Supplementary File

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This document includes the supplemental materials for the paper titled “Enhanced Hidden Moving Target Defense in Smart Grids.”

APPENDIX A
NUMERICAL RESULTS OF PFI-MTD UNDER THE AC MODEL

We apply our PFI-MTD construction algorithm under ac model to the IEEE 14-bus system for the noiseless and steady load case. The original load and generation profile is listed in column “p_{di}”, “q_{di}” and “p_{gi}”, “q_{gi}”, respectively, where p_{di} and q_{di} respectively denote the active and reactive load on bus i; p_{gi} and q_{gi} respectively denote the generator’s active and reactive power output on bus i. The original line parameter profile is listed in column “r_{ij}”, “$\mathcal{X}_{ij}$”, “y_{ij}”, and “τ_{ij}”, respectively. The system state $\langle \mathbf{v}, \boldsymbol{\theta} \rangle$ is listed in column “v_i” and “θ_i”, respectively. The column “p_{ij}”, “q_{ij}”, “p_{ji}”, and “q_{ji}” show the power flow measurements. We choose the voltage phases $\mathring{\boldsymbol{\theta}}$ after MTD, which is listed in column “$\mathring{\theta}_i$”. Then, we solve the nonlinear equations system $\mathbf{h}(\mathbf{v}, \boldsymbol{\theta}, \mathbf{r}, \boldsymbol{\mathcal{X}}, \mathbf{y}, \boldsymbol{\tau}) = \mathbf{h}(\mathbf{v}, \mathring{\boldsymbol{\theta}}, \mathbf{r}', \boldsymbol{\mathcal{X}}', \mathbf{y}', \boldsymbol{\tau}')$ to obtain the new four line parameters after MTD, i.e., $\mathbf{r}'$, $\boldsymbol{\mathcal{X}}'$, $\mathbf{y}'$, and $\boldsymbol{\tau}'$, which is listed in column “r'_{ij}”, “$\mathcal{X}'_{ij}$”, “y'_{ij}”, and “τ'_{ij}”. We apply the new four line parameters to the power system and the result is shown in the table. We can see the power flow is invariant and the system state after MTD $\langle \mathbf{v}', \boldsymbol{\theta}' \rangle = \langle \mathbf{v}, \mathring{\boldsymbol{\theta}} \rangle$, which is consistent with our analysis. In addition, this MTD is stealthy to the attacker since it is PFI-MTD.

APPENDIX B
NUMERICAL RESULTS OF CONSTRAINED PFI-MTD UNDER THE AC MODEL

We apply our constrained PFI-MTD approach under ac model to the IEEE 14-bus system for the noiseless and steady load case. The original load, generation, line parameter and system state profile is the same with that in Table I. We choose the voltage phases $\mathring{\boldsymbol{\theta}}$ after MTD. Then, we solve the optimization problem $\boldsymbol{\mathcal{X}}' = \arg\min_{\boldsymbol{\mathcal{X}}'} \left\| \mathbf{h}(\mathbf{v}, \boldsymbol{\theta}, \mathbf{r}, \boldsymbol{\mathcal{X}}, \mathbf{y}, \boldsymbol{\tau}) - \mathbf{h}(\mathbf{v}, \mathring{\boldsymbol{\theta}}, \mathbf{r}, \boldsymbol{\mathcal{X}}', \mathbf{y}, \boldsymbol{\tau}) \right\|_2$ to obtain the line reactance after MTD, i.e., $\boldsymbol{\mathcal{X}}'$. We apply the new line parameters to the power system and the result is shown in the table. We can see the power flow is nearly invariant and the system state after MTD $\langle \mathbf{v}', \boldsymbol{\theta}' \rangle \approx \langle \mathbf{v}, \mathring{\boldsymbol{\theta}} \rangle$, which is consistent with our analysis.

APPENDIX C
EXISTENCE OF COMPLETE MTD FOR VARIOUS IEEE TEST SYSTEMS

This section examines whether the condition given in Proposition 3 is satisfied for various IEEE test systems. Table III shows the number of state variables (i.e., n) and the number of transmission lines (i.e., m). From the table, we can see that for eight IEEE test systems, the condition $m \geq 2n$ is not satisfied. Thus, from Proposition 3, the complete MTD is not implementable on these systems.

TABLE I
NUMERICAL RESULTS OF PFI-MTD UNDER AC MODEL ON THE 14-BUS SYSTEM.

Bus i	p_{di}	q_{di}	p_{gi}	q_{gi}	v_i	θ_i	$\mathring{\theta}_i$	p'_{di}	q'_{di}	p'_{gi}	q'_{gi}	v'_i	θ'_i
1	0	0	194.33	0.00	1.06	0.00	0.00	0	0	194.33	0.00	1.06	0.00
2	21.7	12.7	36.72	23.69	1.04	-4.02	-3.56	21.7	12.7	36.72	23.69	1.04	-3.56
3	94.2	19	28.74	24.13	1.02	-9.93	-9.93	94.2	19	28.74	24.13	1.02	-9.93
4	47.8	-3.9			1.01	-8.66	-8.81	47.8	-3.9			1.01	-8.81
5	7.6	1.6			1.02	-7.43	-7.46	7.6	1.6			1.02	-7.46
6	11.2	7.5	0.00	11.55	1.06	-12.69	-12.90	11.2	7.5	0.00	11.55	1.06	-12.90
7	0	0			1.05	-11.19	-11.64	0	0			1.05	-11.64
8	0	0	8.49	8.27	1.06	-10.41	-10.85	0	0	8.49	8.27	1.06	-10.85
9	29.5	16.6			1.04	-13.00	-13.29	29.5	16.6			1.04	-13.29
10	9	5.8			1.04	-13.23	-13.54	9	5.8			1.04	-13.54
11	3.5	1.8			1.05	-13.09	-13.41	3.5	1.8			1.05	-13.41
12	6.1	1.6			1.04	-13.53	-13.73	6.1	1.6			1.04	-13.73
13	13.5	5.8			1.04	-13.58	-13.79	13.5	5.8			1.04	-13.79
14	14.9	5			1.02	-14.27	-14.65	14.9	5			1.02	-14.65

Line $i-j$	r_{ij}	$\mathcal{X}_{ij}$	y_{ij}	τ_{ij}	p_{ij}	q_{ij}	p_{ji}	q_{ji}	r'_{ij}	$\mathcal{X}'_{ij}$	y'_{ij}	τ'_{ij}	p'_{ij}	q'_{ij}	p'_{ji}	q'_{ji}
1-2	0.019	0.059	0.026	1.000	129.67	-6.36	-126.77	9.40	0.019	0.052	0.022	1.000	129.67	-6.36	-126.77	9.40
1-5	0.054	0.223	0.025	1.000	64.66	6.37	-62.61	-3.21	0.054	0.224	0.025	1.000	64.66	6.37	-62.61	-3.21
2-3	0.047	0.198	0.022	1.000	55.59	0.47	-54.25	0.56	0.047	0.214	0.024	1.000	55.59	0.47	-54.25	0.56
2-4	0.058	0.176	0.017	1.000	48.92	-0.48	-47.63	0.79	0.058	0.200	0.019	1.000	48.92	-0.48	-47.63	0.79
2-5	0.057	0.174	0.017	1.000	37.28	1.59	-36.54	-3.00	0.057	0.199	0.019	0.999	37.28	1.59	-36.54	-3.00
3-4	0.067	0.171	0.006	1.000	-11.21	4.57	11.31	-5.63	0.067	0.147	0.006	1.001	-11.21	4.57	11.31	-5.63
4-5	0.013	0.042	0.000	1.000	-49.17	11.58	49.50	-10.53	0.013	0.046	0.001	1.000	-49.17	11.58	49.50	-10.53
4-7	0.000	0.209	0.000	0.978	22.85	-3.99	-22.85	5.04	0.000	0.234	0.001	0.979	22.85	-3.99	-22.85	5.04
4-9	0.000	0.556	0.000	0.969	14.84	1.17	-14.84	-0.04	0.000	0.575	0.000	0.969	14.84	1.17	-14.84	-0.04
5-6	0.000	0.252	0.000	0.932	42.06	15.14	-42.06	-10.90	0.000	0.261	0.001	0.931	42.06	15.14	-42.06	-10.90
6-11	0.095	0.199	0.000	1.000	6.09	4.56	-6.04	-4.46	0.095	0.232	0.000	0.999	6.09	4.56	-6.04	-4.46
6-12	0.123	0.256	0.000	1.000	7.65	2.66	-7.58	-2.51	0.123	0.253	0.000	1.000	7.65	2.66	-7.58	-2.51
6-13	0.066	0.130	0.000	1.000	17.12	7.73	-16.91	-7.32	0.066	0.130	0.000	1.000	17.12	7.73	-16.91	-7.32
7-8	0.000	0.176	0.000	1.000	-8.49	-8.05	8.49	8.27	0.000	0.178	0.000	1.000	-8.49	-8.05	8.49	8.27
7-9	0.000	0.110	0.000	1.000	31.34	3.01	-31.34	-2.02	0.000	0.101	0.000	1.000	31.34	3.01	-31.34	-2.02
9-10	0.032	0.085	0.000	1.000	6.49	3.20	-6.47	-3.16	0.032	0.090	0.000	1.000	6.49	3.20	-6.47	-3.16
9-14	0.127	0.270	0.000	1.000	10.20	2.95	-10.06	-2.67	0.127	0.286	0.000	1.000	10.20	2.95	-10.06	-2.67
10-11	0.082	0.192	0.000	1.000	-2.53	-2.64	2.54	2.66	0.082	0.188	0.000	1.000	-2.53	-2.64	2.54	2.66
12-13	0.221	0.200	0.000	1.000	1.48	0.91	-1.47	-0.91	0.221	0.211	0.000	1.000	1.48	0.91	-1.47	-0.91
13-14	0.171	0.348	0.000	1.000	4.88	2.42	-4.84	-2.33	0.171	0.411	0.000	0.999	4.88	2.42	-4.84	-2.33

*Power quantifies in MW; line parameters and voltage magnitude quantify in p.u.; phase quantifies in deg.

TABLE II

NUMERICAL RESULTS OF CONSTRAINED PFI-MTD UNDER AC MODEL ON THE 14-BUS SYSTEM.

Bus i	p_{di}	q_{di}	p_{gi}	q_{gi}	v_i	θ_i	$\mathring{\theta}_i$	p'_{di}	q'_{di}	p'_{gi}	q'_{gi}	v'_i	θ'_i
1	0	0	194.33	0.00	1.06	0.00	0.00	0	0	194.33	-1.45	1.06	0.00
2	21.7	12.7	36.72	23.69	1.04	-4.02	-3.56	21.7	12.7	36.72	24.68	1.04	-3.57
3	94.2	19	28.74	24.13	1.02	-9.93	-9.93	94.2	19	28.74	25.06	1.02	-9.95
4	47.8	-3.9			1.01	-8.66	-8.81	47.8	-3.9			1.01	-8.82
5	7.6	1.6			1.02	-7.43	-7.46	7.6	1.6			1.02	-7.47
6	11.2	7.5	0.00	11.55	1.06	-12.69	-12.90	11.2	7.5	0.00	11.82	1.06	-12.90
7	0	0			1.05	-11.19	-11.64	0	0			1.05	-11.65
8	0	0	8.49	8.27	1.06	-10.41	-10.85	0	0	8.49	8.13	1.06	-10.86
9	29.5	16.6			1.04	-13.00	-13.29	29.5	16.6			1.04	-13.30
10	9	5.8			1.04	-13.23	-13.54	9	5.8			1.04	-13.56
11	3.5	1.8			1.05	-13.09	-13.41	3.5	1.8			1.05	-13.41
12	6.1	1.6			1.04	-13.53	-13.73	6.1	1.6			1.04	-13.74
13	13.5	5.8			1.04	-13.58	-13.79	13.5	5.8			1.04	-13.80
14	14.9	5			1.02	-14.27	-14.65	14.9	5			1.02	-14.66

Line $i-j$	r_{ij}	$\mathcal{X}_{ij}$	y_{ij}	τ_{ij}	p_{ij}	q_{ij}	p_{ji}	q_{ji}	$\mathcal{X}'_{ij}$	p'_{ij}	q'_{ij}	p'_{ji}	q'_{ji}
1-2	0.019	0.059	0.026	1.000	129.67	-6.36	-126.77	9.40	0.052	129.62	-7.91	-126.72	9.90
1-5	0.054	0.223	0.025	1.000	64.66	6.37	-62.61	-3.21	0.224	64.71	6.46	-62.66	-3.25
2-3	0.047	0.198	0.022	1.000	55.59	0.47	-54.25	0.56	0.214	55.69	0.69	-54.34	0.82
2-4	0.058	0.176	0.017	1.000	48.92	-0.48	-47.63	0.79	0.200	48.89	-0.21	-47.60	1.03
2-5	0.057	0.174	0.017	1.000	37.28	1.59	-36.54	-3.00	0.199	37.16	1.60	-36.43	-2.70
3-4	0.067	0.171	0.006	1.000	-11.21	4.57	11.31	-5.63	0.147	-11.12	5.25	11.22	-6.34
4-5	0.013	0.042	0.000	1.000	-49.17	11.58	49.50	-10.53	0.046	-49.19	11.44	49.52	-10.29
4-7	0.000	0.209	0.000	0.978	22.85	-3.99	-22.85	5.04	0.234	22.91	-3.43	-22.91	4.60
4-9	0.000	0.556	0.000	0.969	14.84	1.17	-14.84	-0.04	0.575	14.86	1.20	-14.86	-0.03
5-6	0.000	0.252	0.000	0.932	42.06	15.14	-42.06	-10.90	0.261	41.97	14.64	-41.97	-10.31
6-11	0.095	0.199	0.000	1.000	6.09	4.56	-6.04	-4.46	0.233	6.03	4.35	-5.98	-4.24
6-12	0.123	0.256	0.000	1.000	7.65	2.66	-7.58	-2.51	0.253	7.64	2.63	-7.57	-2.48
6-13	0.066	0.130	0.000	1.000	17.12	7.73	-16.91	-7.32	0.130	17.10	7.64	-16.89	-7.24
7-8	0.000	0.176	0.000	1.000	-8.49	-8.05	8.49	8.27	0.178	-8.49	-7.91	8.49	8.13
7-9	0.000	0.110	0.000	1.000	31.34	3.01	-31.34	-2.02	0.101	31.40	3.31	-31.40	-2.39
9-10	0.032	0.085	0.000	1.000	6.49	3.20	-6.47	-3.16	0.090	6.54	3.43	-6.53	-3.38
9-14	0.127	0.270	0.000	1.000	10.20	2.95	-10.06	-2.67	0.286	10.22	3.09	-10.09	-2.79
10-11	0.082	0.192	0.000	1.000	-2.53	-2.64	2.54	2.66	0.188	-2.47	-2.42	2.48	2.44
12-13	0.221	0.200	0.000	1.000	1.48	0.91	-1.47	-0.91	0.212	1.47	0.88	-1.46	-0.88
13-14	0.171	0.348	0.000	1.000	4.88	2.42	-4.84	-2.33	0.412	4.85	2.32	-4.81	-2.21

*Power quantifies in MW; line parameters and voltage magnitude quantify in p.u.; phase quantifies in deg.

TABLE III

NUMBER OF STATE VARIABLES AND LINES IN THE IEEE TEST SYSTEMS

Test system	Number of state variables (n)	Number of transmission lines (m)	whether $m \geq 2n$ is satisfied
IEEE 9-bus	8	9	×
IEEE 14-bus	13	20	×
IEEE 24-bus	23	38	×
IEEE 30-bus	29	41	×
IEEE 39-bus	38	46	×
IEEE 57-bus	56	80	×
IEEE 118-bus	117	186	×
IEEE 300-bus	299	411	×