Economic Growth Centre
Economics, School of Social Sciences
Nanyang Technological University
14 Nanyang Drive
Singapore 637332

THE MEAN-VARIANCE RULE FOR INVESTORS WITH REVERSE S-SHAPED UTILITY

Wing-Keung Wong
David Yeung
Richard Lu

June 2026

EGC Report No: 2026/01

HSS-04-90A
Tel: +65 67906073
Email: D-EGC@ntu.edu.sg

Economic Growth Centre Working Paper Series

The mean-variance rule for investors with reverse S-shaped utility

Wing-Keung Wong[*]

Department of Finance, Fintech Center, and Big Data Research Center,

Asia University

Department of Medical Research, China Medical University Hospital

Department of Economics and Finance, Hang Seng University of Hong Kong

David Yeung

SRS Consortium for Advanced Study, Shue Yan University, Hong Kong

Center of Game Theory, St Petersburg State University

Department of Finance, Asia University, Taiwan

Richard Lu [†]

Department of Risk Management and Insurance, Feng Chia University

June 13, 2026

The mean-variance rule for investors with reverse S-shaped utility

Abstract:

Our paper contributes to the literature by developing the theory of the mean-variance (MV) rules for investors with reverse S-shaped utility. To do so, we first introduce the definition of the MV rule for investors with reverse S-shaped utility. We then set up the conjecture on the preference for different prospects by using the new MV rule that they could get a higher expected utility for the preferred asset under some conditions. Thereafter, we look for the conditions that the conjecture could hold and construct a theorem for this purpose by showing that when the negative (positive) parts of the assets follow one (another) type of location-scale family or the linear combination of location-scale families, then the preferences of the assets is the same as those by using an expected utility for the investors with reverse S-shaped utility. We then extend the theory by developing some properties of portfolio diversification by using the new MV rule. The theory developed in our paper enables academics and practitioners to apply the theory developed in this paper to analyze some important empirical issues and draw inferences on the preferences of investors with reverse S-shaped utility.

Keywords: Stochastic dominance, the mean-variance rules, expected-utility maximization, risk aversion, risk seeking, investment behaviors, investors with reverse S-shaped utility.

[*]The authors are grateful to Nikolai Sheung-Chi Chow for his substantive comments that have significantly improved this manuscript. The first author would like to thank Robert B. Miller and Howard E. Thompson for their continuous guidance and encouragement. This research has been partially supported by grants from Asia University, China Medical University Hospital, the Hang Seng University of Hong Kong, Shue Yan University, St Petersburg State University, Feng Chia University, the Research Grants Council of Hong Kong (project numbers UGC/IDS14/15 and 12500915), and Ministry of Science and Technology (MOST), R.O.C.

[†]Corresponding author: Email: rclu@fcu.edu.tw

JEL Classification: C00, G11

1 Introduction

Lehmann (1951, 1952, 1955), Quirk and Saposnik (1962), Hadar and Russell (1969), Hanoch and Levy (1969), and others first develop the theory of stochastic dominance (SD) for risk averters while Markowitz (1952), Jean (1975) and others compare the moments with the SD rankings for risk averters. In addition, Hammond (1974), Hershey and Schoemaker (1980), Stoyan and Daley (1983), Myagkov and Plott (1997), Li and Wong (1999), Wong and Li (1999), Anderson (2004), Post and Levy (2005), Wong (2006, 2007), Sriboonchitta, *et al.* (2009), Levy (2015), Guo and Wong (2016), Chan, *et al.* (2020), Bai, *et al.* (2022), and others develop the SD theory for risk lovers while Wong (2007), Guo and Wong (2016), Chan, *et al.* (2022), Bai, *et al.* (2022), and others compare the moments with the SD rankings for risk lovers.

Many empirical studies, see, for example, Thaler and Johnson (1990), find that many investors possess reverse S-shaped utility in their decision-making on their investment. We note that Thaler and Johnson (1990) conducted experiments and found a "house money effect such that investors could be more risk-seeking with a gain prior, implying that investors could be risk-seeking in the gain. On the other hand, they conduct experiments and find the "break-even effect" such that investors prefer to have a chance to break with a prior if losses even gamble is attractive, implying that investors are risk averse to the loss. These two effects join together to form a reverse S-shaped utility. To develop the theory for this purpose, Levy and Wiener (1998), Levy and Levy (2002, 2004)[1], Wong and Chan (2008), and

[1] We note that Wakker (2003) claims that the data of Levy and Levy (2002) supports prospect theory. We note that investors could become more risk-loving and want to make more money while others become more risk-averse because they scare that they will lose the profit they got when they gain profit. On the other hand, we believe most investors could become more risk averse but, of course, some investors, for example,

others have developed the SD theory for investors with reverse S-shaped utility. However, as far as we know, there is no study on the MV rule for investors with reverse S-shaped utility in any previous work. Thus, in this paper, we intend to develop some properties of the MV rule for investors with reverse S-shaped utility. To do so, in this paper, we first introduce the MV rule for investors with reverse S-shaped utility. We then develop some properties of the MV rule for investors with reverse S-shaped utility and extend it by developing some properties of portfolio diversification by using the MV rule for investors with reverse S-shaped utility. We end our paper with discussions of further study and applications of the theory developed in our paper.

The paper is organized as follows. We will state in the next section some definitions and notations. We will develop the theory of the MV rule for investors with reverse S-shaped utility in Section 3 and extend the theory by developing some properties of portfolio diversification of the MV rule for investors with reverse S-shaped utility in Section 4. Section 5 concludes.

2 Definitions and notations

Let $\Omega = [a, b]$ be a subset of $\overline{\mathbb{R}}$ in which $\overline{\mathbb{R}}$ be the set of extended real numbers and $a < b$. For any random variable $Z = X$ and Y with *probability density function* $h = f$ and g;

Nick Leeson and Tom Hayes, become more risk-loving because they want to get back all they have lost when they lose money. Thus, it is possible that one could be risk averse, risk-loving, and have S-shaped and reverse S-shaped utility functions. However, we believe that many investors like Nick Leeson and Tom Hayes, become more risk-loving when they lose money because they use their companies' money to invest, not their own money. So, losing more money means they could lose their jobs while making money will help them keep their jobs and get good bonuses. Thus, we believe that if those using their companies' funds to invest are not considered and only investors using their own funds to invest are considered in the comparison, then compared with investors who become more risk-averse when they lose money, the number of investors who become more risk-loving when they lose money is much less. Thus, we believe the number of investors who are risk-averse or possess reverse S-shaped utility functions is much more than those who possess S-shaped utility functions.

cumulative distribution function" (CDF) $H = F$ and G; and mean $\mu_Z = \mu_H$. The CDF $H_1(x) \equiv H(x) \equiv \mu[a, x]$ for any measure μ defined on the support $\Omega = [a, b] \subset \overline{\mathbb{R}}$ with $\mu(\Omega) = 1$. For any integer $n > 1$,

$$C_H^{(n)} = \int_a^b (x - \mu_H)^n dH(x) \ , \quad H_n(x) = \int_a^x H_{n-1}(t)\, dt \quad , \quad H_n^R(x) = \int_x^b H_{n-1}^R(t)\, dt \ . \tag{2.1}$$

We call H_n (H_n^R) the n^{th}-order (reversed) integral, see, for example, Li and Wong (1999), Wong and Li (1999), Wong (2006, 2007), Sriboonchitta, *et al.* (2009), Levy (2015), Guo and Wong (2016), Chan, *et al.* (2020, 2022), Bai, *et al.* (2022), and the references therein for more information.

In addition, we partition Z into two parts: Z^+ and Z^- with $Z = Z^+ \cup Z^-$ such that

$$Z^+ = Z \cap I_{(0,\infty)} \quad \text{and} \quad Z^- = Z \cap I_{(-\infty,0]} \ ; \tag{2.2}$$

its mean μ_H could be divided into μ_H^+ and μ_H^- with $\mu_H = \mu_H^+ + \mu_H^-$ where

$$\mu_H^+ = \int_0^\infty x dH(x) \quad \text{and} \quad \mu_H^- = \int_{-\infty}^0 x dH(x) \ , \tag{2.3}$$

in which the former is the upper first partial moment and the latter is the lower first partial moment; and its the second moment τ_H could be partitioned into τ_H^+ and τ_H^- with $\tau_H = \tau_H^+ + \tau_H^-$ where

$$\tau_H^+ = \int_0^\infty x^2 dH(x) \ , \quad \text{and} \quad \tau_H^- = \int_{-\infty}^0 x^2 dH(x) \ , \tag{2.4}$$

in which the former is the upper second partial moment and the latter is the lower second partial moment.

In this paper, we extend the MV rule (Markowitz, 1952; Wong, 2007) for both risk averters and risk lovers to the MV rule for investors with reverse S-shaped utility. To do so, we first define the following:

Definition 2.1 *Given CDFs F and G defined in Equation (2.1), for $n = 1, 2$, let U_n^{RA}, U_n^{RL}, $U^R S$, and $U^{RS}(F, G)$ be the sets of the utility functions u such that:*

$$\begin{aligned} U_n^{RA} &= \{u : (-1)^{i+1} u^{(i)} \geq 0 \,, \; i = 1, \cdots, n\} \,; \\ U_n^{RL} &= \{u : u^{(i)} \geq 0 \,, \; i = 1, \cdots, n\} \,; \\ U^{RS} &= \{u : (u^- \in U_2^A) \; \wedge \; (u^+ \in U_2^D)\} \,; \\ U^{RS}(F, G) &= \{u : (u \in U^R) \; \wedge \; (\textstyle\int u dF \textit{ and } \int u dG \textit{ are finite})\} \,. \end{aligned}$$

where $u^{(i)}$ is the i^{th} derivative of the utility function u, $u^+ = u$ ($u^- = u$) restricted for $x > 0$ ($x < 0$).

In Definition 2.1, U_n^{RA} and U_n^{RL} are sets of utility functions for n^{th}-order risk averters and risk lovers, respectively, and $U^{RS}(F, G)$ is the set of utility functions for investors with reverse S-shaped utility who are risk seeking for gains but risk averse for losses. Readers may refer to Markowitz (1952), Tobin (1958), Li and Wong (1999), Wong and Li (1999), Wong (2006, 2007), Sriboonchitta, *et al.* (2009), Levy (2015), Guo and Wong (2016), Chan, *et al.* (2020, 2022), Bai, *et al.* (2022), and the references therein for more information about risk averters and risk lovers and refer to Thaler and Johnson (1990), Levy and Wiener (1998), Levy and Levy (2002, 2004), and Wong and Chan (2008) for more information about investors with reverse S-shaped utility.

There are many empirical studies show evidence that the reverse S-shaped utility functions can be used in many studies, see, for example, Friedman and Savage (1948), Markowitz (1952), Fishburn and Kochenberger (1979), Thaler and Johnson (1990), Wu and Gonzalez (1996), Pennings and Smidts (2003), etc.

In this paper, we use the following Von Neumann-Morgenstern (1944) consistency properties to compare the preference between F and G such that F is (strictly) preferred to G, or equivalently, X is (strictly) preferred to Y if

$$\Delta Eu \equiv u(F) - u(G) \equiv u(X) - u(Y) \geq 0 (> 0), \tag{2.5}$$

where $u(F) \equiv u(X) \equiv \int_a^b udF$ and $u(G) \equiv u(Y) \equiv \int_a^b udG$.

3 Theory

In this paper, we extend the mean-variance (MV) rule for risk averters introduced by Markowitz (1952), Tobin (1958), and others, and the MV rule for risk lovers introduced by Wong (2007) and others to the MV rule for investors with reverse S-shaped utility. For any two prospects X and Y with means μ_X and μ_Y and standard deviations σ_X and σ_Y, readers could refer to Wong (2007) and others to get the definition of the MV rule for risk averters and risk lovers such that X is said to dominate Y by the MV rule for risk averters (lovers), denoted by $X\ MV_{RA}\ Y$ $(X\ MV_{RS}\ Y)$. In this paper, we define the following MV rule for investors with reverse S-shaped utility to check their preferences on assets based on the first two moments of the distributions:

Definition 3.1 *For any two prospects X and Y with means μ_X and μ_Y, the second partial moments, τ_X^+, τ_X^-, τ_Y^+, and τ_Y^-, respectively, X is said to dominate Y by the mean-variance rule for investors with reverse S-shaped utility, denoted by $X\ MV_{RSU}\ Y$, if $\mu_X \geq \mu_Y$, $\tau_X^+ \geq \tau_Y^+$, and $\tau_X^- \leq \tau_Y^-$, in which the inequality holds in at least one of the three.*

In this paper, we set up the following conjecture for the preferences of investors with reverse S-shaped utility on different prospects:

Conjecture 1 *For any two prospects X and Y with means μ_X and μ_Y, the second partial moments, τ_X^+, τ_X^-, τ_Y^+, and τ_Y^-, respectively, under some conditions, if $X\ MV_{RSU}\ Y$, then $E\ [u(X)] \geq E\ [u(Y)]$ for any investor with reverse S-shaped utility.*

One could easily construct an example so that Conjecture 1 does not hold if no additional condition is imposed. To show that Conjecture 1 could hold if we set some proper conditions, we develop the following theorem:

Theorem 3.1 *For any two prospects X and Y with means μ_X and μ_Y, the second partial moments, τ_X^+, τ_X^-, τ_Y^+, and τ_Y^-, respectively, if both X^- and Y^- belong to the same location-scale family or the same linear combination of location-scale families and both X^+ and Y^+ belong to the same location-scale family or the same linear combination of location-scale families, and if X MV_{RSU} Y, then $E\left[u(X)\right] \geq E\left[u(Y)\right]$ for any investor with reverse S-shaped utility u defined in Definition 2.1.*

Remark 3.1 *We note that in Theorem 3.1, both X^- and Y^- belong to the (first and) same location-scale family or the (first and) same linear combination of location-scale families, and both X^+ and Y^+ belong to the (second and) same location-scale family or the (second and) same linear combination of location-scale families, the second location-scale family or the second linear combination of location-scale families could be the same or different from the first linear combination of location-scale families. In another word, both X^- and Y^- and X^+ and Y^+ belong to the same location-scale family or the same linear combination of location-scale families, or X^- and Y^- could belong to one location-scale family or linear combination of location-scale families and X^+ and Y^+ belong to another different type of location-scale family or linear combination of location-scale families.*

4 Portfolio Diversification

We now extend the theory developed in Section 3 by developing some properties of portfolio diversification by using the MV rule for investors with reverse S-shaped utility. Readers may refer to Hadar and Russell (1971), Tesfatsion (1976), Li and Wong (1999), Wong (2007), Egozcue and Wong (2010), Guo and Wong (2016), Chan, *et al.* (2020, 2022), and others for some properties of portfolio diversification by the MV rule for risk averters and risk lovers. Readers may read Broll, *et al.* (2010), Egozcue, *et al.* (2011), and Ortobelli Lozza, *et al.* (2018) for some properties of portfolio diversification for investors with reverse S-shaped utility. In this paper, we extend Theorem 2' in Tesfatsion (1976) and Theorem 5 in Wong

(2007) by using Theorem 3.1 to develop the following properties for portfolio diversification to compare the preferences of two sets of assets for investors with reverse S-shaped utility:

Theorem 4.1 *Let $\{X_1, \cdots, X_m\}$ and $\{Y_1, \cdots, Y_m\}$ be two sets of independent variables, then X_i MV_{RSU} Y_i for $i = 1, \cdots, m$ if and only if $\sum_{i=1}^m \alpha_i X_i$ MV_{RSU} $Y \sum_{i=1}^m \alpha_i Y_i$ for any $\alpha_i \geq 0, i = 1, \cdots, m$.*

The proof of Theorem 4.1 is straightforward. One can easily obtain the following corollary by applying Theorem 4.1:

Corollary 4.1 *Let X and Y be random variables, if X MV_{RSU} Y, then $X + k$ MV_{RSU} $Y + k$ for any real number k.*

5 Concluding remarks

Many empirical studies, see, for example, Thaler and Johnson (1990), find that many investors possess reverse S-shaped utility in their decision-making on their investment. To develop a proper theory for this purpose, in this paper, we extend the theory of the mean-variance (MV) rule for risk averters introduced by Markowitz (1952), Tobin (1958), and others, and the MV rule for risk lovers introduced by Wong (2007) and others to the theory of the MV rule for investors with reverse S-shaped utility. To do so, we first introduce the definition of the MV rule for investors with reverse S-shaped utility. We then set up a conjecture for the preferences of investors with reverse S-shaped utility on different prospects by using the MV rule for investors with reverse S-shaped utility so that they could get a higher expected utility for the preferred asset under some conditions. We note that in general, the conjecture does not hold. However, under some conditions, this conjecture could hold. Thereafter, we look for the conditions that the conjecture could hold and construct a theorem for this purpose to show that when the negative parts of the assets follow one (say, the first) type of location-scale family or the linear combination of location-scale families and when the positive parts of the assets follow another (say, the second) type of location-scale family or the linear combination of location-scale families, then the preferences of the assets by using the MV rule is the same as those by using an expected utility for the investors with reverse S-shaped utility. We then extend the MV theory for investors with reverse S-shaped utility by developing some properties of portfolio diversification by using the MV rule for investors with reverse S-shaped utility.

There are some papers that develop the theories of the MV rule for both risk averters and risk lovers as we discussed before. However, as far as we know, there is no study on the MV rule for investors with reverse S-shaped utility in any previous work. Thus, our paper is the first paper that introduces the MV rule for investors with reverse S-shaped utility and develops some properties for the rule. The theory developed in our paper enables academics and practitioners to apply the theory developed in this paper to analyze some important empirical issues and draw inferences on the preferences of investors with reverse S-shaped utility.

Further studies of our paper could include developing more properties of the MV rule for investors with reverse S-shaped utility. For example, there are some studies have developed some tests that could be useful in the development of the test statistics for the MV rule for investors with reverse S-shaped utility, see, for example, Bai, *et al.* (2011, 2011a), but, so far, there is no study in the literature that has developed any test statistic for the MV rule for investors with reverse S-shaped utility. Thus, further studies could also include developing the test statistics for the MV rule for investors with reverse S-shaped utility. In addition, there are many applications of the theories of the MV rule for both risk averters and/or risk lovers, see, for example, Fong, *et al.* (2005, 2008), Wong, *et al.* (2006, 2008, 2018), Broll, *et al.* (2006, 2015), Gasbarro, *et al.* (2007, 2012), Lean, *et al.* (2007, 2010, 2010a, 2012, 2015), Ma and Wong (2010), Qiao, *et al.* (2010, 2012, 2013, 2015), Chan, *et al.* (2012), Hoang, *et al.* (2015, 2015a, 2018,2019), Clark, *et al.* (2016), Tsang, *et al.* (2016), Niu, *et al.* (2017, 2018), Guo, *et al.* (2017, 2018, 2019, 2019a), Bouri, *et al.* (2018), Valenzuela, *et al.* (2019), Lv, *et al.* (2021, 2022), etc. However, as far as we know, there is no study in the literature to apply the MV rule for investors with reverse S-shaped utility to analyze any important issue. Thus, further studies could apply the MV rule for investors with reverse S-shaped utility to analyze some important issues. Some studies, see, for example, Meyer, *et al.* (2005) comment that stochastic dominance (SD) is better than the mean-variance rule. Further study could examine whether SD outperforms the MV rule for investors with the reverse S-shaped utility developed in this paper.

In addition, this study develops the theory of the MV rule for investors with reverse S-shaped utility but not the theory of the MV rule for investors with S-shaped utility. We note that there are some properties that have been developed for investors with S-shaped utility, see, for example, Kahneman and Tversky (1979), Levy and Wiener (1998), Levy and Levy (2002, 2004), Post and Levy (2005), and Wong and Chan (2008), and others. Thus, academics could extend the theory developed in this paper to develop the theory of the MV rule for investors with S-shaped utility. Moreover, recently, Chan, *et al.* (2022) have developed the theory of the higher-order mean-variance rule for both risk averters and risk lovers but as far as we know, there is no study developing the theory of the higher-order MV rule for investors with (reverse) S-shaped utility. In addition, some studies, for example, Kraus and Litzenberger (1976), Scott and Horvath (1980), Astebro (2003), Cvitanić, *et al.* (2008), Choi and Nam (2008), Martellini and Ziemann (2009), Chiu (2010), Kostakis, *et al.* (2012), Lambert and Hubner (2013), Astebro, *et al.* (2015), Vo and Tran (2021), and many others have studied the effects from the higher-order moments and/or the joint effects of the moments. Thus, extensions of our paper could develop the theory of the higher-order MV rule for investors with (reverse) S-shaped utility and study the relationship with higher-order moments and/or the joint effects of different moments. Extensions of our paper could include applying the theory developed in our paper to analyze many important issues in economics

(Aor, *et al.,* 2021; Aye and Odhiambo, 2021; Levy, 2022; Obrimah and Wong, 2022; Ryu and Slottje, 2022; Wong, *et al.,* 2022), finance (Corradin and Sartore, 2021; Darsono, *et al.,* 2022; Khandelwal and Chotia, 2022; Rey, 2022; Van Der Westhuizen, *et al.,* 2022), and other areas. For instance, scholars could apply the theory developed in our paper to examine the behaviors of different investors, see, for example, Acevedo, *et al.* (2021). There are many other important issues that the theory developed in our paper could be used to analyze. Readers may refer to Wong (2020, 2021) and Moslehpour, *et al.* (2021) for more information.

Appendices

Proof of Theorem 3.1.

We only prove the necessary condition in this paper. To do so, we let two prospects X and Y with means μ_X and μ_Y, the second partial moments, τ_X^+, τ_X^-, τ_Y^+, and τ_Y^-, respectively. We assume that $\mu_X \geq \mu_Y$, $\tau_X^+ \geq \tau_Y^+$, and $\tau_X^- \leq \tau_Y^-$ so that we have $X \ MV_{RSU} \ Y$.

Now, we assume X^* and Y^* with means μ_{X^*} and μ_{Y^*}, the second partial moments, $\tau_{X^*}^+$, $\tau_{X^*}^-$, $\tau_{Y^*}^+$, and $\tau_{Y^*}^-$, respectively, such that $\mu_{X^*} = \mu_X$, $\mu_{Y^*} = \mu_Y$, $\tau_{X^*}^- = \tau_X^-$, $\tau_{Y^*}^- = \tau_Y^-$, $\tau_{X^*}^+ < \tau_{X^*}^+$, and X^* and Y^* belong to the same location-scale family or the same linear combination of location-scale families as X^- and Y^- belong, then, we have $\mu_{X^*} < \mu_{Y^*}$ and $\tau_{X^*} = \tau_{X^*}^+ + \tau_{X^*}^- < \tau_{Y^*} = \tau_{Y^*}^+ + \tau_{Y^*}^-$, and thus, $Var(X^*) < Var(Y^*)$.

Thereafter, applying Theorem 8 in Li and Wong (1999), we have

$E\ [u(X^*)] \ \geq \ E\ [u(Y^*)]$ for any $u \in U_2^{RA}$ defined in Definition 2.1, which implies that $\int_a^x F_{X^*}^A(t)dt \leq \int_a^x F_{Y^*}^A(t)dt$ for any $x \in [a,b]$. Thus, we have

$$\int_a^x F_X^A(t)dt \leq \int_a^x F_Y^A(t)dt \quad \text{for any} \quad x \in [a,0]. \tag{A.1}$$

By using a similar argument, one can easily get

$$\int_x^b F_X^D(t)dt \geq \int_x^b F_Y^D(t)dt \quad \text{for any} \quad x \in [0,b]. \tag{A.2}$$

By Equations (A.2) and (A.2), by Definition 3.1, one can conclude that $E\ [u(X)] \geq E\ [u(Y)]$ for any investor with the utility function $u \in U^{RS}$ and the assertions in Theorem 3.1 hold. $\square$

References

Acevedo, R. A., Aponte, E., Harmath, P., Mora Mora, J. (2021). Rational irrationality: A two stage decision making model. Advances in Decision Science, 25(1).

Anderson, G. (2004). Toward an empirical analysis of polarization. Journal of Econometrics, 122(1), 1-26.

Aor, R. L., Salisu, A. A., Okpe, I. J. (2021). A comparative assessment of the global effects of US monetary and fiscal policy uncertainty shocks. Advances in Decision Sciences, 25(4), 1-25.

Astebro, T. (2003). The return to independent invention: evidence of unrealistic optimism, risk seeking or skewness loving?. Economic Journal 113(484), 226-239.

Astebro, T., Mata, J., & Santos-Pinto, L. (2015). Skewness seeking: risk loving, optimism or overweighting of small probabilities?. Theory and Decision 78, 189-208.

Aye, G. C., Odhiambo, N. M. (2021). Threshold effect of inflation on agricultural growth: evidence from developing countries. Advances in Decision Sciences, 25(2), 1-22.

Bai, Z.D., Guo, X., Li, H., & Wong, W.K. (2022). Stochastic Dominance with Applications in Economics, Finance, and Income Inequality, World Scientific Publishing, forthcoming.

Bai, Z.D., Li, H., Liu, H.X., Wong, W.K., 2011. Test statistics for prospect and Markowitz stochastic dominances with applications. Econometrics Journal, 122, 1-26.

Bai, Z.D., Wang, K.Y., Wong, W.K., 2011. Mean-variance ratio test, a complement to coefficient of variation test and Sharpe ratio test. Statistics and Probability Letters, 81(8), 1078-1085.

Barrett, G. F., & Donald, S. G. (2003). Consistent tests for stochastic dominance. Econometrica, 71(1), 71-104.

Bawa, V.S., Bodurtha Jr.J.N., Rao, M.R., & Suri, H.L. (1985). On determination of stochastic dominance optimal sets. Journal of Finance, 40(2), 417-431.

Borch, K. (1979). Utility and stochastic dominance. In Expected utility hypotheses and the Allais paradox (pp. 193-201). Springer, Dordrecht.

Bouri, E., Gupta, R., Wong, W.K., Zhu, Z.Z. 2018. Is Wine a Good Choice for Investment? Pacific-Basin Finance Journal 51, 171-183.

Brockett, P.L., & Garven, J.R. (1998). A Reexamination of the Relationship Between Preferences and Moment Orderings by Rational Risk-Averse Investors, GENEVA Papers on Risk and Insurance Theory, 23(2), 127-137.

Broll, U., Egozcue, M., Wong, W.K., Zitikis, R., 2010. Prospect theory, indifference curves, and hedging risks. Applied Mathematics Research Express, 2010(2), 142-153.

Broll, U., Guo, X., Welzel, P., Wong, W.K., 2015. The banking firm and risk taking in a two-moment decision model. Economic Modelling, 50, 275-280.

Broll, U., Wahl, J.E., Wong, W.K., 2006. Elasticity of risk aversion and international trade. Economics Letters, 91(1), 126-130.

Buckle, M., Chen, J., & Williams, J. M. (2016). Realised higher moments: Theory and practice. European Journal of Finance 22(13), 1272-1291.

Chan, C.-Y., de Peretti, C., Qiao, Z. Wong, W.K., 2012. Empirical test of the efficiency of the UK covered warrants market: Stochastic dominance and likelihood ratio test approach, Journal of Empirical Finance, 19(1), 162-174.

Chan, R. H., Chow, S. C., Guo, X., & Wong, W. K. (2022). Central moments, stochastic dominance, moment rule, and diversification with an application. Chaos, Solitons &

Fractals, 161, 112251.

Chan, R.H., Clark, E., Guo, X., & Wong, W.K. (2020). New Development on the Third Order Stochastic Dominance for Risk-Averse and Risk-Seeking Investors with Application in Risk Management, Risk Management 22, 108-132.

Charness, G., Karni, E., & Levin, D. (2007). Individual and group decision making under risk: An experimental study of Bayesian updating and violations of first-order stochastic dominance. Journal of Risk and uncertainty, 35(2), 129-148.

Chiu, W. H. (2010). Skewness Preference, Risk Taking and Expected Utility Maximisation, Geneva Risk and Insurance Review 35(2), 108-129.

Choi, P., & Nam, K. (2008). Asymmetric and leptokurtic distribution for heteroscedastic asset returns: The SU-normal distribution. Journal of Empirical Finance,15(1), 41-63.

Clark, E., Qiao, Z., & Wong, W. K. (2016). Theories of risk: Testing investor behavior on the taiwan stock and stock index futures markets. Economic Inquiry, 54(2), 907-924.

Corradin, F., Sartore, D. (2021). Non-Central Moments Of The Truncated Normal Variable In Finance. Annals of Financial Economics, 16(04), 2150017.

Cvitanić, J., Polimenis, V., & Zapatero, F. (2008). Optimal portfolio allocation with higher moments. Annals of Finance, 4(1), 1-28.

Darsono, S. N. A. C., Wong, W. K., Nguyen, T. T. H., Jati, H. F., Dewanti, D. S. (2022). Good Governance and Sustainable Investment: The Effects of Governance Indicators on Stock Market Returns. Advances in Decision Sciences, 26(1), 69-101.

Denuit, M., Eeckhoudt, L., Tsetlin, I., & Winkler, R.L. (2013). Multivariate concave and convex stochastic dominance. Biagini, F., Richter, A., Schlesinger, H., eds. Risk Measures and Attitudes (Springer, London), 11-32).

Do, H.X., Brooks, R., Treepongkaruna, S., & Wu, E. (2016). Stock and currency market linkages: New evidence from realized spillovers in higher moments. International Review of Economics Finance 42, 167-185.

Egozcue, M., Fuentes García, L., Wong, W.K., & Zitikis, R. (2011). Do Investors Like to Diversify? A Study of Markowitz Preferences, European Journal of Operational Research 215(1), 188-193.

Egozcue, M., & Wong, W.K. (2010). Gains from diversification: A majorization and stochastic dominance approach. European Journal of Operational Research, 200(3), 893-900.

Ekern, S. (1980). Increasing Nth degree risk, Economics Letters 6(4), 329-333.

Fang, Y., & Post, T. (2017). Higher-degree stochastic dominance optimality and efficiency. European Journal of Operational Research, 261(3), 984-993.

Fang, Y., & Post, T. (2022). Optimal portfolio choice for higher-order risk averters. Journal of Banking & Finance, 106429.

Fishburn, P. C. (1980). Stochastic dominance and moments of distributions. Mathematics of Operations Research, 5(1), 94-100.

Fishburn, P. C., & Kochenberger, G. A. (1979). Two-piece von Neumann-Morgenstern utility functions. Decision Sciences, 10(4), 503-518.

Fong, W.M., H.H. Lean, W.K. Wong, 2008. Stochastic dominance and behavior towards risk: The market for internet stocks. Journal of Economic Behavior and Organization, 68(1), 194-208.

Fong, W.M., Wong, W.K., Lean, H.H., 2005. International momentum strategies: A stochastic dominance approach. Journal of Financial Markets, 8, 89-109.

Friedman, M., & Savage, L. J. (1948). The utility analysis of choices involving risk. Journal of political Economy, 56(4), 279-304.

Gasbarro, D., Wong, W.K., Zumwalt, J.K., 2007. Stochastic dominance analysis of iShares. European Journal of Finance, 13, 89-101.

Gasbarro, D., Wong, W.K., Zumwalt, J.K., 2012. Stochastic dominance and behavior towards risk: The market for iShares. Annals of Financial Economics, 7(1), 1250005-1-20.

Goeman, J. J. and Solari, A. (2014). Multiple hypothesis testing in genomics. Statistics in

Medicine, 33(11):1946-1978.

Grigoletto, M., & Lisi, F. (2011). Practical implications of higher moments in risk management. Statistical Methods and Applications,20(4), 487-506.

Guo, X., Chan, R.H., Wong, W.K., Zhu, L.X., 2019, Mean-Variance, Mean-VaR, Mean-CvaR Models for Portfolio Selection with Background Risk, Risk Management, 21: 73-98.

Guo, X., Jiang, X.J., Wong, W.K., 2017, Stochastic Dominance and Omega Ratio: Measures to Examine Market Efficiency, Arbitrage Opportunity, and Anomaly, Economies, 5, no. 4: 38.

Guo, X., Niu, C.Z., Wong, W.K. 2019, Farinelli and Tibiletti Ratio and Stochastic Dominance, Risk Management, 21(3), 201-213.

Guo, X., Wagener, A., Wong, W.K., 2018. The Two-Moment Decision Model with Additive Risks, Risk Management 20(1), 77-94.

Guo, X., & Wong, W.K. (2016). Multivariate Stochastic Dominance for Risk Averters and Risk Seekers, RAIRO - Operations Research 50(3), 575-586.

Hadar, J., & Russell, W.R. (1969). Rules for Ordering Uncertain Prospects, American Economic Review 59, 25-34.

Hadar, J., & Russell, W.R. (1971). Stochastic Dominance and Diversification. Journal of Economic Theory 3(3), 288-305.

Hammond III, J. S. (1974). Simplifying the choice between uncertain prospects where preference is nonlinear. Management Science, 20(7), 1047-1072.

Hanoch G., & Levy, H. (1969). The Efficiency Analysis of Choices Involving Risk. Review of Economic studies 36, 335-346.

Hardy, G.H., Littlewood, J.E., & Pólya, G. (1934). Inequalities, Cambridge University Press, Cambridge, MA.

Harvey, C.R., Liechty, J.C., Liechty, M.W., & Müller, P. (2010). Portfolio selection with higher moments. Quantitative Finance 10(5), 469-485.

Hershey, J.C., Schoemaker, P.J.H., 1980. Risk taking and problem context in the domain of losses: an expected utility analysis. Journal of Risk and Insurance 47(1), 111-132.

Hoang, T.H.V., Lean, H.H., Wong, W.K., (2015). Is gold good for portfolio diversification? A stochastic dominance analysis of the Paris stock exchange, International Review of Financial Analysis, 42, 98-108.

Hoang, V.T.H., Wong, W.K., Zhu, Z.Z., 2015a. Is gold different for risk-averse and risk-seeking investors? An empirical analysis of the Shanghai Gold Exchange. Economic Modelling, 50, 200-211.

Hoang, V.T.H., Zhu, Z.Z., Khamlichi, A.E., Wong, W.K. 2019, Does the Shari'ah Screening Impact the Gold-Stock Nexus? A Sectorial Analysis, Resources Policy 61, 617-626.

Hoang, V.T.H., Zhu, Z.Z., Xiao, B., Wong, W.K. 2018, The seasonality of gold prices in China: Does the risk-aversion level matter? Accounting and Finance 60 (3), 2617-2664.

Jean, W.H. (1975). Comparison of moment and stochastic dominance ranking methods, Journal of Financial and Quantitative Analysis 10(1), 151-161.

Jondeau, E., & Rockinger, M. (2003). Conditional volatility, skewness, and kurtosis: Existence, persistence, and comovements. Journal of Economic dynamics and Control 27(10), 1699-1737.

Jondeau, E., & Rockinger, M. (2006). Optimal portfolio allocation under higher moments. European Financial Management,12(1), 29-55.

Kallio, M., Hardoroudi, N. D. (2019). Advancements in stochastic dominance efficiency tests. European Journal of Operational Research, 276(2), 790-794.

Kostakis, A., & Muhammad, K., Siganos, A. (2012). Higher co-moments and asset pricing on London stock exchange. Journal of Banking Finance 36, 913-922.

Kraus, A., & Litzenberger, R. (1976). Skewness preference and the valuation of risk assets. The Journal of Finance,31, 1085-1100.

Kahneman, D., & Tversky, A. (1979). Prospect Theory: An Analysis of Decision under

Risk. Econometrica, 47(2), 263-292.

Khandelwal, V., Chotia, V. (2022). Is there a Beta Anomaly? – Evidence from the India. Annals of Financial Economics, 2250020.

Lambert, M., & Hubner, G. (2013). Comoment risk and stock returns. Journal of Empirical Finance 23, 191-205.

Lean, H.H., Lien, D., Wong, W.K. (2010), Preferences of Futures or Stocks? A Stochastic Dominance Study in Malaysian Markets, Advances in Investment Analysis and Portfolio Management 4, 49-80.

Lean, H.H., McAleer, M., Wong, W.K., 2010a. Market efficiency of oil spot and futures: A mean-variance and stochastic dominance approach. Energy Economics, 32, 979-986.

Lean, H.H., McAleer, M., Wong, W.K., 2015. Preferences of risk-averse and risk-seeking investors for oil spot and futures before, during and after the Global Financial Crisis. International Review of Economics and Finance, 40, 204-216.

Lean, H.H., Phoon, K.F., Wong, W.K. 2012. Stochastic dominance analysis of CTA funds. Review of Quantitative Finance and Accounting, 40(1), 155-170.

Lean, H.H., Smyth, R. Wong, W.K., 2007. Revisiting calendar anomalies in Asian stock markets using a stochastic dominance approach. Journal of Multinational Financial Management, 17(2), 125-141.

Lehmann, E.L. (1951). Consistency and Unbiasedness of Certain Nonparametric Tests, Annals of Mathematical Statistics 22(2), 165-179.

Lehmann, E.L. (1952). Testing Multiparameter Hypotheses, Annals of Mathematical Statistics 23(4), 541-552.

Lehmann, E.L. (1955). Ordered Families of Distributions, Annals of Mathematical Statistics 26(3), 399-419.

Levy, H. (2015). Stochastic dominance: Investment decision making under uncertainty. Third Edition, Springer, New York.

Levy, H. (2022). Risk-Aversion: Mathematical And Economic Perspectives. Annals of Financial Economics (AFE), 17(02), 1-23.

Levy, H., & Levy, M. (2004). Prospect Theory and Mean-Variance Analysis. The Review of Financial Studies, 17(4), 1015-1041.

Levy, H., & Wiener, Z. (1998). Stochastic dominance and prospect dominance with subjective weighting functions. Journal of Risk and Uncertainty, 16(2), 147-163.

Levy, M., & Levy, H. (2002). Prospect Theory: Much Ado About Nothing? Management Science, 48(10), 1334-1349.

Li, C.K., & Wong, W.K. (1999). Extension of stochastic dominance theory to random variables. RAIRO - Operations Research 33(4), 509-524.

Lim, K. (1989). A new test of the three-moment capital asset pricing model. Journal of Financial and Quantitative Analysis,24, 205-216.

Lv, Z., Chu, A. M., Wong, W. K., & Chiang, T. C. (2021). The maximum-return-and-minimum-volatility effect: evidence from choosing risky and riskless assets to form a portfolio. Risk Management, 23(1), 97-122.

Lv, Z.H., Tsang, C.-K., Wagner, N.F., Wong, W.K. 2022. What is an Optimal Allocation in Hong Kong Stock, Real Estate, and Money Markets: An Individual Asset, Efficient Frontier Portfolios, or a Na ive Portfolio? Emerging Markets Finance and Trade, forthcoming.

Ma, C., & Wong, W.K. (2010). Stochastic dominance and risk measure: A decision-theoretic foundation for VaR and C-VaR. European Journal of Operational Research, 207(2), 927-935.

Markowitz, H. (1952). The utility of wealth. Journal of political Economy, 60(2), 151-158.

Martellini, L., & Ziemann, V. (2009). Improved estimates of higher-order comoments and implications for portfolio selection. Review of Financial Studies 23(4), 1467-1502.

Meyer, T.O., Li, X.M., & Rose, L.C. (2005). Comparing mean variance tests with stochastic dominance tests when assessing international portfolio diversification benefits. Financial Services Review 14(2), 149-168.

Moslehpour, M., Pan, S. H., Tiwari, A. K., Wong, W. K. (2021). Editorial in Honour of Professor Michael McAleer. Advances in Decision Sciences, 25(4), 1-14.

Muliere, P., & Scarsini, M. (1989). A note on stochastic dominance and inequality measures? Journal of Economic Theory 49(2), 314-323.

Myagkov, M., & Plott, C. R. (1997). Exchange economies and loss exposure: experiments exploring prospect theory and competitive equilibria in market environments. American Economic Review 87(5), 801-828.

Niu, C.Z., Guo, X., McAleer, M., Wong, W.K. 2018. Theory and Application of an Economic Performance Measure of Risk, International Review of Economics & Finance 56, 383-396.

Niu, C.Z., Wong, W.K., Xu, Q.F. 2017, Kappa Ratios and (Higher-Order) Stochastic Dominance, Risk Management 19(3), 245-253.

Obrimah, O.A., Wong, W.K. 2022. Modeling of stock returns in continuous vis-á-vis discrete time is equivalent, respectively, to the conditioning of stock returns on a random walk process for trade imbalances vis-?-vis a random walk process for evolution of information. Annals of Financial Economics 17(02), 2250010.

Ogryczak, W., & Ruszczyński, A. (1999). From stochastic dominance to mean-risk models: Semideviations as risk measures. European Journal of Operational Research 116(1), 33-50.

Ortobelli Lozza, S., Wong, W. K., Fabozzi, F. J., & Egozcue, M. (2018). Diversification versus optimality: is there really a diversification puzzle?. Applied Economics, 50(43), 4671-4693.

Pennings, J.M., & Smidts, A. (2003). The shape of utility functions and organizational behavior. Management Science, 49(9), 1251-1263.

Perez-Quiros, G., & Timmermann, A. (2001). Business cycle asymmetries in stock returns:

Evidence from higher order moments and conditional densities. Journal of Econometrics 103(1), 259-306.

Post, T. (2003). Empirical tests for stochastic dominance efficiency. Journal of Finance, 58(5), 1905-1931.

Post, T., & Kopa, M. (2017). Portfolio choice based on third-degree stochastic dominance. Management Science, 63(10), 3381-3392.

Post, T., & Kopa, M. (2013). General linear formulations of stochastic dominance criteria. European Journal of Operational Research, 230(2), 321-332.

Post, T., & Levy, H. (2005). Does risk seeking drive stock prices? A stochastic dominance analysis of aggregate investor preferences and beliefs. Review of Financial Studies, 18(3), 925-953.

Post, T., & Versijp, P. (2007). Multivariate tests for stochastic dominance efficiency of a given portfolio. Journal of Financial and Quantitative Analysis, 42(2), 489-515.

Qiao, Z., Clark, E., & Wong, W. K. (2014). Investors preference towards risk: Evidence from the Taiwan stock and stock index futures markets. Accounting & Finance, 54(1), 251-274.

Qiao, Z., Qiao, W.W., Wong, W.K., 2010, Examining the Day-of-the-Week Effects in Chinese Stock Markets: New Evidence from a Stochastic Dominance Approach, Global Economic Review 39(3), 225-246.

Qiao, Z., Wong, W.K., 2015. Which is a better investment choice in the Hong Kong residential property market: A big or small property? Applied Economics, 47(16), 1670-1685.

Qiao, Z., Wong, W.K., Fung, J.K.W., 2013. Stochastic dominance relationships between stock and stock index futures markets: International evidence. Economic Modelling, 33, 552-559.

Quirk, J. P., & Saposnik, R. (1962). Admissibility and measurable utility functions. The Review of Economic Studies, 29(2), 140-146.

Rey, S. A. (2022). A Linkage between the Financial and the Real Economy. Annals of Financial Economics, 17(03), 2250021.

Ritchken, P., & Kuo, S. (1989). On stochastic dominance and decreasing absolute risk averse option pricing bounds. Management Science, 35(1), 51-59.

Rothschild, M., & Stiglitz, J. E. (1970). Increasing risk: I. A definition. Journal of Economic theory, 2(3), 225-243.

Ryu, H. K., Slottje, D. J. (2022). Ten Ways to Specify a Gini Coefficient Using Entropy. Annals of Financial Economics, 2140001.

Samuelson, P.A., 1967. General Proof that Diversification Pays. Journal of Financial and Quantitative Analysis 2(1), 1-13.

Scott, R. C., & Horvath, P. A. (1980). On the direction of preference for moments of higher order than the variance. Journal of Finance 35(4), 915-919.

Slovic, P. (1964). Assessment of risk taking behavior. Psychological Bulletin, 61(3), 220-233.

Sriboonchitta, S., W.K. Wong, S. Dhompongsa, H.T. Nguyen, 2009. Stochastic Dominance and Applications to Finance, Risk and Economics, Chapman and Hall/CRC, Taylor and Francis, Boca Raton, Florida, USA.

Stoyan, D., & Daley, D. J. (1983). Comparison Methods for Queues and Other Stochastic Models. JOHN WILEY & SONS, INC., NEW YORK.

Tehranian, H. (1980). Empirical Studies in Portfolio Performance Using Higher Degrees of Stochastic Dominance, Journal of Finance 35(1), 159-171.

Tesfatsion, L. (1976). Stochastic Dominance and Maximization of Expected Utility. Review of Economic Studies 43, 301-315.

Thaler, R. H., Johnson, E. J. (1990). Gambling with the house money and trying to break even: The effects of prior outcomes on risky choice. Management Science, 36(6), 643-660.

Tobin, J. (1958). Liquidity Preference as Behavior towards Risk. Review of Economic Studies 25,65-86.

Tsang, C. K., Wong, W. K., & Horowitz, I. (2016). Arbitrage opportunities, efficiency, and the role of risk preferences in the Hong Kong property market. Studies in Economics and Finance 33(4), 735-754.

Valenzuela, M.R., Wong, W.K., Zhu, Z.Z. 2019, Sources of inequality in the Philippines: Insights from stochastic dominance tests for richness and poorness, World Economy, https://doi.org/10.1111/twec.12772

Van Der Westhuizen, C., Van Eyden, R., Aye, G. (2022). Contagion across financial markets during COVID-19: A look at volatility spillovers between the stock and foreign exchange markets in South Africa. Annals of Financial Economics, 17(1), 2250002.

Vinod, H. D. (2004). Ranking mutual funds using unconventional utility theory and stochastic dominance. Journal of Empirical Finance, 11(3), 353-377.

Vo, X.V., & Tran, T.T.A. (2021). Higher-order comoments and asset returns: evidence from emerging equity markets. Annals of Operations Research, 297(1), 323-340.

von Neumann, J., & Morgenstern, O. (1944). Theory of Games and Economic Behavior. Princeton University Press, Princeton N.J.

Wakker, P. P. (2003). The data of Levy and Levy (2002)Prospect theory: Much ado about nothing? actually support prospect theory. Management Science, 49(7), 979-981.

Wong, W.K. (2006), Stochastic Dominance Theory for Location-Scale Family. Journal of Applied Mathematics and Decision Sciences, now Advances in Decision Sciences 2006, 1-10.

Wong, W.K. (2007). Stochastic dominance and mean-variance measures of profit and loss for business planning and investment. European Journal of Operational Research, 182(2), 829-843.

Wong, W. K. (2020). Review on behavioral economics and behavioral finance. Studies in Economics and Finance. 37(4), 625-672. https://doi.org/10.1108/SEF-10-2019-0393

Wong, W.K. 2021. Editorial Statement and Research Ideas for Behavioral Financial Economics in Emerging Market. International Journal of Emerging Markets 16(5), 946-951. https://doi.org/10.1108/IJOEM-07-2021-991.

Wong, W.K., Broll, U., Qiao, Z., Ma, C. 2022. New stochastic dominance theory for investors with risk-averse and risk-seeking utilities with applications. Risk Management, forthcoming.

Wong, W.K., & Chan, R. (2008). Prospect and Markowitz Stochastic Dominance. Annals of Finance, 4(1), 105-129.

Wong, W.K., & Li., C.K. (1999). A note on convex stochastic dominance. Economics Letters 62, 293-300.

Wong, W.K., & Ma, C. (2008). Preferences over Location-Scale Family, Economic Theory 37(1), 119-146.

Wong, W. K., Phoon, K. F., & Lean, H. H. (2008). Stochastic dominance analysis of Asian hedge funds. Pacific-Basin finance journal, 16(3), 204-223.

Wong, W.K., H.E. Thompson, S. Wei, Y.F. Chow (2006). Do Winners perform better than Losers? A Stochastic Dominance Approach, Advances in Quantitative Analysis of Finance and Accounting, 4, 219-254.

Wu, G., & Gonzalez, R. (1996). Curvature of the probability weighting function. Management science, 42(12), 1676-1690.

Yalonetzky, G. (2013). Stochastic dominance with ordinal variables: Conditions and a test. Econometric Reviews, 32(1), 126-163.