assess how jurisdictional regulatory divergence affects arbitrage efficiency and portfolio resilience. (3) Monetary Policy Transmission: Analyze how central bank digital currencies or quantitative tightening influence correlations between bonds and crypto assets.

There are many possible extensions of our paper. for example, one could consider including other assets in the portfolios, for example, gold (Hoang, Lean, and Wong, 2015; Hoang, Wong, and Zhu, 2015; Hoang,Zhu, Xiao, and Wong, 2020), oil (Lean, McAleer, and Wong, 2010, 2015), real estate (Qiao and Wong, 2015; Tsang, Wong, and Horowitz, 2016; Lv, Tsang, Wagner, and Wong, 2023), wine (Bouri, Gupta, Wong, and Zhu, 2018), warrants (Chan, de Peretti, Qiao, and Wong, 2012), futures (Qiao, Wong, and Fung, 2013; Clark, Qiao, and Wong, 2016), etc. Extensions also include using more advance portfolio estimation, see, for example, Bai, Liu, and Wong (2009), Leung, Ng, and Wong (2012), Li, Li, Hui, and Wong (2018), Guo, Chan, Wong, and

Zhu (2019), Li, Bai, Wong, and McAleer (2022), Li, Jiang, Chen, and Wong (2022), and Chan, Chow, Guo, and Wong (2022), Li, Hui, Wong, Lin (2024), and Hui, Shi, Wong, and Zheng (2024). Extensions could also include preferences of different investors on the issue, see, for example, Fong, Lean, and Wong (2008), Egozcue, Fuentes García, Wong, and Zitikis (2011), Qiao, Clark, and Wong (2012), Lozza, Wong, Fabozzi, and Egozcue (2018), and Wong, Yeung, and Lu (2023), and preferences of investors on different events, for example, crisis (Lean, McAleer, and Wong, 2015; Zhu, Bai, Vieito, and Wong, 2019).

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Table 1. Descriptions Statistics of return series

	Mean	Std. Dev.	Skewness	Kurtosis	Jarque-Bera
Bitcoin	0.002985	**0.056591**	-1.0655***	23.0988***	46348.31***
ETF	0.000026	0.003880	-0.4226***	73.9832***	571754.17***
SP GSCI Commodity	-0.000169	0.012811	-0.8838***	13.1011***	11930.84***
SP GSCI Gold	-0.000026	0.010155	-0.6710***	10.4187***	6448.70***
SP GSCI Crude Oil	-0.000430	0.027655	-3.9009***	85.2627***	774696.47***
MSCI EUROPE	0.000158	0.011510	-1.0216***	16.4753***	21075.70***
MSCI PACIFIC	0.000140	0.009598	-0.2269***	6.6427***	1528.86***
MSCI NORTH AMERICA	0.000466	0.010422	-0.9702***	23.2683***	47036.46***
FTSE EPRA Nareit GLOBAL	0.000146	0.009271	-2.1659***	38.0656***	141637.03***
US DOLLAR INDEX DXY	0.000094	0.004128	0.0319	5.0601***	481.97***
SP 500	0.000492	0.010472	-0.8789***	22.5008***	43496.59***
MSCI World	0.000317	0.008826	-1.1882***	21.8246***	40846.29***
3MTBILL	**0.005753**	0.007951	1.2536***	3.0076	713.19***

Note: This table reports the summary statistics including the mean, standard deviation (Std. Dev.), skewness, excess kurtosis, and Jarque-Bera (JB) test. The sample period is August 18, 2011 to January 25, 2022. PIMCO Investment Grade Corporate Bond Index Exchange-Traded Fund (ETF), Commodity market (SP GSCI Commodity), gold market (SP GSCI Gold), crude oil market (SP GSCI Crude Oil), real estate market (FTSE EPRA Nareit GLOBAL), exchange rate market (US DOLLAR INDEX DXY). MSCI represent stock market indices for the world and major graphical regions. The symbols *, **, and *** denote the significance at the 10%, 5%, and 1% levels, respectively.

Table 2. SD results for individual assets

	ETF	S&P GSCI Commodity	S&P GSCI Gold	S&P GSCI Crude Oil	MSCI EUROPE	MSCI PACIFIC	MSCI NORTH AMERICA	FTSE EPRA Nareit GLOBAL	US DOLLAR INDEX DXY	S&P 500	MSCI World	3MTBILL
Bitcoin	$\prec_{3}$	$\prec_{3}$	$\prec_{3}$	$\prec_{3}$	$\prec_{3}$	$\prec_{3}$	$\prec_{2,3}$	$\prec_{3}$	$\prec_{2,3}$	$\prec_{2,3}$	$\prec_{3}$	$\prec_{2,3}$
ETF		$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{2,3}$	$\neq$	$\succ_{2,3}$	$\succ_{2,3}$	$\prec_{1,2,3}$
S&P GSCI Commodity			$\prec_{2,3}$	$\succ_{2,3}$	$\neq$	$\prec_{2,3}$	$\prec_{2,3}$	$\prec_{2,3}$	$\prec_{2,3}$	$\prec_{2,3}$	$\prec_{2,3}$	$\prec_{2,3}$
S&P GSCI Gold				$\succ_{2,3}$	$\neq$	$\neq$	$\neq$	$\neq$	$\prec_{2,3}$	$\neq$	$\prec_{2,3}$	$\prec_{2,3}$
S&P GSCI Crude Oil					$\prec_{2,3}$	$\prec_{2,3}$	$\prec_{2,3}$	$\prec_{2,3}$	$\prec_{2,3}$	$\prec_{2,3}$	$\prec_{2,3}$	$\prec_{2,3}$
MSCI EUROPE						$\prec_{2,3}$	$\neq$	$\prec_{2,3}$	$\prec_{2,3}$	$\neq$	$\prec_{2,3}$	$\prec_{2,3}$
MSCI PACIFIC							$\neq$	$\neq$	$\prec_{2,3}$	$\neq$	$\neq$	$\prec_{2,3}$
MSCI NORTH AMERICA								$\neq$	$\prec_{2,3}$	$\neq$	$\neq$	$\prec_{2,3}$
FTSE EPRA Nareit GLOBAL									$\prec_{2,3}$	$\neq$	$\neq$	$\prec_{2,3}$
US DOLLAR INDEX DXY										$\succ_{2,3}$	$\succ_{2,3}$	$\prec_{1,2,3}$
S&P 500											$\neq$	$\prec_{2,3}$
MSCI World												$\prec_{2,3}$

Table 3. Stochastic dominance analysis between portfolios with and without Bitcoin for both the long-only and short-allowed strategies

With-Bitcoin	**Long-only**	**Short-allowed**	Without-Bitcoin
PF1	=	=	PF1
PF2	=	=	PF2
PF3	=	=	PF3
PF4	=	=	PF4
PF5	=	=	PF5
PF6	=	=	PF6
PF7	=	=	PF7
PF8	=	=	PF8
PF9	=	=	PF9
PF10	=	=	PF10
PF11	=	=	PF11
PF12	=	=	PF12
PF13	=	=	PF13
PF14	=	=	PF14
PF15	=	=	PF15
PF16	=	=	PF16

Note: This table reports the stochastic-dominance results to test whether with-Bitcoin portfolios strictly dominate without-Bitcoin portfolios at the j-order stochastic dominance for j = 1, 2, and 3. "Long-only" means the short selling is not allowed and "short-allowed" means the short selling is allowed.

Table 4. Stochastic dominance analysis between portfolios with and without a 3-month T-Bill for both the long-only and short-allowed strategies

With T-Bill	**Long-only**	**Short-allowed**	Without T-Bill
PF1	$\succ_{1,2,3}$	$\succ_{1,2,3}$	PF1
PF2	$\succ_{2,3}$	$\succ_{2,3}$	PF2
PF3	$\succ_{2,3}$	$\succ_{2,3}$	PF3
PF4	$\succ_{2,3}$	$\succ_{2,3}$	PF4
PF5	$\succ_{2,3}$	$\succ_{2,3}$	PF5
PF6	$\succ_{2,3}$	$\succ_{2,3}$	PF6
PF7	$\succ_{2,3}$	$\succ_{2,3}$	PF7
PF8	$\succ_{2,3}$	$\succ_{2,3}$	PF8
PF9	$\succ_{2,3}$	$\succ_{2,3}$	PF9
PF10	$\succ_{2,3}$	$\succ_{2,3}$	PF10
PF11	$\succ_{2,3}$	$\succ_{2,3}$	PF11
PF12	$\succ_{2,3}$	$\succ_{2,3}$	PF12
PF13	$\succ_{2,3}$	$\succ_{2,3}$	PF13
PF14	$\succ_{2,3}$	$\succ_{2,3}$	PF14
PF15	$\succ_{2,3}$	$\succ_{2,3}$	PF15
PF16	$=$	$=$	PF16

Note: T-Bill means 3-month T-Bill. This table reports the stochastic-dominance results to test whether portfolios with-T-Bill strictly dominate portfolios without T-Bill at the j-order stochastic dominance for $j = 1, 2, 3$. "Long-only" means the short selling is not allowed and "short-allowed" means the short selling is allowed.

Table 5. Stochastic dominance analysis between portfolios with and without Bitcoin and a 3-month T-Bill for both the long-only and short-allowed strategies

With both Bitcoin and T-Bill	**Long-only**	**Short-allowed**	Without both Bitcoin and T-Bill
PF1	$\succ_{1,2,3}$	$\succ_{1,2,3}$	PF1
PF2	$\succ_{1,2,3}$	$\succ_{1,2,3}$	PF2
PF3	$\succ_{1,2,3}$	$\succ_{1,2,3}$	PF3
PF4	$\succ_{1,2,3}$	$\succ_{1,2,3}$	PF4
PF5	$\succ_{1,2,3}$	$\succ_{1,2,3}$	PF5
PF6	$\succ_{1,2,3}$	$\succ_{1,2,3}$	PF6
PF7	$\succ_{1,2,3}$	$\succ_{1,2,3}$	PF7
PF8	$\succ_{1,2,3}$	$\succ_{1,2,3}$	PF8
PF9	$\succ_{1,2,3}$	$\succ_{1,2,3}$	PF9
PF10	$\succ_{1,2,3}$	$\succ_{1,2,3}$	PF10
PF11	$\succ_{1,2,3}$	$\succ_{1,2,3}$	PF11
PF12	$\succ_{1,2,3}$	$\succ_{1,2,3}$	PF12
PF13	$\succ_{1,2,3}$	$\succ_{1,2,3}$	PF13
PF14	$\succ_{1,2,3}$	$\succ_{1,2,3}$	PF14
PF15	$\succ_{1,2,3}$	$\succ_{1,2,3}$	PF15
PF16	$\succ_{1,2,3}$	$\succ_{1,2,3}$	PF16

Note: T-Bill means 3-month T-Bill. This table reports the stochastic-dominance results to test whether portfolios with both Bitcoin and a T-Bill strictly dominate those without Bitcoin and a T-Bill at the *j*-order stochastic dominance for $j = 1, 2, 3$. "Long-only" means the short selling is not allowed and "short-allowed" means the short selling is allowed.

Table 6A. Stochastic dominance analysis between portfolios with and without assets for the long-only strategy (sub-period 1)

With	**Long-only**			Without
PF1	=	$\succ_{1,2,3}$	$\succ_{1,2,3}$	PF1
PF2	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF2
PF3	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF3
PF4	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF4
PF5	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF5
PF6	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF6
PF7	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF7
PF8	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF8
PF9	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF9
PF10	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF10
PF11	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF11
PF12	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF12
PF13	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF13
PF14	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF14
PF15	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF15
PF16	=	=	$\succ_{1,2,3}$	PF16

Note: This table reports the stochastic-dominance results to test whether with-asset portfolios strictly dominate without-asset portfolios at the j-order stochastic dominance for j = 1, 2, 3. "Long-only" means the short selling is not allowed and "short-allowed" means the short selling is allowed. The asset in the second column refers to Bitcoin, the asset in the third column refers to a 3-month T-Bill, and the asset in the fourth column refers to both Bitcoin and a 3-month T-Bill.

Table 6B. Stochastic dominance analysis between portfolios with and without assets for both the short-allowed strategy (sub-period 1)

With	**Short-allowed**			Without
PF1	=	$\succ_{1,2,3}$	$\succ_{1,2,3}$	PF1
PF2	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF2
PF3	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF3
PF4	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF4
PF5	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF5
PF6	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF6
PF7	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF7
PF8	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF8
PF9	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF9
PF10	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF10
PF11	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF11
PF12	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF12
PF13	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	PF13
PF14	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	PF14
PF15	$\succ_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	PF15
PF16	=	=	$\succ_{1,2,3}$	PF16

Note: This table reports the stochastic-dominance results to test whether with-asset portfolios strictly dominate without-asset portfolios at the *j*-order stochastic dominance for j = 1, 2, 3. "Long-only" means the short selling is not allowed and "short-allowed" means the short selling is allowed. The asset in the second column refers to Bitcoin, the asset in the third column refers to a 3-month T-Bill, and the asset in the fourth column refers to both Bitcoin and a 3-month T-Bill.

Table 7A. Stochastic dominance analysis between portfolios with and without assets for both the long-only strategy (sub-period 2)

With	Long-only			Without
PF1	=	$\succ_{2,3}$	$\succ_{2,3}$	PF1
PF2	=	$\succ_{2,3}$	$\succ_{2,3}$	PF2
PF3	=	$\succ_{2,3}$	$\succ_{2,3}$	PF3
PF4	=	=	$\succ_{2,3}$	PF4
PF5	=	=	$\succ_{2,3}$	PF5
PF6	=	=	$\succ_{2,3}$	PF6
PF7	=	=	$\succ_{2,3}$	PF7
PF8	=	=	$\succ_{2,3}$	PF8
PF9	=	=	$\succ_{2,3}$	PF9
PF10	=	=	$\succ_{2,3}$	PF10
PF11	=	=	$\succ_{2,3}$	PF11
PF12	=	=	$\succ_{2,3}$	PF12
PF13	=	=	$\succ_{2,3}$	PF13
PF14	=	=	$\succ_{2,3}$	PF14
PF15	=	=	$\succ_{2,3}$	PF15
PF16	=	=	=	PF16

Note: This table reports the stochastic-dominance results to test whether with-asset portfolios strictly dominate without-asset portfolios at the j-order stochastic dominance for j = 1, 2, 3. "Long-only" means the short selling is not allowed and "short-allowed" means the short selling is allowed. The asset in the second column refers to Bitcoin, the asset in the third column refers to a 3-month T-Bill, and the asset in the fourth column refers to both Bitcoin and a 3-month T-Bill.

Table 7B. Stochastic dominance analysis between portfolios with and without assets for both the short-allowed strategy (sub-period 2)

With	**Short-allowed**			Without
PF1	=	$\succ_{2,3}$	$\succ_{2,3}$	PF1
PF2	=	$\succ_{2,3}$	$\succ_{2,3}$	PF2
PF3	=	$\succ_{2,3}$	$\succ_{2,3}$	PF3
PF4	=	$\succ_{2,3}$	$\succ_{2,3}$	PF4
PF5	=	$\succ_{2,3}$	$\succ_{2,3}$	PF5
PF6	=	$\succ_{2,3}$	$\succ_{2,3}$	PF6
PF7	=	$\succ_{2,3}$	$\succ_{2,3}$	PF7
PF8	=	$\succ_{2,3}$	$\succ_{2,3}$	PF8
PF9	=	$\succ_{2,3}$	$\succ_{2,3}$	PF9
PF10	=	$\succ_{2,3}$	$\succ_{2,3}$	PF10
PF11	=	$\succ_{2,3}$	$\succ_{2,3}$	PF11
PF12	=	$\succ_{2,3}$	$\succ_{2,3}$	PF12
PF13	=	$\succ_{2,3}$	$\succ_{2,3}$	PF13
PF14	=	$\succ_{2,3}$	$\succ_{2,3}$	PF14
PF15	=	$\succ_{2,3}$	$\succ_{2,3}$	PF15
PF16	=	=	=	PF16

Note: This table reports the stochastic-dominance results to test whether with-asset portfolios strictly dominate without-asset portfolios at the j-order stochastic dominance for j = 1, 2, 3. "Long-only" means the short selling is not allowed and "short-allowed" means the short selling is allowed. The asset in the second column refers to Bitcoin, the asset in the third column refers to a 3-month T-Bill, and the asset in the fourth column refers to both Bitcoin and a 3-month T-Bill.

Table 8A. Stochastic dominance analysis between portfolios with and without assets for the long-only strategy (exclude outliers)

With	**Long-only**			Without
PF1	=	$\succ_{1,2,3}$	$\succ_{1,2,3}$	PF1
PF2	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF2
PF3	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF3
PF4	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF4
PF5	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF5
PF6	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF6
PF7	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF7
PF8	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF8
PF9	$\prec_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	PF9
PF10	$\prec_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	PF10
PF11	$\prec_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	PF11
PF12	$\prec_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	PF12
PF13	$\prec_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	PF13
PF14	$\prec_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	PF14
PF15	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF15
PF16	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF16

Note: This table reports the stochastic-dominance results to test whether with-asset portfolios strictly dominate without-asset portfolios at the *j*-order stochastic dominance for *j* = 1, 2, 3. "Long-only" means the short selling is not allowed and "short-allowed" means the short selling is allowed. The asset in the second column refers to Bitcoin, the asset in the third column refers to a 3-month T-Bill, and the asset in the fourth column refers to both Bitcoin and a 3-month T-Bill.

Table 8B. Stochastic dominance analysis between portfolios with and without assets for both the short-allowed strategy (exclude outliers)

With	**Short-allowed**			Without
PF1	=	$\succ_{1,2,3}$	$\succ_{1,2,3}$	PF1
PF2	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF2
PF3	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF3
PF4	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF4
PF5	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF5
PF6	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF6
PF7	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF7
PF8	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF8
PF9	$\prec_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	PF9
PF10	$\prec_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	PF10
PF11	$\prec_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	PF11
PF12	$\prec_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	PF12
PF13	$\prec_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	PF13
PF14	$\prec_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	PF14
PF15	$\prec_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	PF15
PF16	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF16

Note: This table reports the stochastic-dominance results to test whether with-asset portfolios strictly dominate without-asset portfolios at the *j*-order stochastic dominance for *j* = 1, 2, 3. "Long-only" means the short selling is not allowed and "short-allowed" means the short selling is allowed. The asset in the second column refers to Bitcoin, the asset in the third column refers to a 3-month T-Bill, and the asset in the fourth column refers to both Bitcoin and a 3-month T-Bill.

Table 9A. Stochastic dominance analysis between portfolios with and without assets for the long-only strategy (different portfolio sizes)

With	**Long-only**			Without
PF1	=	$\succ_{1,2,3}$	$\succ_{1,2,3}$	PF1
PF2	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF2
PF3	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF3
PF4	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF4
PF5	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF5
PF6	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF6
PF7	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF7
PF8	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF8
PF9	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF9
PF10	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF10

Note: This table reports the stochastic-dominance results to test whether with-asset portfolios strictly dominate without-asset portfolios at the j-order stochastic dominance for j = 1, 2, 3. "Long-only" means the short selling is not allowed and "short-allowed" means the short selling is allowed. The asset in the second column refers to Bitcoin, the asset in the third column refers to a 3-month T-Bill, and the asset in the fourth column refers to both Bitcoin and a 3-month T-Bill.

Table 9B. Stochastic dominance analysis between portfolios with and without assets for both the short-allowed strategy (different portfolio sizes)

With	**Short-allowed**			Without
PF1	=	$\succ_{1,2,3}$	$\succ_{1,2,3}$	PF1
PF2	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF2
PF3	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF3
PF4	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF4
PF5	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF5
PF6	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF6
PF7	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF7
PF8	=	$\succ_{2,3}$	$\succ_{1,2,3}$	PF8
PF9	$\prec_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	PF9
PF10	$\prec_{2,3}$	$\succ_{2,3}$	$\succ_{1,2,3}$	PF10

Note: This table reports the stochastic-dominance results to test whether with-asset portfolios strictly dominate without-asset portfolios at the j-order stochastic dominance for j = 1, 2, 3. "Long-only" means the short selling is not allowed and "short-allowed" means the short selling is allowed. The asset in the second column refers to Bitcoin, the asset in the third column refers to a 3-month T-Bill, and the asset in the fourth column refers to both Bitcoin and a 3-month T-Bill.

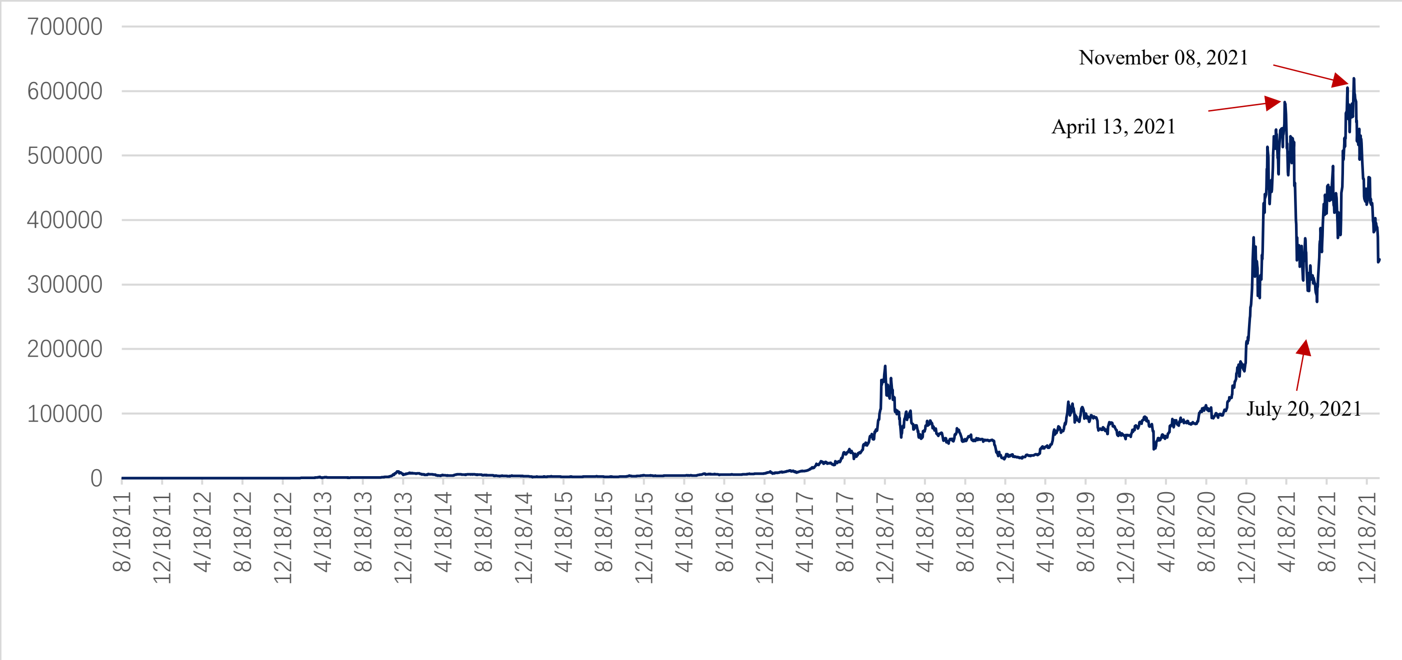


Figure 1A. Bitcoin prices from August 18, 2011 to January 25, 2022

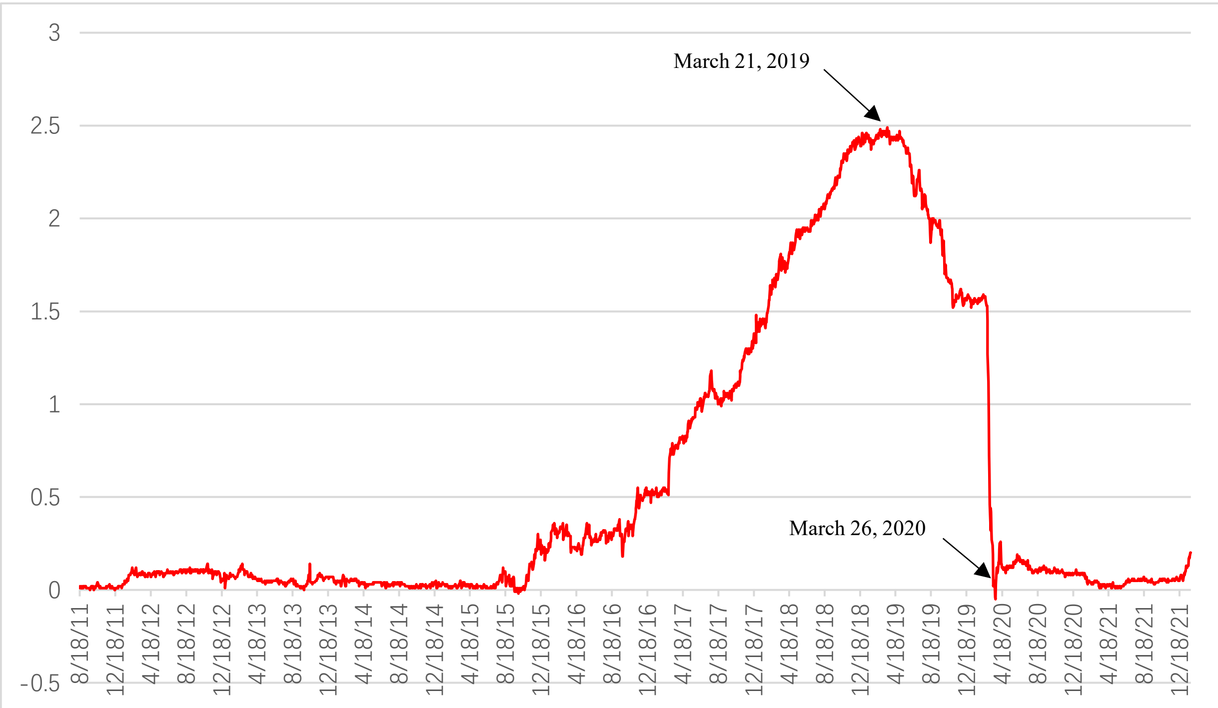


Figure 1B. The 3-month Treasury Bill yields from secondary market rate (percent) from August 18, 2011 to January 25, 2022

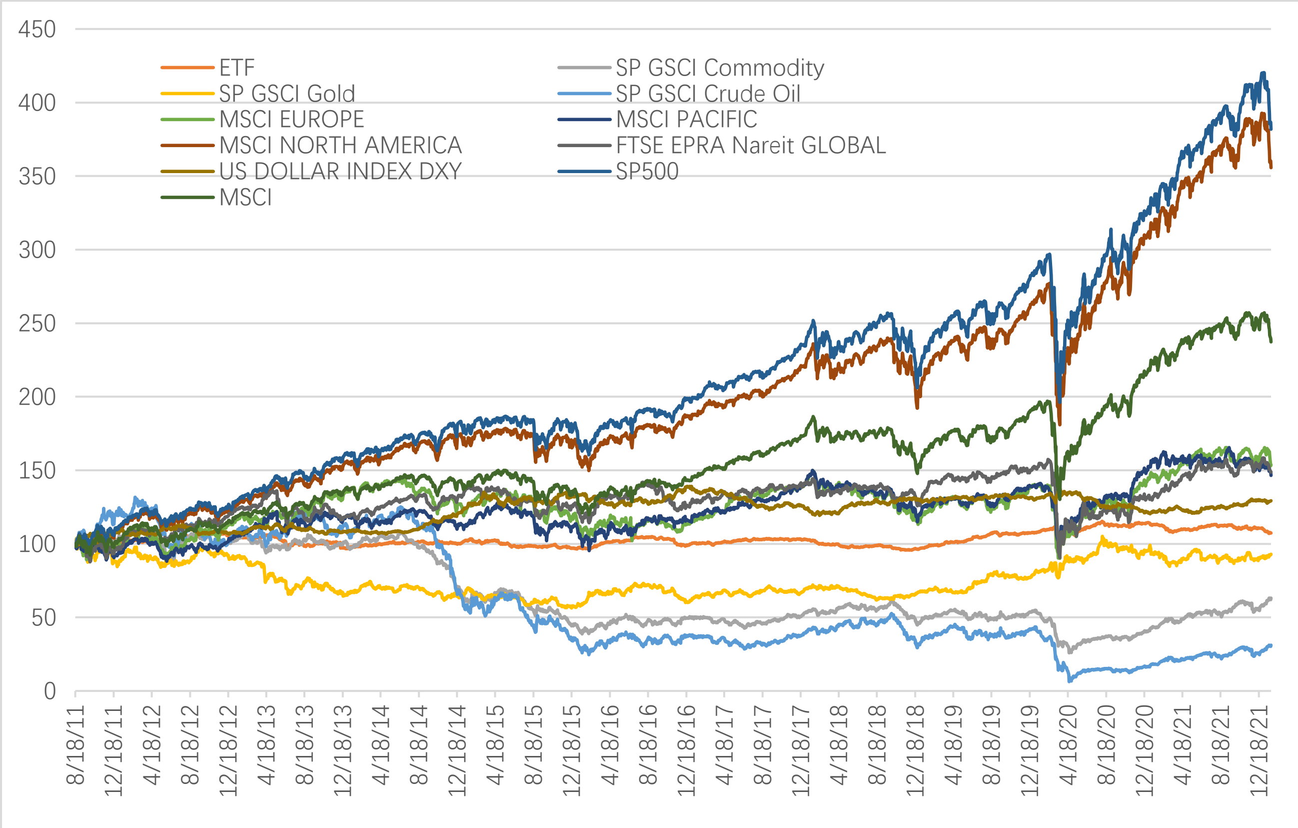
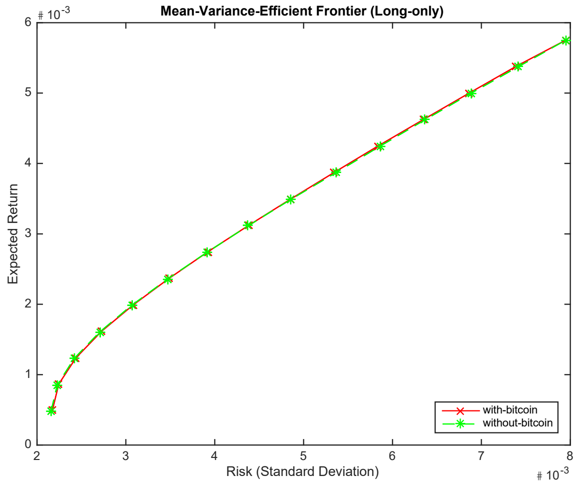
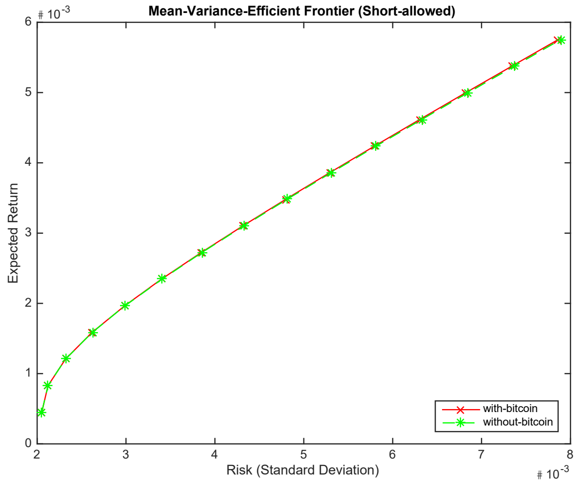


Figure 1C. Levels of indices from August 18, 2011 to January 25, 2022

Figure 2. Efficient Frontiers between portfolios with and without Bitcoin

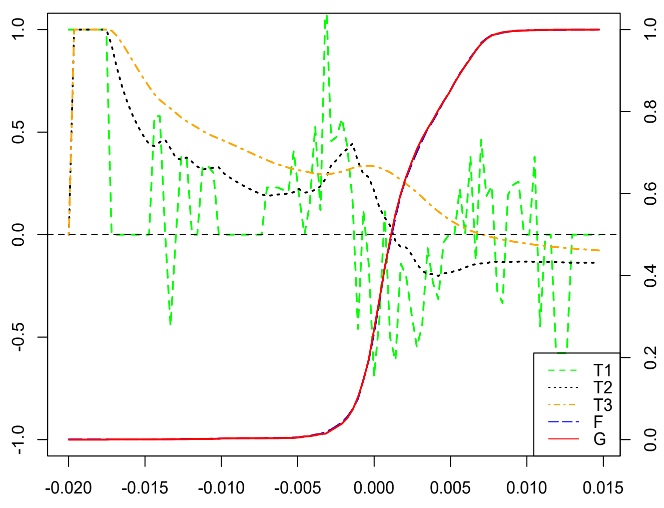




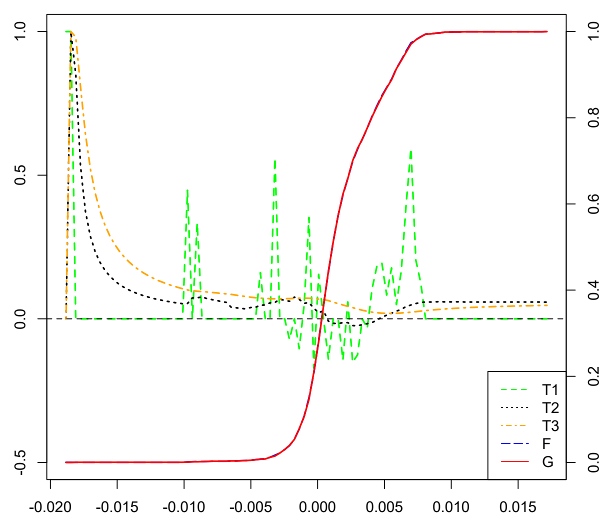
Note: The red line represents portfolios with Bitcoin, the green line denotes portfolios without Bitcoin, and the asterisks indicate the 15 selected portfolios with varying proportions. The left and right panels of the figure show the Mean-Variance Efficient frontiers when a short sale is allowed and not allowed, respectively.

Figure 3. CDF and SD statistics for PF4 with and without Bitcoin

Panel A Long-only

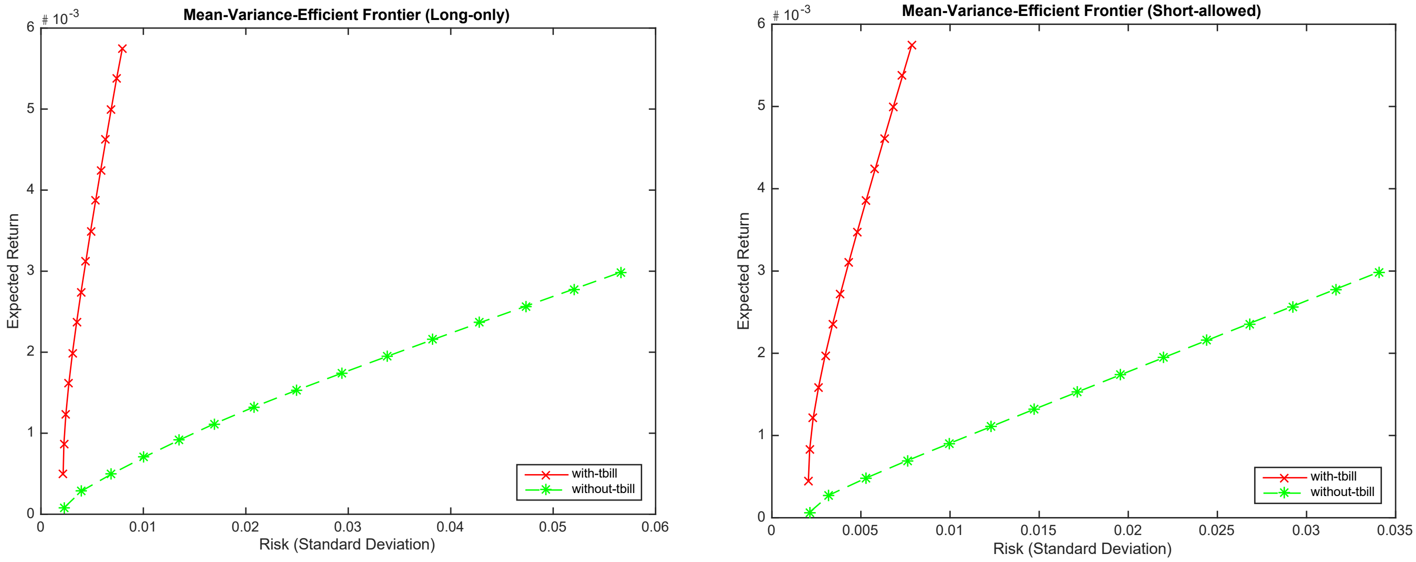


Panel B Short-allowed



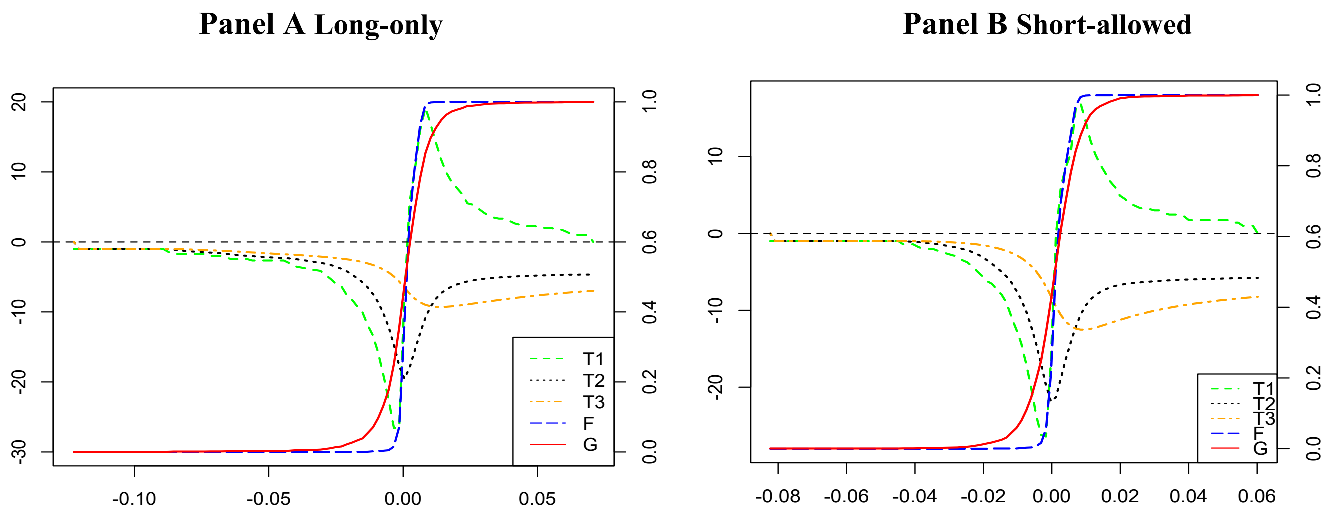
Note: F=CDF of portfolios with both Bitcoin while G=CDF of portfolios without both Bitcoin. T1, T2 and T3 denote the SD statistics.

Figure 4. Efficient Frontiers between portfolios with and without 3-month T-Bill



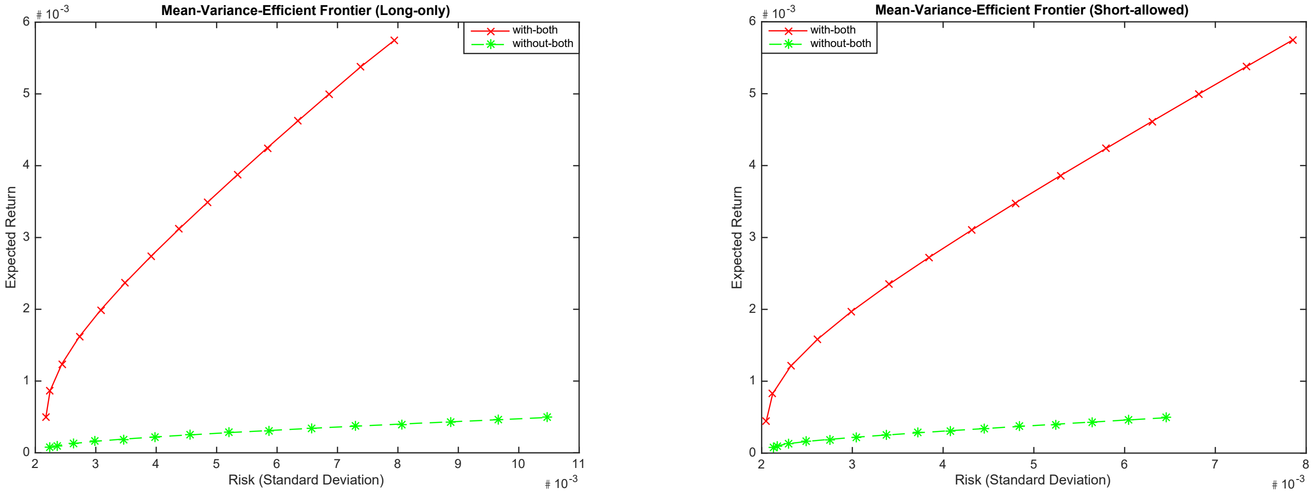
Note: The red line represents portfolios with 3-month US Treasury Bill, the green line denotes portfolios without 3-month US Treasury Bill, and the asterisks indicate the 15 selected portfolios with varying proportions. The left and right panels of the figure show the Mean-Variance Efficient frontiers when a short sale is allowed and not allowed, respectively.

Figure 5. CDF and SD statistics for PF4 with and without a 3-month T-Bill



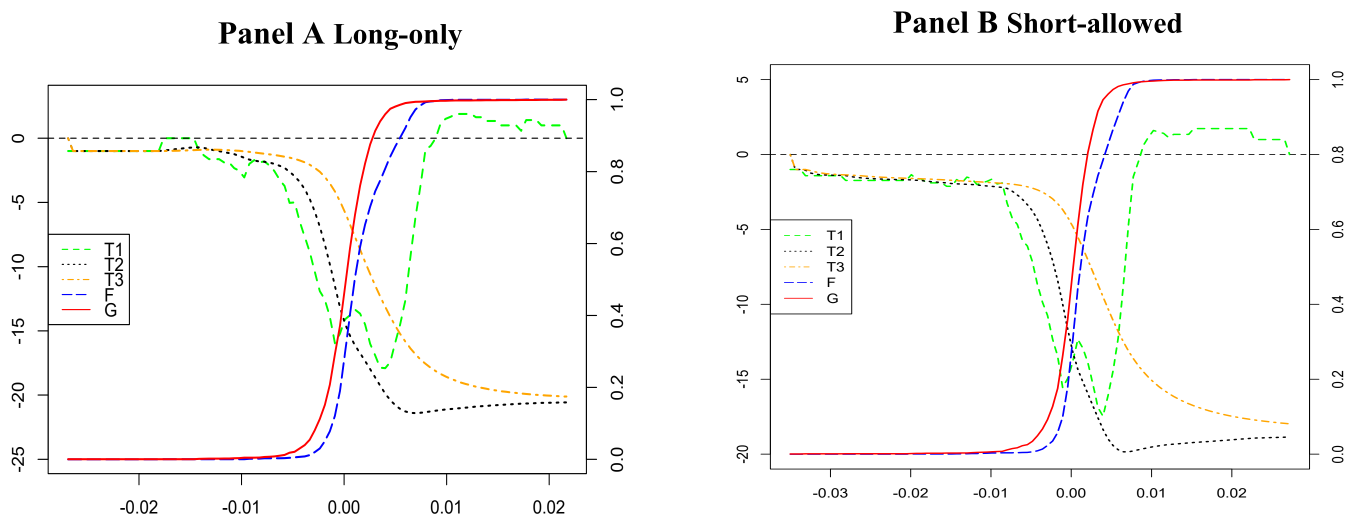
Note: F=CDF of portfolios with a 3-month T-Bill while G=CDF of portfolios without a 3-month T-Bill. T1, T2 and T3 denote the SD statistics.

Figure 6: Efficient Frontiers between portfolios with and without both Bitcoin and a 3-month T-Bill



Note: The red line represents portfolios with both (Bitcoin and 3-month US Treasury Bill) , the green line denotes portfolios without both (Bitcoin and 3-month US Treasury Bill), and the asterisks indicate the 15 selected portfolios with varying proportions. The left and right panels of the figure show the Mean-Variance Efficient frontiers when a short sale is allowed and not allowed, respectively.

Figure 7. CDF and SD statistics for PF4 with and without Bitcoin and a 3-month T-Bill



Note: F=CDF of portfolios with both Bitcoin and a 3-month T-Bill while G=CDF of portfolios without both Bitcoin and a 3-month T-Bill. T1, T2 and T3 denote the SD statistics.

Figure 8. Bitcoin return marked with outliers

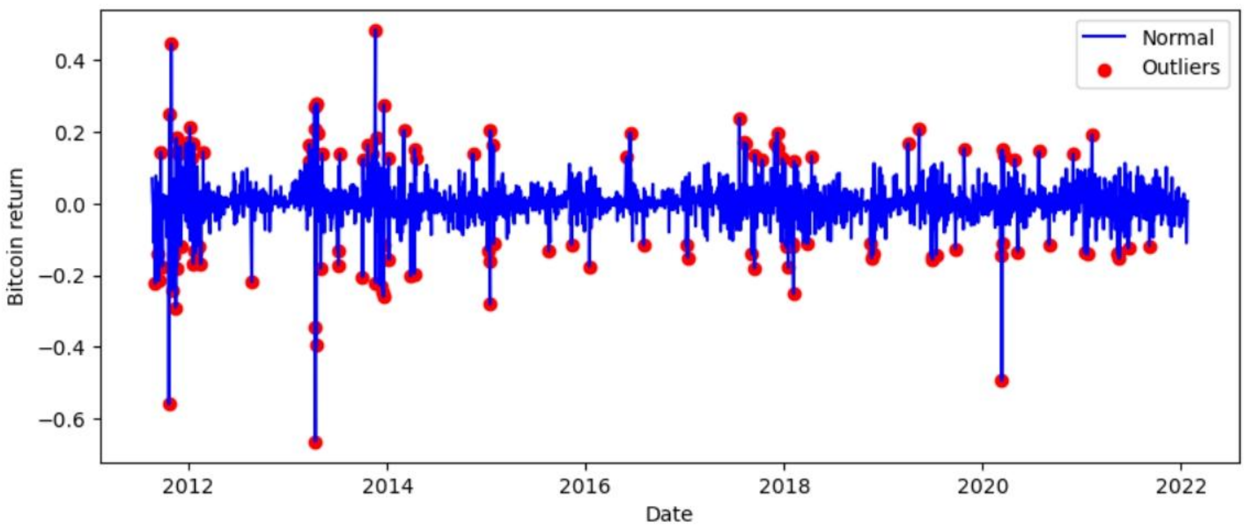
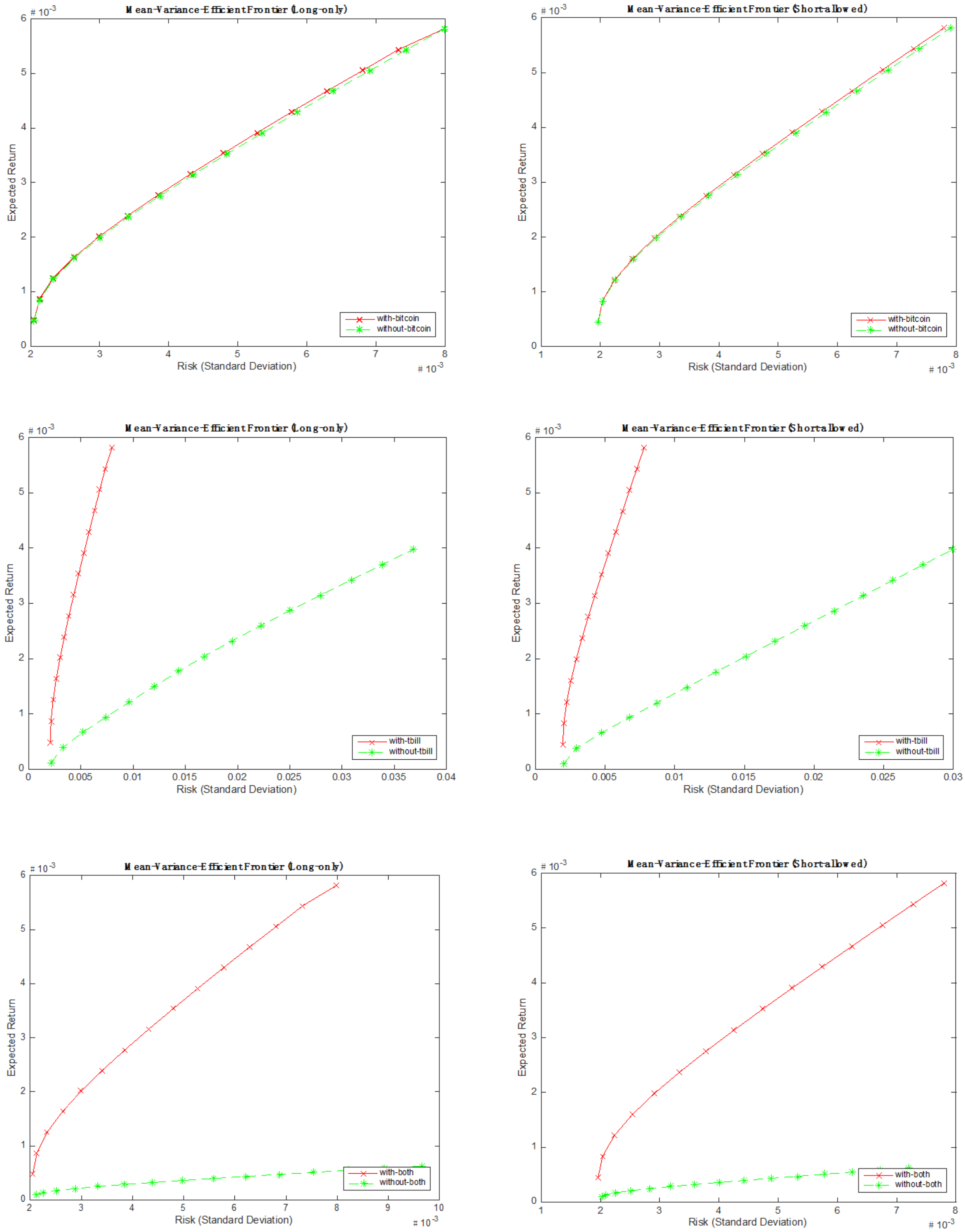


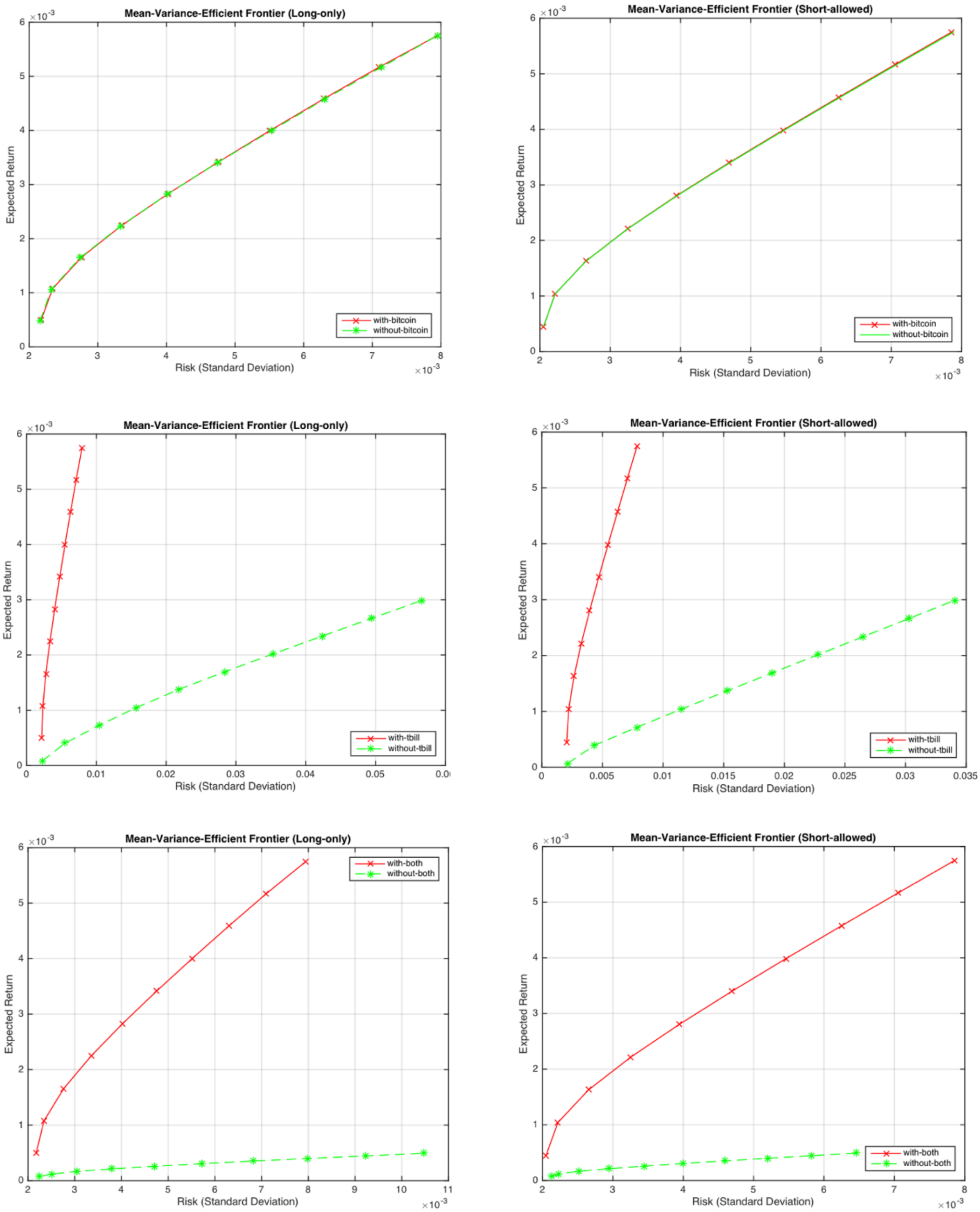
Figure 9. Efficient Frontiers between portfolios with and without Bitcoin (3-month T-Bill, and both) for excluding the outliers



Note: The red line represents portfolios with Bitcoin, 3-month US Treasury Bill and both, the green line denotes portfolios without Bitcoin, 3-month US Treasury Bill and both, and the asterisks indicate the 15 selected portfolios with varying proportions. The left and right panels of the figure show the Mean-Variance Efficient frontiers when a short sale is allowed and not allowed, respectively.

Figure 10. Efficient Frontiers between portfolios with and without Bitcoin (3-month T-Bill, and both)

for 10 portfolios (from PF1 to PF10)



Note: The red line represents portfolios with Bitcoin, 3-month US Treasury Bill and both, the green line denotes portfolios without Bitcoin, 3-month US Treasury Bill and both, and the asterisks indicate the 10 selected portfolios with varying proportions. The left and right panels of the figure show the Mean-Variance Efficient frontiers when a short sale is allowed and not allowed, respectively.

Appendix

Table A1. Descriptive statistics for return series (sub-period 1: August 18, 2011 - April 13, 2021)

	Mean	Std. Dev.	Skewness	Kurtosis	Jarque-Bera
Bitcoin	0.003444	**0.057318**	-1.1030***	23.6332***	45176.55***
ETF	0.000037	0.003948	-0.4293***	74.5814***	537659.79***
SP GSCI Commodity	-0.000297	0.012840	-0.8416***	13.3925***	11628.61***
SP GSCI Gold	-0.000050	0.010274	-0.6472***	10.4486***	5996.81***
SP GSCI Crude Oil	-0.000624	0.028122	-3.9435***	85.8168***	726111.00***
MSCI EUROPE	0.000169	0.011678	-1.0283***	16.6635***	20030.66***
MSCI PACIFIC	0.000180	0.009681	-0.2384***	6.7987***	1537.85***
MSCI NORTH AMERICA	0.000490	0.010600	-0.9869***	23.4354***	44222.65***
FTSE EPRA Nareit GLOBAL	0.000144	0.009448	-2.1961***	38.0767***	131110.68***
US DOLLAR INDEX DXY	0.000085	0.004199	0.0300	5.0345***	434.64***
SP 500	0.000512	0.010657	-0.8921***	22.6143***	40697.41***
MSCI	0.000341	0.008982	-1.2072***	21.9120***	38136.40***
3MTBILL	**0.006179**	**0.008120**	1.1369***	2.7196***	550.68***

Table A2. Descriptive statistics for return series (sub-period 2: April 14, 2021-January 25, 2022)

	Mean	Std. Dev.	Skewness	Kurtosis	Jarque-Bera
Bitcoin	-0.002643	**0.046499**	-0.3286*	3.7366*	8.32**
ETF	-0.000111	0.002925	-0.2535	3.0349	2.21
SP GSCI Commodity	0.001404	0.012373	-1.4605***	9.4842***	432.01***
SP GSCI Gold	0.000264	0.008583	-1.0925***	7.8201***	239.23***
SP GSCI Crude Oil	0.001960	0.020993	-1.7572***	12.9198***	946.01***
MSCI EUROPE	0.000024	0.009228	-0.7678***	4.9417***	52.34***
MSCI PACIFIC	-0.000352	0.008526	-0.0652	2.9993	0.15
MSCI NORTH AMERICA	0.000179	0.007934	-0.3931**	3.5403	7.77**
FTSE EPRA Nareit GLOBAL	0.000161	0.006733	-0.5595***	3.7839**	15.94***
US DOLLAR INDEX DXY	0.000213	0.00314	0.1748	3.1071	1.14
SP 500	0.000247	0.007876	-0.3830**	3.4373	6.65**
MSCI	0.00003	0.006613	-0.4759***	4.0320**	16.83***
3MTBILL	**0.000519**	0.000315	2.3106***	10.4068***	651.01***

Figure A1A. Efficient Frontiers between portfolios with and without Bitcoin for the first sub-period

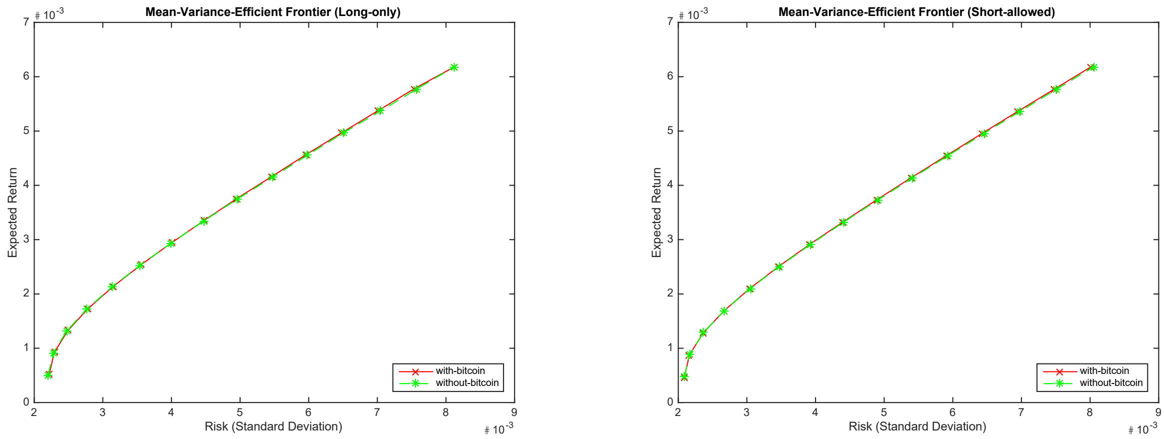


Figure A1B. Efficient Frontiers between portfolios with and without a 3-month T-Bill for the first sub-period

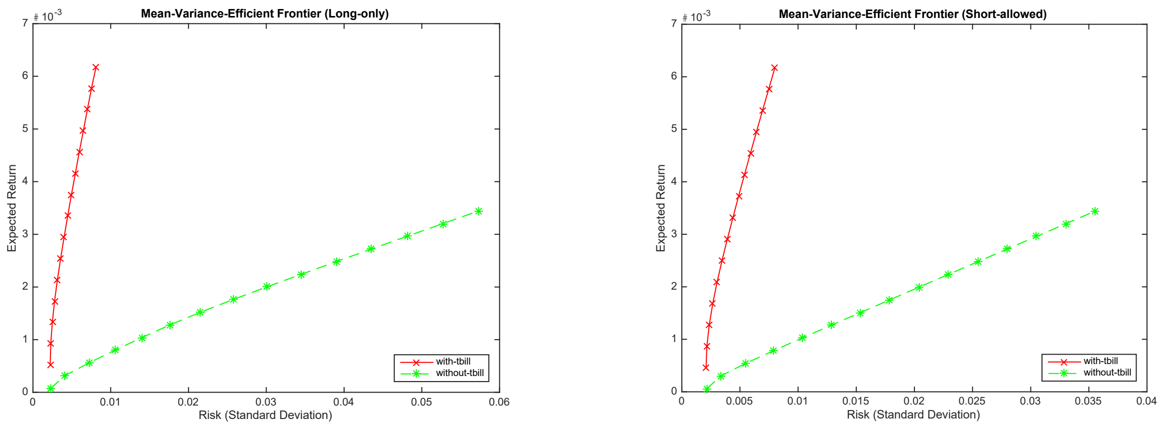


Figure A1C. Efficient Frontiers between portfolios both with and without Bitcoin and 3-month T Bill for the first sub-period

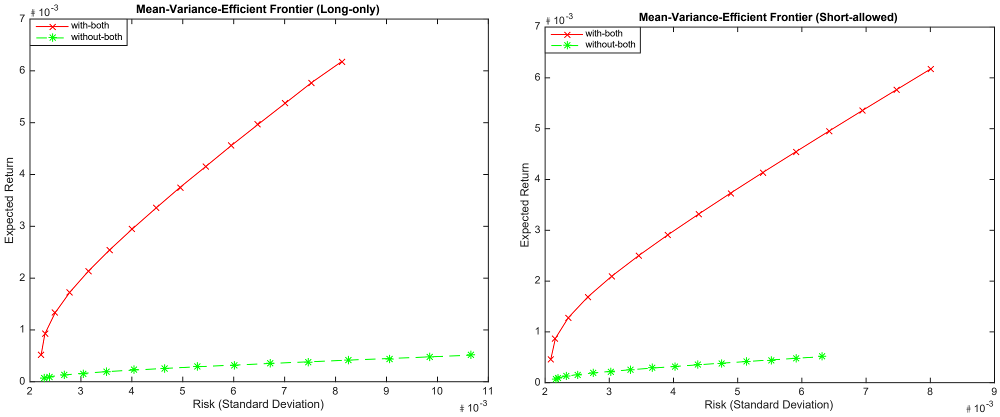
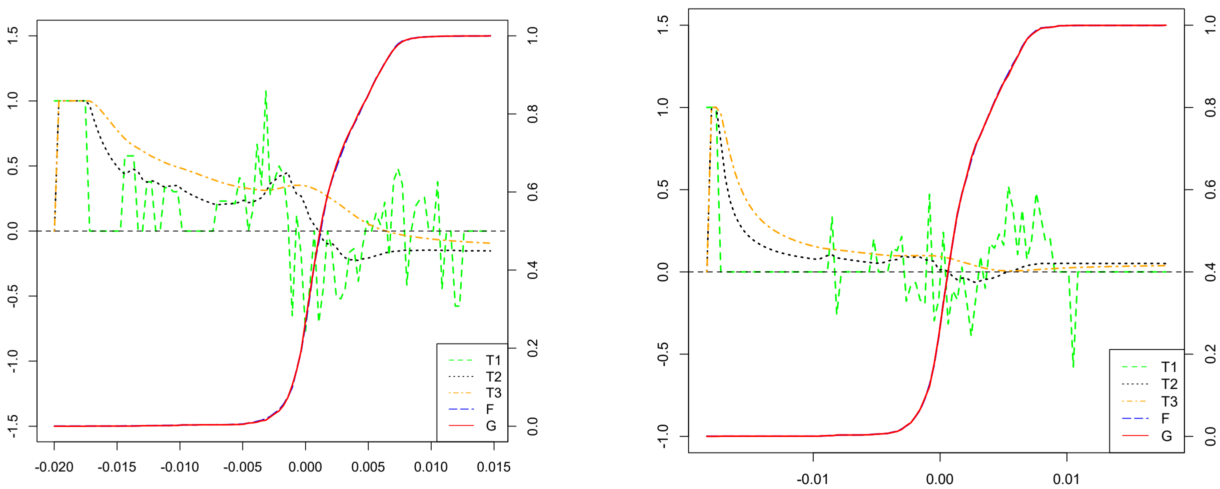


Figure A2A. CDF and SD statistics for PF4 with and without Bitcoin for the first sub-period

Panel A: Long-only **Panel B: Short-allowed**

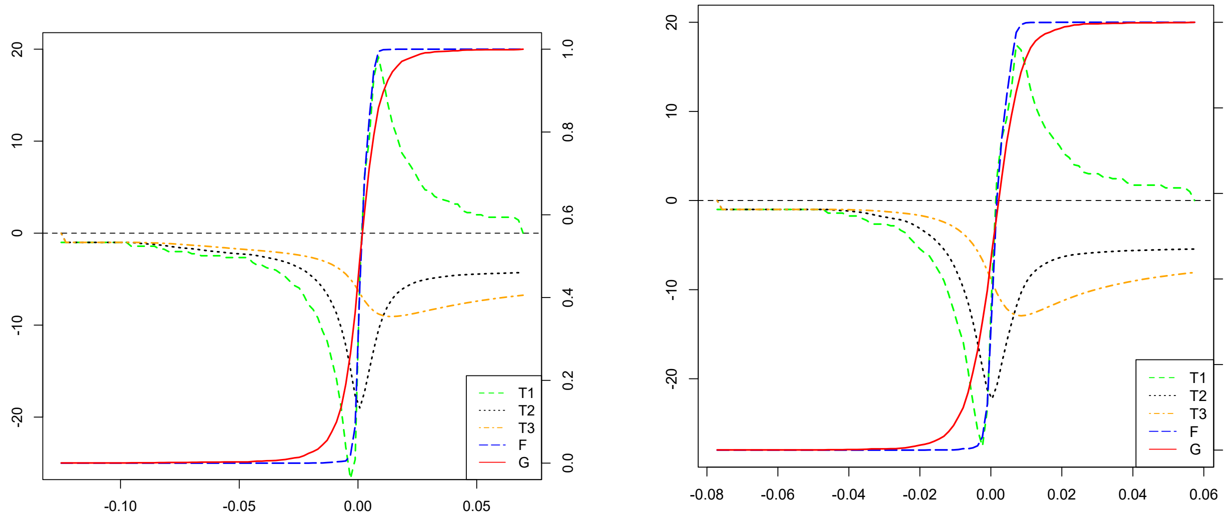


Note: F=CDF of portfolios with both Bitcoin while G=CDF of portfolios without both Bitcoin. T1, T2, and T3 denote the T1, T2, and T3 SD statistics.

Figure A2B. CDF and SD statistics for PF4 with and without a 3-month T-Bill for the first sub-period

Panel A: Long-only

Panel B Short-allowed

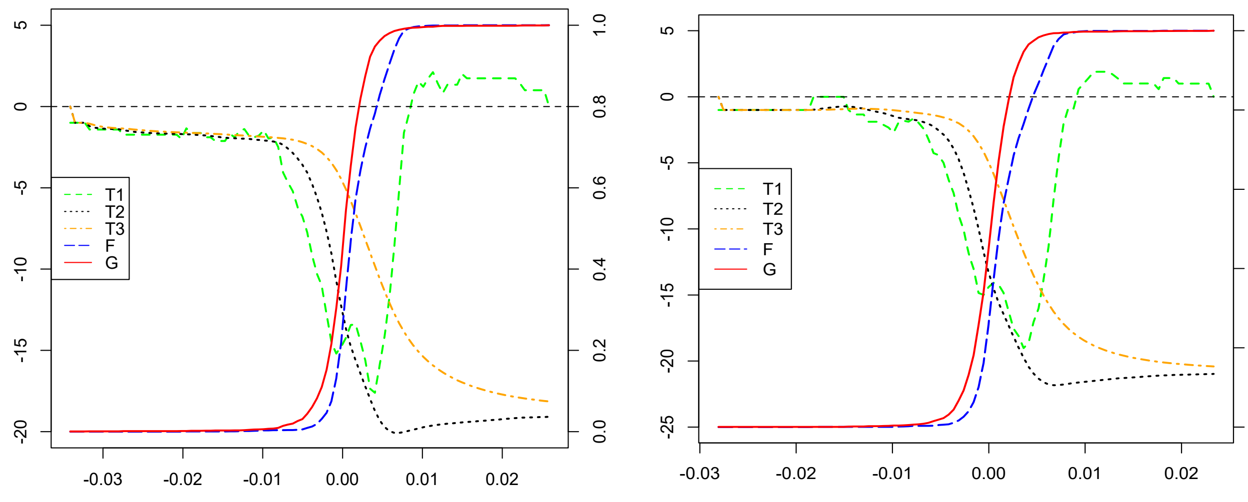


Note: F=CDF of portfolios with a 3-month T-Bill while G=CDF of portfolios without a 3-month T-Bill. T1, T2 and T3 denote the SD statistics.

Figure A2C. CDF and SD statistics for PF4 with and without Bitcoin and a 3-month T-Bill for the first sub-period

Panel A : Long-only

Panel B: Short-allowed



Note: F=CDF of portfolios with both Bitcoin and a 3-month T-Bill while G=CDF of portfolios without both Bitcoin and a 3-month T-Bill. T1, T2 and T3 denote the SD statistics.

Figure A3A. Efficient Frontiers between portfolios with and without Bitcoin for the second **sub-period**

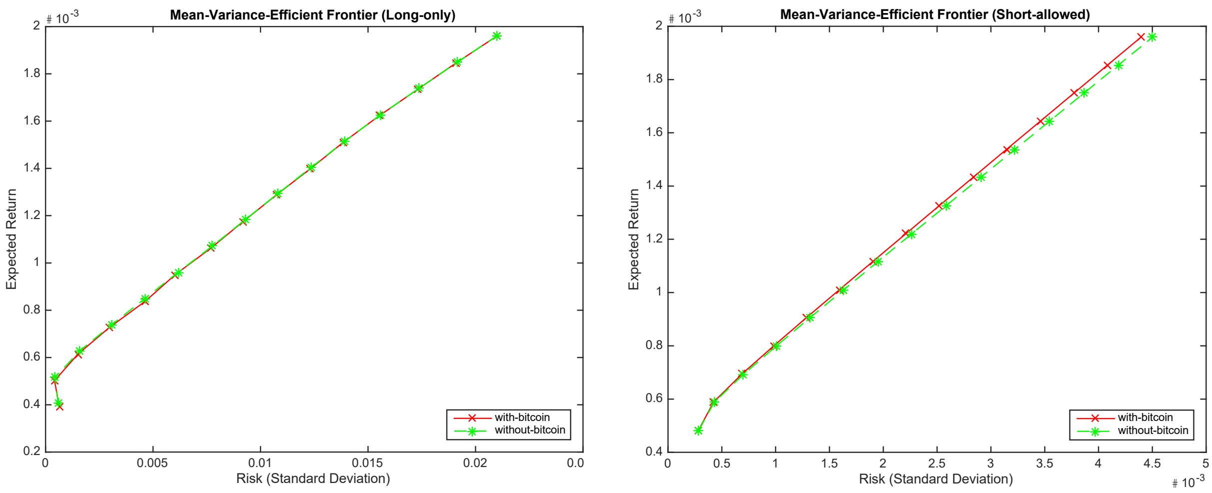


Figure A3B. Efficient Frontiers between portfolios with and without a 3-month T-Bill for the second sub-period

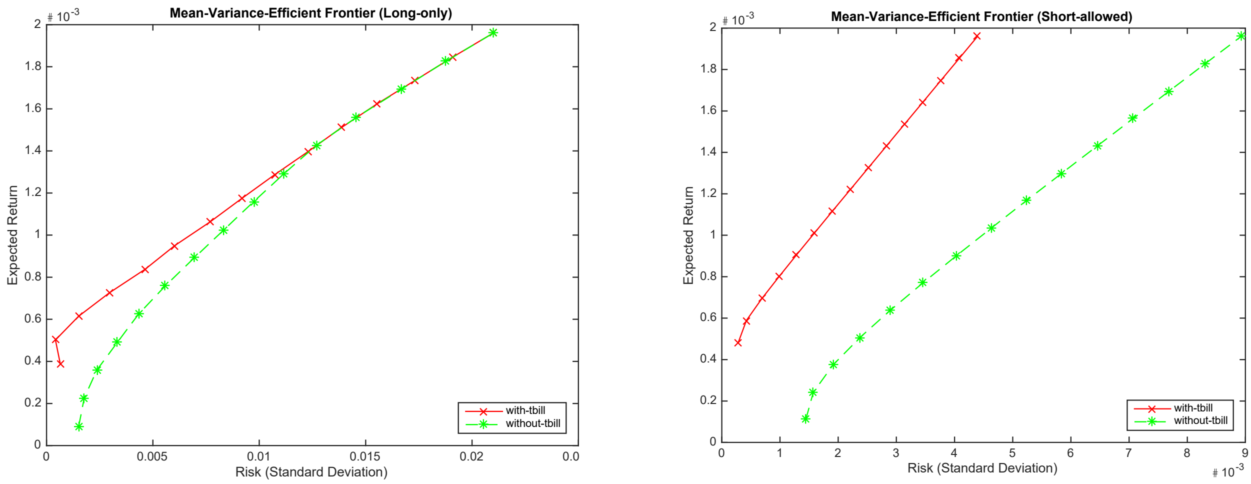
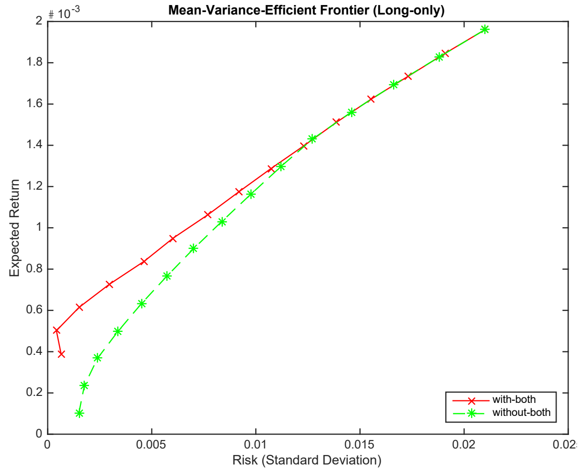


Figure A3C. Efficient Frontiers between portfolios with and without both Bitcoin and a 3-month T-Bill for the second sub-period



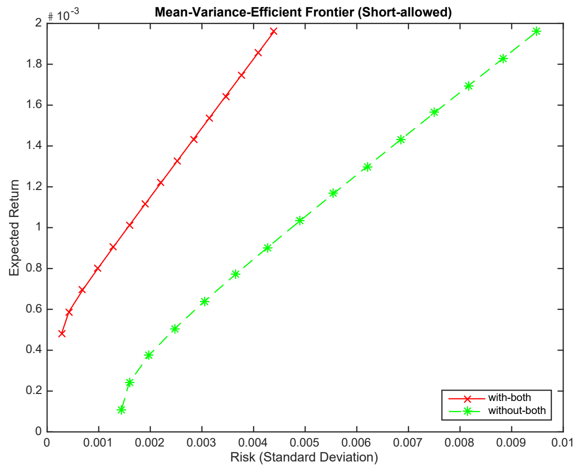
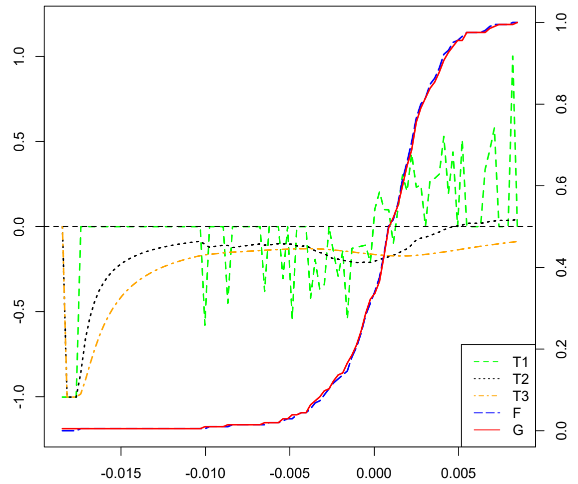
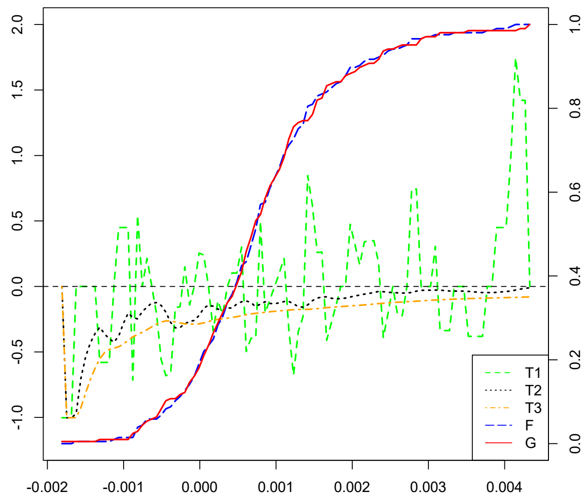


Figure A4A. CDF and SD statistics for PF4 with and without Bitcoin for the second sub-period

Panel A: Long-only



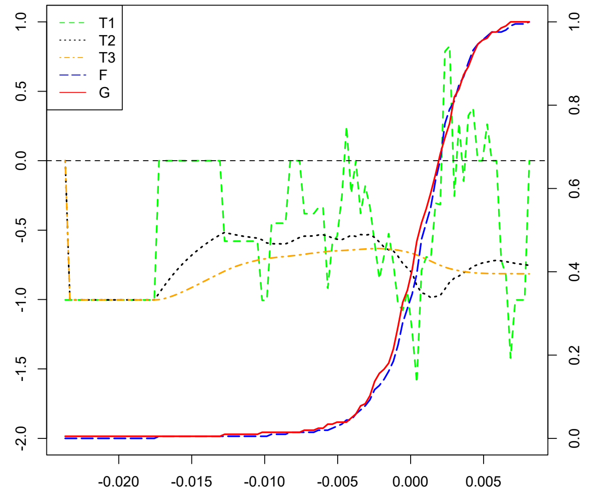
Panel B: Short-allowed



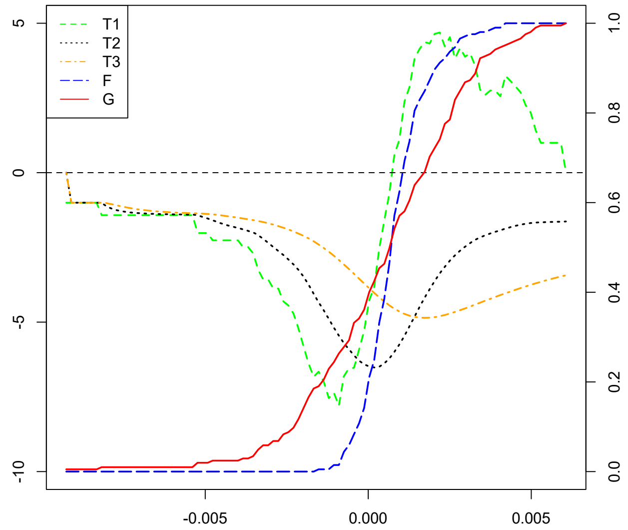
Note: F=CDF of portfolios with both Bitcoin while G=CDF of portfolios without both Bitcoin. T1, T2 and T3 denote the SD statistics.

Figure A4B. CDF and SD statistics for PF4 with and without a 3-month T-Bill for the second sub-period

Panel A: Long-only



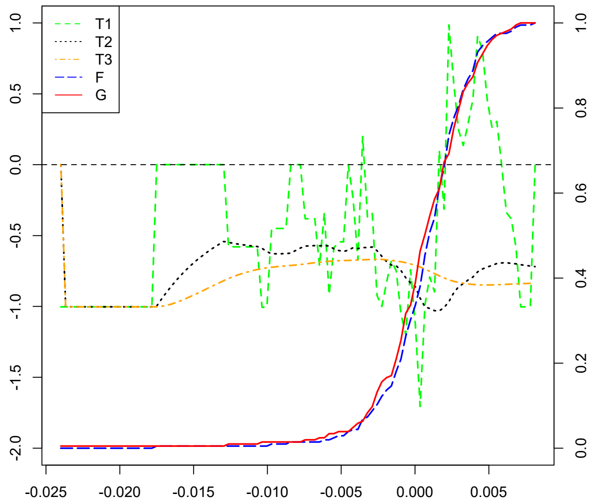
Panel B: Short-allowed



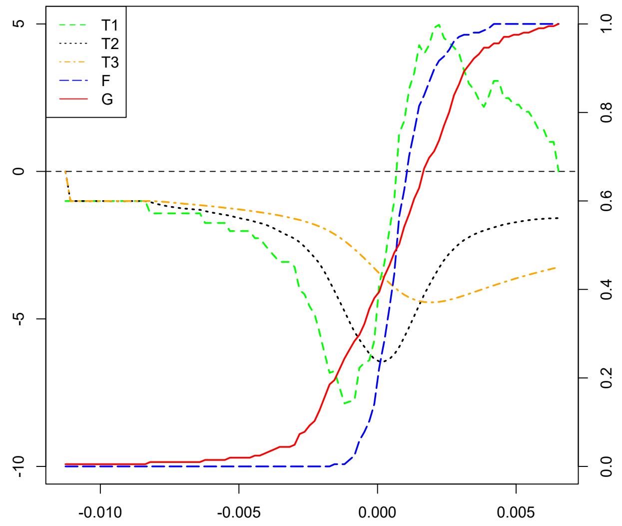
Note: F=CDF of portfolios with a 3-month T-Bill while G=CDF of portfolios without a 3-month T-Bill. T1, T2 and T3 denote the SD statistics.

Figure A4C. CDF and SD statistics for PF4 with and without Bitcoin and a 3-month T-Bill for the second sub-period

Panel A Long-only



Panel B Short-allowed



Note: F=CDF of portfolios with both Bitcoin and a 3-month T-Bill while G=CDF of portfolios without both Bitcoin and a 3-month T-Bill. T1, T2, and T3 denote the SD statistics.

Table A3. Performance measures for the portfolios in the entire period under the long-only Strategy

Panel A

Withboth	Sharpe ratio	Rachev ratio	Information ratio	Alpha	Average return
PF1	0.2255	1.1598	0.0250	0.0005	0.000490
PF2	0.3858	1.4892	0.0696	0.0008	0.000866
PF3	0.5090	2.0062	0.1134	0.0012	0.001242
PF4	0.5929	2.7215	0.1556	0.0016	0.001618
PF5	0.6464	3.6248	0.1960	0.0020	0.001994
PF6	0.6794	4.7432	0.2341	0.0023	0.002370
PF7	0.6997	6.1256	0.2697	0.0027	0.002746
PF8	0.7121	7.8405	0.3028	0.0031	0.003121
PF9	0.7198	10.0507	0.3331	0.0035	0.003497
PF10	0.7248	12.8478	0.3614	0.0039	0.003873
PF11	0.7276	17.0471	0.3869	0.0042	0.004249
PF12	0.7285	19.6689	0.4114	0.0046	0.004625
PF13	0.7285	24.3057	0.4321	0.0050	0.005001
PF14	0.7281	31.0104	0.4490	0.0054	0.005377
PF15	0.7236	-154.4509	0.4552	0.0058	0.005753
PF16	0.0994	0.9670	0.0732	0.0006	0.000766

Panel B

Without Bitcoin	Sharpe ratio	Rachev ratio	Information ratio	Alpha	Average return
PF1	0.2212	1.1639	0.0235	0.0005	0.000477
PF2	0.3833	1.5027	0.0683	0.0008	0.000854
PF3	0.5087	2.0462	0.1120	0.0012	0.001231
PF4	0.5938	2.8150	0.1542	0.0016	0.001608
PF5	0.6474	3.8233	0.1944	0.0020	0.001985
PF6	0.6800	5.1441	0.2323	0.0023	0.002361
PF7	0.6997	6.9062	0.2679	0.0027	0.002738
PF8	0.7116	9.2925	0.3009	0.0031	0.003115
PF9	0.7188	12.6300	0.3315	0.0035	0.003492
PF10	0.7215	15.1804	0.3642	0.0038	0.003869
PF11	0.7245	21.7460	0.3882	0.0042	0.004246
PF12	0.7261	32.3432	0.4104	0.0046	0.004622
PF13	0.7260	50.6069	0.4320	0.0050	0.004999
PF14	0.7249	78.5580	0.4505	0.0054	0.005376
PF15	0.7236	-154.4509	0.4552	0.0058	0.005753
PF16	0.0904	0.9489	0.0599	0.0004	0.000581

Panel C

Without T-Bill	Sharpe ratio	Rachev ratio	Information ratio	Alpha	Average return
PF1	0.0355	0.9593	-0.0241	0.0001	0.000080
PF2	0.0723	0.9701	0.0002	0.0002	0.000288
PF3	0.0721	0.9888	0.0265	0.0003	0.000495
PF4	0.0701	0.9905	0.0431	0.0005	0.000703
PF5	0.0679	0.9844	0.0492	0.0006	0.000910
PF6	0.0660	0.9876	0.0478	0.0008	0.001118
PF7	0.0636	0.9983	0.0434	0.0010	0.001325
PF8	0.0613	1.0030	0.0396	0.0012	0.001533
PF9	0.0593	1.0046	0.0360	0.0015	0.001740
PF10	0.0577	1.0041	0.0327	0.0017	0.001948
PF11	0.0563	1.0044	0.0296	0.0019	0.002155
PF12	0.0552	1.0042	0.0264	0.0021	0.002363
PF13	0.0542	1.0042	0.0233	0.0023	0.002570
PF14	0.0534	1.0045	0.0202	0.0025	0.002778
PF15	0.0528	1.0050	0.0170	0.0028	0.002985
PF16	0.0420	0.9159	0.0058	0.0001	0.000350

Panel D

Withoutboth	Sharpe ratio	Rachev ratio	Information ratio	Alpha	Average return
PF1	0.0316	0.9609	-0.0252	0.0000	0.000071
PF2	0.0428	0.9609	-0.0227	0.0001	0.000101
PF3	0.0497	0.9517	-0.0201	0.0001	0.000131
PF4	0.0538	0.9449	-0.0172	0.0001	0.000161
PF5	0.0554	0.9457	-0.0140	0.0001	0.000191
PF6	0.0556	0.9481	-0.0104	0.0001	0.000221
PF7	0.0551	0.9430	-0.0064	0.0001	0.000251
PF8	0.0542	0.9383	-0.0019	0.0001	0.000281
PF9	0.0531	0.9340	0.0034	0.0001	0.000312
PF10	0.0520	0.9316	0.0096	0.0001	0.000342
PF11	0.0509	0.9301	0.0165	0.0001	0.000372
PF12	0.0498	0.9265	0.0238	0.0001	0.000402
PF13	0.0487	0.9209	0.0309	0.0001	0.000432
PF14	0.0478	0.9189	0.0354	0.0001	0.000462
PF15	0.0470	0.9168	0.0381	0.0001	0.000492
PF16	0.0158	0.8801	-0.0425	-0.0001	0.000110

Table A4. Performance measures and the results for short selling is allowed portfolios in the entire period

Panel A

Withboth	Sharpe ratio	Rachev ratio	Information ratio	Alpha	Average return
PF1	0.2197	1.1774	0.0199	0.0004	0.000451
PF2	0.3909	1.5195	0.0638	0.0008	0.000830
PF3	0.5207	2.0858	0.1069	0.0012	0.001208
PF4	0.6062	2.8786	0.1488	0.0016	0.001587
PF5	0.6586	3.8972	0.1891	0.0020	0.001966
PF6	0.6897	5.1777	0.2275	0.0023	0.002344
PF7	0.7082	6.7561	0.2636	0.0027	0.002723
PF8	0.7192	8.6123	0.2975	0.0031	0.003102
PF9	0.7256	10.7109	0.3289	0.0035	0.003481
PF10	0.7293	12.9496	0.3579	0.0038	0.003859
PF11	0.7313	14.9912	0.3846	0.0042	0.004238
PF12	0.7322	16.6206	0.4090	0.0046	0.004617
PF13	0.7325	17.8210	0.4313	0.0050	0.004996
PF14	0.7323	18.4493	0.4516	0.0054	0.005374
PF15	0.7319	18.5043	0.4700	0.0057	0.005753
PF16	0.0994	0.9670	0.0732	0.0006	0.000766

Panel B

Without Bitcoin	Sharpe ratio	Rachev ratio	Information ratio	Alpha	Average return
PF1	0.2215	1.1785	0.0203	0.0004	0.000455
PF2	0.3923	1.5234	0.0642	0.0008	0.000833
PF3	0.5215	2.0982	0.1073	0.0012	0.001212
PF4	0.6063	2.9060	0.1491	0.0016	0.001590
PF5	0.6580	3.9566	0.1893	0.0020	0.001968
PF6	0.6886	5.3037	0.2276	0.0023	0.002347
PF7	0.7067	7.0254	0.2637	0.0027	0.002725
PF8	0.7174	9.1972	0.2974	0.0031	0.003104
PF9	0.7236	11.9046	0.3287	0.0035	0.003482
PF10	0.7271	15.1200	0.3576	0.0038	0.003861
PF11	0.7289	18.5593	0.3841	0.0042	0.004239
PF12	0.7298	21.7817	0.4084	0.0046	0.004618
PF13	0.7299	24.2462	0.4306	0.0050	0.004996
PF14	0.7297	25.2871	0.4507	0.0054	0.005375
PF15	0.7292	25.3553	0.4690	0.0057	0.005753
PF16	0.0904	0.9489	0.0599	0.0004	0.000581

Panel C

Without T-Bill	Sharpe ratio	Rachev ratio	Information ratio	Alpha	Average return
PF1	0.2215	1.1785	0.0203	0.0004	0.000455
PF2	0.3923	1.5234	0.0642	0.0008	0.000833
PF3	0.5215	2.0982	0.1073	0.0012	0.001212
PF4	0.6063	2.9060	0.1491	0.0016	0.001590
PF5	0.6580	3.9566	0.1893	0.0020	0.001968
PF6	0.6886	5.3037	0.2276	0.0023	0.002347
PF7	0.7067	7.0254	0.2637	0.0027	0.002725
PF8	0.7174	9.1972	0.2974	0.0031	0.003104
PF9	0.7236	11.9046	0.3287	0.0035	0.003482
PF10	0.7271	15.1200	0.3576	0.0038	0.003861
PF11	0.7289	18.5593	0.3841	0.0042	0.004239
PF12	0.7298	21.7817	0.4084	0.0046	0.004618
PF13	0.7299	24.2462	0.4306	0.0050	0.004996
PF14	0.7297	25.2871	0.4507	0.0054	0.005375
PF15	0.7292	25.3553	0.4690	0.0057	0.005753
PF16	0.0904	0.9489	0.0599	0.0004	0.000581

Panel D

Withoutboth	Sharpe ratio	Rachev ratio	Information ratio	Alpha	Average return
PF1	0.0336	0.9787	-0.0244	0.0001	0.000071
PF2	0.0468	0.9991	-0.0213	0.0001	0.000102
PF3	0.0573	1.0150	-0.0181	0.0001	0.000132
PF4	0.0648	1.0249	-0.0148	0.0001	0.000162
PF5	0.0697	1.0341	-0.0113	0.0001	0.000192
PF6	0.0728	1.0433	-0.0077	0.0002	0.000222
PF7	0.0747	1.0501	-0.0041	0.0002	0.000252
PF8	0.0758	1.0559	-0.0004	0.0002	0.000282
PF9	0.0764	1.0613	0.0033	0.0002	0.000312
PF10	0.0767	1.0638	0.0069	0.0003	0.000342
PF11	0.0767	1.0653	0.0105	0.0003	0.000372
PF12	0.0767	1.0661	0.0140	0.0003	0.000402
PF13	0.0766	1.0670	0.0173	0.0003	0.000432
PF14	0.0764	1.0680	0.0205	0.0004	0.000462
PF15	0.0762	1.0684	0.0236	0.0004	0.000492
PF16	0.0158	0.8801	-0.0425	-0.0001	0.000110

Table A5. Performance measures and the results for long-only portfolios in the sub-period 1

Panel A

Withboth	Sharpe ratio	Rachev ratio	Information ratio	Alpha	Average return
PF1	0.2336	1.1601	0.0254	0.0005	0.000518
PF2	0.4025	1.4863	0.0725	0.0009	0.000923
PF3	0.5327	1.9981	0.1187	0.0013	0.001327
PF4	0.6216	2.7026	0.1634	0.0017	0.001731
PF5	0.6784	3.5905	0.2060	0.0021	0.002136
PF6	0.7137	4.6850	0.2463	0.0025	0.002540
PF7	0.7354	6.0180	0.2841	0.0029	0.002944
PF8	0.7488	7.6555	0.3191	0.0033	0.003349
PF9	0.7583	10.4188	0.3496	0.0037	0.003753
PF10	0.7619	11.3892	0.3825	0.0041	0.004157
PF11	0.7657	16.2911	0.4079	0.0045	0.004562
PF12	0.7666	18.3293	0.4351	0.0049	0.004966
PF13	0.7666	21.7241	0.4550	0.0054	0.005370
PF14	0.7666	27.8078	0.4746	0.0058	0.005775
PF15	0.7609	-158.1661	0.4792	0.0062	0.006179
PF16	0.1047	0.9772	0.0761	0.0006	0.000816

Panel B

Without Bitcoin	Sharpe ratio	Rachev ratio	Information ratio	Alpha	Average return
PF1	0.2282	1.1632	0.0236	0.0005	0.000503
PF2	0.3991	1.4980	0.0708	0.0009	0.000909
PF3	0.5318	2.0368	0.1171	0.0013	0.001314
PF4	0.6220	2.7971	0.1617	0.0017	0.001719
PF5	0.6789	3.7954	0.2042	0.0021	0.002125
PF6	0.7138	5.1001	0.2443	0.0025	0.002530
PF7	0.7349	6.8379	0.2819	0.0029	0.002936
PF8	0.7477	9.1851	0.3168	0.0033	0.003341
PF9	0.7554	12.4384	0.3492	0.0037	0.003747
PF10	0.7583	15.0885	0.3838	0.0041	0.004152
PF11	0.7617	21.4617	0.4093	0.0045	0.004557
PF12	0.7634	31.4055	0.4327	0.0049	0.004963
PF13	0.7616	27.4613	0.4640	0.0053	0.005368
PF14	0.7623	82.0115	0.4739	0.0058	0.005774
PF15	0.7609	-158.1661	0.4792	0.0062	0.006179
PF16	0.0916	0.9501	0.0576	0.0004	0.000597

Panel C

Without T-Bill	Sharpe ratio	Rachev ratio	Information ratio	Alpha	Average return
PF1	0.0339	0.9595	-0.0265	0.0000	0.000078
PF2	0.0771	0.9797	0.0013	0.0002	0.000319
PF3	0.0775	1.0042	0.0305	0.0004	0.000559
PF4	0.0758	1.0040	0.0480	0.0006	0.000799
PF5	0.0742	1.0006	0.0544	0.0007	0.001040
PF6	0.0726	1.0040	0.0536	0.0010	0.001280
PF7	0.0706	1.0132	0.0496	0.0012	0.001521
PF8	0.0684	1.0176	0.0459	0.0015	0.001761
PF9	0.0665	1.0189	0.0425	0.0017	0.002001
PF10	0.0650	1.0185	0.0392	0.0020	0.002242
PF11	0.0636	1.0187	0.0360	0.0022	0.002482
PF12	0.0625	1.0183	0.0329	0.0025	0.002722
PF13	0.0616	1.0181	0.0298	0.0027	0.002963
PF14	0.0608	1.0181	0.0267	0.0030	0.003203
PF15	0.0601	1.0185	0.0235	0.0032	0.003444
PF16	0.0438	0.9229	0.0051	0.0001	0.000369

Panel D

Withoutboth	Sharpe ratio	Rachev ratio	Informamtion ratio	Alpha	Average return
PF1	0.0295	0.9606	-0.0278	0.000038	0.000068
PF2	0.0415	0.9574	-0.0251	0.000056	0.000099
PF3	0.0492	0.9441	-0.0227	0.000068	0.000131
PF4	0.0535	0.9375	-0.0199	0.000080	0.000163
PF5	0.0555	0.9376	-0.0168	0.000092	0.000195
PF6	0.0559	0.9398	-0.0133	0.000101	0.000226
PF7	0.0555	0.9363	-0.0094	0.000110	0.000258
PF8	0.0548	0.9326	-0.0048	0.000116	0.000290
PF9	0.0534	0.9281	0.0007	0.000117	0.000322
PF10	0.0525	0.9263	0.0075	0.000122	0.000353
PF11	0.0516	0.9250	0.0152	0.000126	0.000385
PF12	0.0506	0.9226	0.0231	0.000130	0.000417
PF13	0.0495	0.9204	0.0300	0.000132	0.000449
PF14	0.0488	0.9196	0.0341	0.000137	0.000480
PF15	0.0481	0.9187	0.0365	0.000140	0.000512
PF16	0.0127	0.8785	-0.0513	-0.000142	0.000090

Table A6. Performance measures and the results for both the long-only and short-allowed strategies in sub-period 1

Panel A

Withboth	Sharpe ratio	Rachev ratio	Information ratio	Alpha	Average return
PF1	0.2228	1.1752	0.0187	0.0004	0.000466
PF2	0.4043	1.5128	0.0652	0.0009	0.000874
PF3	0.5424	2.0729	0.1109	0.0013	0.001282
PF4	0.6337	2.8551	0.1553	0.0017	0.001690
PF5	0.6899	3.8556	0.1981	0.0021	0.002098
PF6	0.7236	5.1001	0.2388	0.0025	0.002506
PF7	0.7438	6.6148	0.2772	0.0029	0.002914
PF8	0.7559	8.3820	0.3131	0.0033	0.003322
PF9	0.7631	10.3523	0.3465	0.0037	0.003730
PF10	0.7674	12.4006	0.3773	0.0041	0.004139
PF11	0.7698	14.1749	0.4056	0.0045	0.004547
PF12	0.7711	15.6003	0.4316	0.0049	0.004955
PF13	0.7716	16.5912	0.4553	0.0053	0.005363
PF14	0.7716	17.1551	0.4768	0.0058	0.005771
PF15	0.7713	17.2082	0.4964	0.0062	0.006179
PF16	0.1047	0.9772	0.0761	0.0006	0.000816

Panel B

Without Bitcoin	Sharpe ratio	Rachev ratio	Information ratio	Alpha	Average return
PF1	0.2251	1.1769	0.0192	0.0005	0.000470
PF2	0.4061	1.5177	0.0657	0.0009	0.000878
PF3	0.5433	2.0874	0.1113	0.0013	0.001286
PF4	0.6337	2.8855	0.1557	0.0017	0.001694
PF5	0.6891	3.9247	0.1984	0.0021	0.002101
PF6	0.7221	5.2545	0.2389	0.0025	0.002509
PF7	0.7417	6.9402	0.2772	0.0029	0.002917
PF8	0.7534	9.0572	0.3130	0.0033	0.003325
PF9	0.7603	11.6723	0.3462	0.0037	0.003732
PF10	0.7644	14.7436	0.3768	0.0041	0.004140
PF11	0.7666	17.9609	0.4050	0.0045	0.004548
PF12	0.7677	20.8627	0.4308	0.0049	0.004956
PF13	0.7681	23.0694	0.4543	0.0054	0.005364
PF14	0.7680	24.1552	0.4756	0.0058	0.005771
PF15	0.7677	24.2026	0.4951	0.0062	0.006179
PF16	0.0916	0.9501	0.0576	0.0004	0.000597

Panel C

Without T-Bill	Sharpe ratio	Rachev ratio	Information ratio	Alpha	Average return
PF1	0.0274	0.9744	-0.0279	0.0000	0.000059
PF2	0.0904	1.0321	-0.0006	0.0002	0.000301
PF3	0.0985	1.0600	0.0268	0.0005	0.000543
PF4	0.0992	1.0728	0.0480	0.0007	0.000784
PF5	0.0990	1.0777	0.0616	0.0009	0.001026
PF6	0.0986	1.0795	0.0696	0.0011	0.001268
PF7	0.0983	1.0812	0.0740	0.0013	0.001510
PF8	0.0980	1.0828	0.0764	0.0015	0.001751
PF9	0.0977	1.0841	0.0775	0.0017	0.001993
PF10	0.0975	1.0852	0.0778	0.0019	0.002235
PF11	0.0974	1.0860	0.0777	0.0021	0.002477
PF12	0.0972	1.0866	0.0772	0.0024	0.002718
PF13	0.0971	1.0871	0.0765	0.0026	0.002960
PF14	0.0970	1.0875	0.0757	0.0028	0.003202
PF15	0.0969	1.0878	0.0748	0.0030	0.003444
PF16	0.0438	0.9229	0.0051	0.0001	0.000369

Panel D

Withoutboth	Sharpe ratio	Rachev ratio	Information ratio	Alpha	Average return
PF1	0.0295	0.9760	-0.0274	0.0000	0.000064
PF2	0.0434	0.9917	-0.0242	0.0001	0.000096
PF3	0.0550	1.0029	-0.0209	0.0001	0.000128
PF4	0.0637	1.0145	-0.0174	0.0001	0.000160
PF5	0.0698	1.0221	-0.0138	0.0001	0.000192
PF6	0.0740	1.0249	-0.0100	0.0002	0.000224
PF7	0.0767	1.0306	-0.0062	0.0002	0.000256
PF8	0.0785	1.0402	-0.0024	0.0002	0.000288
PF9	0.0796	1.0486	0.0015	0.0002	0.000320
PF10	0.0803	1.0561	0.0053	0.0003	0.000352
PF11	0.0808	1.0607	0.0091	0.0003	0.000384
PF12	0.0810	1.0635	0.0129	0.0003	0.000416
PF13	0.0811	1.0661	0.0165	0.0003	0.000448
PF14	0.0812	1.0683	0.0200	0.0004	0.000480
PF15	0.0812	1.0697	0.0233	0.0004	0.000512
PF16	0.0127	0.8785	-0.0513	-0.0001	0.000090

Table A7. Performance measures and the results for both the long-only and short-allowed strategies in the sub-period 2

Panel A

Withboth	Sharpe ratio	Rachev ratio	Information ratio	Alpha	Average return
PF1	0.6064	1.5533	0.0337	0.0004	0.000390
PF2	1.1936	5.5404	0.0427	0.0005	0.000502
PF3	0.4078	1.2460	0.0516	0.0006	0.000614
PF4	0.2457	0.9908	0.0595	0.0007	0.000726
PF5	0.1806	0.9136	0.0657	0.0008	0.000838
PF6	0.1577	0.8848	0.0711	0.0009	0.000950
PF7	0.1383	0.8645	0.0744	0.0011	0.001063
PF8	0.1279	0.8528	0.0771	0.0012	0.001175
PF9	0.1200	0.8444	0.0787	0.0013	0.001287
PF10	0.1136	0.8378	0.0793	0.0014	0.001399
PF11	0.1090	0.8368	0.0796	0.0015	0.001511
PF12	0.1045	0.8390	0.0789	0.0016	0.001623
PF13	0.1002	0.8387	0.0774	0.0017	0.001736
PF14	0.0966	0.8427	0.0758	0.0018	0.001848
PF15	0.0934	0.8489	0.0739	0.0020	0.001960
PF16	0.0227	0.8185	0.0113	0.0001	0.000146

Panel B

Without Bitcoin	Sharpe ratio	Rachev ratio	Information ratio	Alpha	Average return
PF1	0.6933	1.8619	0.0350	0.0004	0.000405
PF2	1.1357	5.2847	0.0439	0.0005	0.000516
PF3	0.3958	1.2333	0.0526	0.0006	0.000627
PF4	0.2397	0.9848	0.0603	0.0007	0.000739
PF5	0.1838	0.9151	0.0666	0.0008	0.000850
PF6	0.1557	0.8830	0.0715	0.0010	0.000961
PF7	0.1380	0.8635	0.0748	0.0011	0.001072
PF8	0.1275	0.8522	0.0773	0.0012	0.001183
PF9	0.1197	0.8440	0.0788	0.0013	0.001294
PF10	0.1136	0.8378	0.0795	0.0014	0.001405
PF11	0.1089	0.8371	0.0796	0.0015	0.001516
PF12	0.1044	0.8390	0.0789	0.0016	0.001627
PF13	0.1001	0.8388	0.0774	0.0017	0.001738
PF14	0.0966	0.8427	0.0758	0.0018	0.001849
PF15	0.0934	0.8489	0.0739	0.0020	0.001960
PF16	0.0746	0.9108	0.0301	0.0004	0.000378

Panel C

Without T-Bill	Sharpe ratio	Rachev ratio	Information ratio	Alpha	Average return
PF1	0.0605	0.9271	0.0096	0.0001	0.000092
PF2	0.1270	0.9688	0.0203	0.0002	0.000225
PF3	0.1516	0.9009	0.0306	0.0004	0.000359
PF4	0.1474	0.8536	0.0406	0.0005	0.000492
PF5	0.1443	0.8550	0.0498	0.0006	0.000625
PF6	0.1359	0.8398	0.0578	0.0008	0.000759
PF7	0.1285	0.8286	0.0644	0.0009	0.000892
PF8	0.1230	0.8323	0.0698	0.0010	0.001026
PF9	0.1190	0.8329	0.0741	0.0012	0.001159
PF10	0.1157	0.8329	0.0773	0.0013	0.001293
PF11	0.1125	0.8346	0.0795	0.0014	0.001426
PF12	0.1071	0.8389	0.0794	0.0016	0.001560
PF13	0.1016	0.8385	0.0779	0.0017	0.001693
PF14	0.0973	0.8419	0.0762	0.0018	0.001826
PF15	0.0934	0.8489	0.0739	0.0020	0.001960
PF16	0.0165	0.8126	0.0086	0.0001	0.000115

Panel D

Withoutboth	Sharpe ratio	Rachev ratio	Informamtion ratio	Alpha	Average return
PF1	0.0684	0.9487	0.0105	0.000102	0.000102
PF2	0.1334	0.9830	0.0211	0.000235	0.000235
PF3	0.1535	0.9014	0.0314	0.000367	0.000368
PF4	0.1475	0.8564	0.0412	0.000500	0.000500
PF5	0.1405	0.8469	0.0501	0.000632	0.000633
PF6	0.1337	0.8346	0.0580	0.000765	0.000766
PF7	0.1283	0.8297	0.0647	0.000897	0.000898
PF8	0.1233	0.8326	0.0701	0.001030	0.001031
PF9	0.1192	0.8331	0.0743	0.001162	0.001164
PF10	0.1158	0.8327	0.0775	0.001294	0.001296
PF11	0.1125	0.8347	0.0795	0.001427	0.001429
PF12	0.1070	0.8389	0.0794	0.001559	0.001562
PF13	0.1018	0.8383	0.0781	0.001691	0.001695
PF14	0.0973	0.8419	0.0761	0.001824	0.001827
PF15	0.0934	0.8489	0.0739	0.001956	0.001960
PF16	0.0661	0.9012	0.0286	0.000364	0.000365

Table A8. Performance measures and the results for both the long-only and short-allowed strategies in sub-period 2

Panel A: With both

Withboth	Sharpe ratio	Rachev ratio	Information ratio	Alpha	Average return
PF1	1.6983	-14.5011	0.0412	0.0005	0.000483
PF2	1.3909	-99.8265	0.0496	0.0006	0.000589
PF3	1.0086	6.3633	0.0580	0.0007	0.000694
PF4	0.8142	3.9356	0.0662	0.0008	0.000800
PF5	0.7042	3.1834	0.0743	0.0009	0.000905
PF6	0.6346	2.8201	0.0824	0.0010	0.001011
PF7	0.5869	2.6084	0.0902	0.0011	0.001116
PF8	0.5523	2.4688	0.0980	0.0012	0.001222
PF9	0.5260	2.3713	0.1056	0.0013	0.001327
PF10	0.5054	2.2993	0.1130	0.0014	0.001432
PF11	0.4888	2.2437	0.1202	0.0015	0.001538
PF12	0.4752	2.1995	0.1273	0.0016	0.001643
PF13	0.4638	2.1646	0.1342	0.0017	0.001749
PF14	0.4541	2.1358	0.1409	0.0019	0.001854
PF15	0.4458	2.1115	0.1475	0.0020	0.001960
PF16	0.0227	0.8185	0.0113	0.0001	0.000146

Panel B

Without Bitcoin	Sharpe ratio	Rachev ratio	Information ratio	Alpha	Average return
PF1	1.6911	-16.4784	0.0411	0.0005	0.000482
PF2	1.3685	-185.1128	0.0495	0.0006	0.000588
PF3	0.9866	5.9988	0.0579	0.0007	0.000693
PF4	0.7951	3.7747	0.0661	0.0008	0.000799
PF5	0.6873	3.0821	0.0742	0.0009	0.000904
PF6	0.6193	2.7424	0.0823	0.0010	0.001010
PF7	0.5727	2.5445	0.0901	0.0011	0.001115
PF8	0.5389	2.4125	0.0978	0.0012	0.001221
PF9	0.5133	2.3212	0.1054	0.0013	0.001327
PF10	0.4932	2.2516	0.1128	0.0014	0.001432
PF11	0.4770	2.1980	0.1200	0.0015	0.001538
PF12	0.4637	2.1554	0.1271	0.0016	0.001643
PF13	0.4526	2.1208	0.1339	0.0017	0.001749
PF14	0.4432	2.0921	0.1406	0.0019	0.001854
PF15	0.4351	2.0680	0.1470	0.0020	0.001960
PF16	0.0746	0.9108	0.0301	0.0004	0.000378

Panel C

Without T-Bill	Sharpe ratio	Rachev ratio	Information ratio	Alpha	Average return
PF1	0.0768	1.0025	0.0111	0.0001	0.000111
PF2	0.1542	1.0993	0.0218	0.0002	0.000243
PF3	0.1958	1.1005	0.0324	0.0004	0.000375
PF4	0.2134	1.0873	0.0427	0.0005	0.000507
PF5	0.2203	1.0909	0.0527	0.0006	0.000639
PF6	0.2228	1.0940	0.0624	0.0008	0.000771
PF7	0.2235	1.0958	0.0717	0.0009	0.000903
PF8	0.2234	1.1028	0.0806	0.0010	0.001035
PF9	0.2229	1.1076	0.0890	0.0012	0.001167
PF10	0.2223	1.1110	0.0969	0.0013	0.001299
PF11	0.2217	1.1157	0.1044	0.0014	0.001432
PF12	0.2211	1.1222	0.1113	0.0016	0.001564
PF13	0.2206	1.1284	0.1177	0.0017	0.001696
PF14	0.2200	1.1345	0.1237	0.0018	0.001828
PF15	0.2195	1.1398	0.1292	0.0020	0.001960
PF16	0.0165	0.8126	0.0086	0.0001	0.000115

Panel D

Without both	Sharpe ratio	Rachev ratio	Information ratio	Alpha	Average return
PF1	0.0761	0.9999	0.0111	0.0001	0.000110
PF2	0.1520	1.1086	0.0217	0.0002	0.000242
PF3	0.1900	1.0956	0.0323	0.0004	0.000374
PF4	0.2046	1.0648	0.0426	0.0005	0.000506
PF5	0.2098	1.0722	0.0526	0.0006	0.000638
PF6	0.2113	1.0832	0.0622	0.0008	0.000771
PF7	0.2114	1.0916	0.0714	0.0009	0.000903
PF8	0.2109	1.1015	0.0800	0.0010	0.001035
PF9	0.2103	1.1095	0.0882	0.0012	0.001167
PF10	0.2096	1.1157	0.0959	0.0013	0.001299
PF11	0.2089	1.1217	0.1030	0.0014	0.001431
PF12	0.2082	1.1267	0.1096	0.0016	0.001563
PF13	0.2076	1.1308	0.1157	0.0017	0.001696
PF14	0.2071	1.1344	0.1213	0.0018	0.001828
PF15	0.2066	1.1374	0.1264	0.0020	0.001960
PF16	0.0661	0.9012	0.0286	0.0004	0.000365