Annexe A: New/Revised Course Content in OBTL+ Format

Course Overview

Expected Implementation in Academic Year (New format)	AY2026-2027
Semester/Trimester/Others (specify approx. Start/End date)	Semester 2
Course Author * Faculty proposing/revising the course	Chew Lock Yue
Course Author Email	lockyue@ntu.edu.sg
Course Title	Dynamical System Theory with Chaos and Fractals
Course Code	PH3502/MH3320
Academic Units	4
Contact Hours	52
Research Experience Components	Not Applicable

Course Requisites (if applicable)

Pre-requisites	MH1803 and MH2802 Or MH1201 Linear Algebra II and MH2100 Calculus III (for PHY); MH1201 Linear Algebra II and MH2100 Calculus III (for MAS)
Co-requisites	
Pre-requisite to	
Mutually exclusive to	MH3320/PH3502, PH7011
Replacement course to	
Remarks (if any)	

Course Aims

This course aims to equip you with the basic concepts of determinism and randomness in the physical world. You will develop a basic understanding of dynamical system theory which is an essential component in physics, engineering, chemistry, biology, finance, and also the social sciences. You will also gain basic computational and analytical skills to solve and understand real-world problems involving chaotic and non-linear systems.

Course's Intended Learning Outcomes (ILOs)

Upon the successful completion of this course, you (student) would be able to:

ILO 1	Mathematical Formalism (MAT): analyze and solve problems in linear system and linear dynamical system in N-dimensions mathematically and computationally
ILO 2	Mathematical Formalism (MAT): analyze and solve problems in nonlinear system and nonlinear dynamical system in N-dimensions mathematically and computationally
ILO 3	Mathematical Formalism (MAT): formulate first-order differential equations to model the evolution of diverse continuous-time dynamical phenomena and solve the equations mathematically and computationally
ILO 4	Mathematical Formalism (MAT): formulate recurrence equations to model the evolution of diverse discrete-time dynamical phenomena and solve the equations mathematically and computationally
ILO 5	Mathematical Formalism (MAT): Perform geometric analysis on the phase portrait of linear and nonlinear dynamical systems.
ILO 6	Mathematical Formalism (MAT): Perform mathematical definition and analysis of self-similar and fractal sets
ILO 7	Dynamical System Theory (DYN): determine the fixed points, limit cycles/periodic orbits, strange and non-strange attractors of the dynamical system under-study analytically and numerically
ILO 8	Dynamical System Theory (DYN): determine the stability properties of the fixed points and limit cycles/periodic orbits of the dynamical system under-study analytically and numerically
ILO 9	Dynamical System Theory (DYN): gain the concept of stable manifold and unstable manifold as geometric structures in phase space that guide the flow of the dynamical trajectories
ILO 10	Dynamical System Theory (DYN): master the use of the phase portrait (as a geometric picture of phase space) that contains the set of fixed points, limit cycles, strange attractors, stable and unstable manifolds, as the solution to dynamical system problems
ILO 11	Dynamical System Theory (DYN): analyze dynamical systems that are chaotic, compute its trajectories, and yield the level of chaos by evaluating its Lyapunov exponents numerically or analytically
ILO 12	Dynamical System Theory (DYN): account for the different type of bifurcations that occur in nonlinear dynamical systems
ILO 13	Fractals (FRA): explain and describe the concepts of countable and uncountable sets
ILO 14	Fractals (FRA): describe and distinguish sets that have fractional dimension
ILO 15	Fractals (FRA): construct and analyze fractal sets that are self-similar and non-self-similar
ILO 16	Fractals (FRA): determine diverse fractal dimensions of fractal sets that are self-similar and non-self-similar

Course Content

The course consists of the following topics:

Introduction

- A dynamical view of the world
- What is nonlinear dynamics?

Linear Dynamical System

- Examples
- General formulation

Ingredients of a Dynamical System

- Phase space
- Evolution equations
- Initial conditions

Stability Properties of Linear Dynamical System

- Two-dimensional linear dynamical system
- N-dimensional linear dynamical system

Phase Portraits from the Stability Properties of Manifolds of Fixed Points in Continuous-Time Nonlinear Dynamical System

- Phase portraits
- Existence and uniqueness, no-intersection theorem
- Stability properties of fixed points in nonlinear dynamical systems
- Stable and unstable manifolds

Bifurcations

- Saddle-node bifurcation
- Transcritical bifurcation
- Pitchfork bifurcation
- Hopf bifurcation
- Global bifurcation of cycles

Lorenz Equations

- Linear stability of Poincaŕe orbits via Poincaŕe map
- Homoclinic and Heteroclinic orbits

- Bifurcations, chaos, and strange attractors

Nonlinear Mapping and their Dynamical Properties

- Fixed points and cobwebs
- Periodic points
- Logistic map
- The fully chaotic logistic map at A = 4
- Symbolic dynamics and the Bernoulli shift map

Fractals

- Countable and uncountable sets
- Cantor set
- Dimension of self-similar fractals
- Box dimension
- Hausdorff dimension

Reading and References (if applicable)

1. Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering, 2nd Edition, Steven Strogatz (2014). ISBN: 978-0738204536

NOTE: The above listing comprises the foundational readings for the course and more up-to-date relevant readings will be provided when they become available.

Planned Schedule

Week or Session	Topics or Themes	ILO	Delivery Mode	Activities	Readings
1	Introduction	MAT1	In-person	Lecture	
2	Linear Dynamical System and Ingredients of Dynamical System,	MAT1	In-person	Lecture, Tutorial	
3	Stabilities of Linear Dynamical System	MAT1, MAT3, MAT5, DYN 7-8	In-person	Lecture, Tutorial	
4	Phase Portraits	MAT 2-3, MAT5, DYN 7-8	In-person	Lecture, Tutorial	
5	Bifurcations	DYN 12	In-person	Lecture, Tutorial	
6	Poincaŕe map, Homoclinic and Heteroclinic orbits	DYN 9	In-person	Lecture, Tutorial	
7	Lorenz equations	MAT 2-3, 5, DYN 9-11	In-person	Lecture, Tutorial	
8	Lorenz equations	MAT 2-3, 5, DYN 9-11	In-person	Lecture, Tutorial	
9	Nonlinear Mappings	MAT 2, 4-5, DYN 7-8, 10-11	In-person	Lecture, Tutorial	
10	Nonlinear Mappings	MAT 2, 4-5, DYN 7-8, 10-11	In-person	Lecture, Tutorial	
11	Fractals	MAT6, FRA 13-14	In-person	Lecture, Tutorial	
12	Fractals, Revision	MAT6, FRA 15-16	In-person	Lecture, Tutorial	
13	Project Presentation	All	In-person	Project	

Learning and Teaching Approach

Approach	How does this approach support you in achieving the learning outcomes?
Problem formulation and solution (lecture and tutorial)	Develop your competence in mathematical formulation in diverse inter-disciplinary contexts and the application of dynamical system theory to analyze and solve mathematical problems
Problem sets (tutorial)	Apply the theory and mathematical formulation learnt in class to solve problems in dynamical systems which develops your understanding, competence, and intuition on the topic, as well as attaining both analytical and computational skills at the same time.
Projects (lecture and tutorial)	Sharpen your knowledge in dynamical system theory through creatively working on a project in a team of two/three persons. The project enhances your analytical and computational skillsets as you work to deliver the requirements of the project. Furthermore, it trains you on your presentation and communication skills through project presentation and the answering of probing questions from peers and seniors during the question-and-answer session.

Assessment Structure

Assessment Components (includes both continuous and summative assessment)

No.	Component	ILO	Related PLO or Accreditation	Weightage	Description of Assessment Component	Team/Individual	Rubrics	Level of Understanding
1	Summative Assessment (EXAM): Final exam(Final Examination)	All		50		Individual	Analytic	Multistructural
2	Continuous Assessment (CA): Assignment(Continuous Assessment 1 (CA1): Problem Sets)	All		10		Individual	Analytic	Multistructural
3	Continuous Assessment (CA): Project(CA2: Project)	All		40		Team	Holistic	Relational

Description of Assessment Components (if applicable)

For CA2 on project, the rubric based assessment is done on individual student while taking into account the teamwork and collaboration between the students who work in a team of 2 or at most 3. Each student in the team needs to indicate and submit their role in the project, perform a peer evaluation, and the rubric based marking is done on each student and not on each team.

Formative Feedback

You will receive formative feedback which will be given through discussion within tutorial lessons.

Feedback will also be provided for each marked problem set, where any particularly problematic areas will be identified in the marked scripts.

Finally, feedback will be given constantly during lectures and tutorials on the common mistakes and level of difficulty of the course materials and applied examples/problems. Past exam questions and examiner's report are also made available for you, and will be discussed near the end of the course.

NTU Critical Core Skills+ Mapping

This course intends to develop the following Critical Core Skills+ (maximum 5 most relevant)

Critical Core Skills+	Level
Creative Thinking	Advanced
Problem Solving	Intermediate

Course Policy

Policy (Academic Integrity)

Good academic work depends on honesty and ethical behaviour. The quality of your work as a student relies on adhering to the principles of academic integrity and to the NTU Honour Code, a set of values shared by the whole university community. Truth, Trust and Justice are at the core of NTU's shared values. As a student, it is important that you recognize your responsibilities in understanding and applying the principles of academic integrity in all the work you do at NTU. Not knowing what is involved in maintaining academic integrity does not excuse academic dishonesty. You need to actively equip yourself with strategies to avoid all forms of academic dishonesty, including plagiarism, academic fraud, collusion and cheating. If you are uncertain of the definitions of any of these terms, you should go to the academic integrity website for more information. On the use of technological tools (such as Generative AI tools), different courses / assignments have different intended learning outcomes. Students should refer to the specific assignment instructions on their use and requirements and/or consult your instructors on how you can use these tools to help your learning. Consult your instructor(s) if you need any clarification about the requirements of academic integrity in the course.

Policy (General)

You are expected to complete all assigned readings, activities, assignments, attend all classes punctually and complete all scheduled assignments by due dates. You are expected to take responsibility to follow up with assignments and course related announcements. You are expected to participate in all project critiques, class discussions and activities.

Policy (Absenteeism)

In-class activities make up a significant portion of your course grade. Absence from class without a valid reason will affect your participation grade. Valid reasons include falling sick supported by a medical certificate and participation in NTU's approved activities supported by an excuse letter from the relevant bodies. There will be no make-up opportunities for in-class activities.

Absence Due to Medical or Other Reasons

If you are sick and unable to attend your class, you have to:

1. Send an email to the instructor regarding the absence and request for a replacement class.
2. Submit the Medical Certificate* or official letter of excuse to administrator.
3. Attend the assigned replacement class (subject to availability).

* The medical certificate mentioned above should be issued in Singapore by a medical practitioner registered with the Singapore Medical Association.

Policy (Others, if applicable)

Diversity and inclusion policy

Integrating a diverse set of experiences is important for a more comprehensive understanding of science.

It is our goal to create an inclusive and collaborative learning environment that supports a diversity of perspectives and learning experiences, and that honours your identities; including ethnicity, gender, socioeconomic status, sexual

orientation, religion or ability.

To help accomplish this:

- If you are neuroatypical or neurodiverse, have dyslexia or ADHD (for example), or have a social anxiety disorder or social phobia;
- If you feel like your performance in the class is being impacted by your experiences outside of class;
- If something was said in class (by anyone, including the instructor) that made you feel uncomfortable;

Please speak to your teaching team, our school pastoral officer or a peer or senior (either in-person or via email) about how we can help facilitate your learning experience.

As a participant in course discussions, you should also strive to honour the diversity of your classmates. You can do this by: using preferred pronouns and names; being respectful of others opinions and actively making sure all voices are being heard; and refraining from the use of derogatory or demeaning speech or actions.

All members of the class are expected to adhere to the NTU anti-harassment policy. if you witness something that goes against this or have any other concerns, please speak to your instructors or a faculty member.

Technical Skills To Be Developed

All skills to be developed for this course:

Computational Modelling
Engineering Problem Solving
Physics Concepts Application
Mathematical Concepts Application
Systems Design
Technical Analysis

Appendix 1: Rubrics for Project

The assessment criteria will be based on a score of 4:

- Excellently demonstrated – 4
- Mostly demonstrated – 3
- Moderately demonstrated – 2
- Somewhat demonstrated – 1
- Never demonstrated – 0

Criteria Description	**Assessment**					**Score**
	Poor **(0)**	**Somewhat** **(1)**	**Moderately** **(2)**	**Good** **(3)**	**Excellent** **(4)**	
CREATIVITY AND RESOURCEFULNESS (10%) Ability to make new connections under constraint.	Never Demonstrated	Somewhat Demonstrated	Moderately Demonstrated	Mostly Demonstrated	Excellently Demonstrated	Max 4
KNOWLEDGE (7%) Ability to correctly apply the knowledge in the course.	Never Demonstrated	Somewhat Demonstrated	Moderately Demonstrated	Mostly Demonstrated	Excellently Demonstrated	Max 4
ANALYTICAL SKILL (7%) Ability to conduct rigorous mathematical and numerical analysis as well as perform computational simulation.	Never Demonstrated	Somewhat Demonstrated	Moderately Demonstrated	Mostly Demonstrated	Excellently Demonstrated	Max 4
PREPARATION (8%) Ability to systematically organize the idea, concepts, approach, analysis, results and	Never Demonstrated	Somewhat Demonstrated	Moderately Demonstrated	Mostly Demonstrated	Excellently Demonstrated	Max 4

conclusion in the presentation materials.						
CLASS PRESENTATION (8%) Ability to communicate clearly and effectively, and to give appropriate response and answers to questions raised on the project	Never Demonstrated	Somewhat Demonstrated	Moderately Demonstrated	Mostly Demonstrated	Excellently Demonstrated	Max 4
					Total	Max 20

Note: This rubric is for the assessment of individual student. Although the students work in a team of 2 or at most 3, each student in the team needs to indicate and submit their role in the project, and the rubric based marking is done on each student and not on each team.

Peer Review for teamwork evaluation

Criteria (Weighs)	**Score from 1 to 9*** (**1**: Never; **3**: Rarely; **5**: Occasionally; **7**: Frequently; **9**: Always)				
(For 6-members team)	*Member A*	*Member B*	*Member C*	*Member D*	*Member E*
Member name					
a. Fulfilling one's responsibilities duly (15%)					
Behaved responsibly--such as attend meetings punctually and regularly; participate in discussion; complete assigned tasks/roles punctually.	*Score from 1 to 9*				
	Qualitative comments/reasons				
b. Fulfilling one's responsibilities effectively (25%)					
Behaved and contributed effectively--such as quality of work produced;	*Score from 1 to 9*				
	Qualitative comments/reasons				

creativity of ideas; extensiveness of research and thinking.					
c. Managing interpersonal relationships (30%)					
Listened attentively to and sought inputs from others; helped team resolve conflicts and achieved common understanding to function effectively; promoted respect for others and differences; fostered camaraderie.	*Score from 1 to 9*				
	Qualitative comments/reasons				
d. Providing support to others to achieve goals (30%)					
Behaved fairly and ethically—such as sharing responsibilities and giving credits. Exhibited group citizenship behavior--such as helping others to learn and complete their work through guidance and encouragement; standing up for others when needed.	*Score from 1 to 9*				
	Qualitative comments/reasons				

* Score of 1 should be given only when a team member does not really deserve to be awarded any mark for the team assignment (i.e., zero mark) because the member either has not or has barely participated and/or contributed to the team assignment in any meaningful manner.