Supervisory Control under Partial Observation

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Outline

• Motivation
• The Concept of Observability
• Supervisor Synthesis under Partial Observation
• Example
• Conclusions
Three Main Concepts in Control

• Controllability
  – allows you to improve the dynamics of a system by feedback
  – e.g. controllability in the RW supervisory control theory

• Observability
  – allows you to deploy such feedback by using the system's output

• Optimality
  – gives rise to formal methods of control synthesis
  – e.g. supremality in the RW supervisory control theory
Example

\[ \Sigma = \{a, b, c, d\} \]
\[ \Sigma_c = \{b, d\} \]
Example (cont.)

\[ \Sigma = \{a, b, c, d\} \]

\[ \Sigma_c = \{b, d\} \]
Some Intuitions

- Supervisor can only act upon receiving observable events
- Partial observation forces a supervisor to be conservative
- We can enable or disable a unobservable event
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Observability

• Given $G \in \phi(\Sigma)$, let $\Sigma_o \subseteq \Sigma$ and $P: \Sigma^* \rightarrow \Sigma_o^*$ be the natural projection.

• A language $K \subseteq L(G)$ is $(G,P)$-observable, if

  $$(\forall s \in \overline{K})(\forall \sigma \in \Sigma) \ s\sigma \in L(G)-\overline{K} \Rightarrow P^{-1}P(s)\sigma \cap \overline{K} = \emptyset$$

\[ \Sigma_o = \{b\} \]
Or equivalently ...

- $K \subseteq L(G)$ is $(G, P)$-observable, iff for any $s \in K$, $s' \in \Sigma^*$ and $\sigma \in \Sigma$,
  
  $s \sigma \in L(G) - \bar{K} \land s' \sigma \in L(G) \land P(s) = P(s') \implies s' \sigma \in L(G) - \bar{K}$

  or equivalently,

  $s \sigma \in \bar{K} \land s' \sigma \in L(G) \land P(s) = P(s') \implies s' \sigma \in \bar{K}$

  (Think about why they are equivalent)
Example 1

- $\Sigma = \{a, b, c, d\}$
- $\Sigma_o = \{c\}$
- $K = \{ac, bc\}$

Question: is $K (G,P)$-observable? yes
Example 2

- $\Sigma = \{a,b,c,d\}$
- $\Sigma_o = \{c\}$
- $K = \{ac, bc\}$

Question: is $K \ (G,P)$-observable? no
Example 3

- $\Sigma = \{a, b, c, d\}$
- $\Sigma_o = \{a, c\}$
- $K = \{ac, bc\}$

Question: is $K$ $(G, P)$-observable? yes
(G,P)-observability is *decidable*. But how?
Procedure of Checking Observability: Step 1

- Let $G = (X, \Sigma, \xi, x_0, X_m)$
- Suppose $K$ is recognized by $A = (Y, \Sigma, \eta, y_0, Y_m)$, i.e. $K = L_m(A)$
- Let $A' = G \times A = (X \times Y, \Sigma, \xi \times \eta, (x_0, y_0), X_m \times Y_m)$
  - Since $K = L(A) \subseteq L(G)$, we have $L(G \times A) = L(A)$
- A state $(x, y) \in X \times Y$ is a boundary state of $A'$ w.r.t. $G$, if
  - $(\exists s \in L(A')) \xi \times \eta((x_0, y_0), s) = (x, y)$, i.e. $(x, y)$ is reachable from $(x_0, y_0)$
  - $(\exists \sigma \in \Sigma) \xi(x, \sigma)! \land \neg \eta(y, \sigma)!$, where “!” denotes “is defined”
- Let $B$ be the collection of all boundary states of $A'$ w.r.t. $G$
  - $B$ is a finite set. (Why?)
Procedure of Checking Observability: Step 2

- For each boundary state \((x,y) \in B\), we define two sets
  - \(T(x,y) := \{s \in L(A') \mid \xi \times \eta((x_0, y_0), s) = (x, y)\}\) (\(T(x,y)\) is regular, why?)
  - \(\Sigma(x,y) := \{\sigma \in \Sigma \mid \xi(x, \sigma) \land \neg \eta(y, \sigma)\}\)

- Theorem
  - \(K\) is observable w.r.t. \(G\) and \(P\), iff for any boundary state \((x,y) \in B\),
    \[
    P^{-1}P(T(x,y))\Sigma(x,y) \cap \overline{K} = \emptyset
    \]
Example

- $\Sigma = \{a, b, c, d\}$
- $\Sigma_o = \{c\}$
- $K = \{ac, bc\}$
Example – Step 1

- \( \Sigma = \{a, b, c, d\} \)
- \( \Sigma_o = \{c\} \)
- \( K = \{ac, bc\} \)

- \( B = \{(1,1), (2,2)\} \)

\[ A' = G \times A \]
Example – Step 2

- For the boundary state (1,1) we have
  - $T(1,1) = \{b\}$
  - $\Sigma(1,1) = \{c\}$
  - $P^{-1}P(T(1,1))\Sigma(1,1) \cap \overline{K} = \{bc, ac\} \cap \{ac, ba\} = \{ac\} \neq \emptyset$

- For the boundary state (2,2) we have
  - $T(2,2) = \{a\}$
  - $\Sigma(2,2) = \{d\}$
  - $P^{-1}P(T(2,2))\Sigma(2,2) \cap \overline{K} = \{ad\} \cap \{ac, ba\} = \emptyset$

$K$ is not observable w.r.t. $G$ and $P$
Properties of Observable Languages

• Suppose $K_1$ and $K_2$ are closed, observable w.r.t. $G$ and $P$. Then
  – $K_1 \cap K_2$ is observable w.r.t. $G$ and $P$
  – $K_1 \cup K_2$ may not be observable w.r.t. $G$ and $P$

• Given a plant $G$, let

\[ O(G) := \{ K \subseteq L(G) | K \text{ is closed and observable w.r.t. } G \text{ and } P \} \]

• The partially ordered set (poset) $(O(G), \subseteq)$ is a meet-semi-lattice
  – The greatest element may not exist (i.e. no supremal observable sublanguage)
Example

$\Sigma = \{a, b, c, d, e\}$

$\Sigma_o = \{c\}$

$K_1 \cap K_2$ is observable, but $K_1 \cup K_2$ is not. (Why?)
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Main Existence Result

- Theorem 1
  - Let $K \subseteq \mathcal{L}_m(G)$ and $K \neq \emptyset$. There exists a proper supervisor iff
    - $K$ is controllable with respect to $G$
    - $K$ is observable with respect to $G$ and $P$
    - $K$ is $\mathcal{L}_m(G)$-closed, i.e. $K = K \cap \mathcal{L}_m(G)$
Supervision under Partial Observation

• Suppose $K$ is controllable, observable and $L_m(G)$-closed.
• Let $A=(Y,\Sigma_o,\eta,y_0,Y_m)$ be the canonical recognizer of $P(K)$.
• We construct a new automaton $S=(Y,\Sigma,\lambda,y_0,Y_m)$ as follow:
  – For any $y\in Y$, an event $\sigma\in\Sigma-\Sigma_o$ is control-relevant w.r.t. $y$ and $K$, if
    \[(\exists s\in K) \; \eta(y_0,P(s))=y \land s\sigma\in K\]
  – Let $\Sigma(y)$ be the collection of all events in $\Sigma-\Sigma_o$ control-relevant w.r.t. $y$, $K$
  – We define the transition map $\lambda:Y\times\Sigma\rightarrow Y$ as follows:
    • $\lambda$ is the same as $\eta$ over $Y\times\Sigma_o$
    • For any $y\in Y$ and $\sigma\in\Sigma(y)$, define $\lambda(y,\sigma):=y$ (i.e. selfloop all events of $\Sigma(y)$ at $y$)
    • For all other $(y,\sigma)$ pairs, $\lambda(y,\sigma)$ is undefined

$S$ is a proper supervisor of $G$ under PO such that $L_m(S/G)=K$
Example

- $\Sigma = \{a,b,c,d\}$
- $\Sigma_o = \{c\}$
- $K = \{ac, bc\}$

$L_m(S/G) = K$?
Difficulty of Synthesis

• Given a plant $G$ and a specification $SPEC$, let

$O(G,SPEC) := \{K \subseteq L_m(G) \cap L_m(SPEC) | K$ is controllable and observable$\}$

• Unfortunately, there is no supremal element in $O(G,SPEC)$. 
Solution 1: A New Supervisory Control Problem

- Given $G$, suppose we have $A \subseteq E \subseteq L(G)$ and $\Sigma = \Sigma_o \cup \Sigma_c$.
- To synthesize a supervisor $S$ under partial observation such that
  $$A \subseteq L(S/G) \subseteq E \quad (*)$$
- Let $O(A) := \{K \subseteq A | K \text{ is closed and observable w.r.t. } G \text{ and } P\}$
- Let $C(E) := \{K \subseteq E | K \text{ is closed and controllable w.r.t. } G\}$
- Theorem (Feng Lin)
  - Assume $A \neq \emptyset$. The (*) problem has a solution $S$ iff $\inf O(A) \subseteq \sup C(E)$
Solution 2 : The Concept of Normality

- Given \( N \subseteq M \subseteq \Sigma^* \), we say \( N \) is \((M,P)\)-normal if
  \[
  N = M \cap P^{-1}P(N)
  \]
  - In particular, take \( N=M\cap P^{-1}(K) \) for any \( K \subseteq \Sigma_{o^*} \). Then \( N \) is \((M,P)\)-normal.

- \((\forall s_1, s_2 \in M) (s_1, s_2) \in \ker P \iff P(s_1) = P(s_2)\)
- \( N/\ker P \subseteq M/\ker P \)
Properties of Normality

• Let $\mathcal{N}(E ; M) := \{N \subseteq E \mid N \text{ is } (M,P)\text{-normal}\}$ for some $E \subseteq \Sigma^*$
  
  – The poset $(\mathcal{N}(E ; M), \subseteq)$ is a complete lattice
    • The union of $(M,P)$-normal sublanguages is normal (intuitive explanation ?)
    • The intersection of $(M,P)$-normal sublanguages is normal (intuitive explanation ?)
  
  – Lin-Brandt formula: $\sup \mathcal{N}(E ; M) = E - P^{-1}P(M - E)$
    • In TCT: $N = \text{Supnorm}(E,M,\text{Null/Image})$

• Let $E \subseteq L_m(G)$, and $\mathcal{N}(E ; L(G)) := \{N \subseteq E \mid N \text{ is } (L(G),P)\text{-normal}\}$
  
  – $\mathcal{N}(E ; M)$ is closed under arbitrary unions, but not under intersections
Relationship between Normality and Observability

- Let $K \subseteq L_m(G)$. Then

  \[
  \bar{K} \text{ is } (L(G), P)\text{-normal} \Rightarrow K \text{ is observable w.r.t. } G \text{ and } P
  \]

- Let $\Sigma(K) := \{ \sigma \in \Sigma \mid (\exists s \in \bar{K}) s \sigma \in L(G) - \bar{K} \}$
  - $\Sigma(K)$ is the collection of all boundary events of $K$ w.r.t. $G$

  $K$ is observable w.r.t. $G$, $P \land \Sigma(K) \subseteq \Sigma_o \Rightarrow \bar{K}$ is $(L(G), P)$-normal
Supervisory Control under Normality

- Given a plant $G$ and a specification $E$, let
  
  $$\mathcal{C}(G,E) := \{K \subseteq L_m(G) \cap L_m(E) | K \text{ is controllable w.r.t. } G\}$$

- We define a new set
  
  $$\mathcal{S}(G,E) := \{K \subseteq \Sigma^* | K \in \mathcal{C}(G,E) \land \mathcal{N}(L_m(E),L(G)) \land L_m(G)\text{-closed}\}$$
  
  - $\mathcal{S}(G,E)$ is nonempty and closed under arbitrary unions. $\sup \mathcal{S}(G,E)$ exists

- Supervisory Control and Observation Problem (SCOP)
  
  - to compute a proper supervisor $S$ under partial observation such that
    
    $$L_m(S/G) = \sup \mathcal{S}(G,E)$$
The TCT Procedure for SCOP

• Given a plant $G$ and a specification $E$, let

$$A = \text{Supscop}(E, G, \text{Null/Image})$$

– $L_m(A) = \sup S(G, E)$

– Based on $A$, we construct a proper supervisor $S$ under partial observation

• Why can we do that? Because $\sup S(G, E)$ is controllable and observable
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Warehouse Collision Control

- Car 1
- Car 2
- Traffic Light
- Track 1
- Track 2
- Track 3
- Track 4
- Receiving Dock
- Dispatching Dock
- Sensor
Plant Model

- $\Sigma_1 = \{11, 12, 13, 15\}$, $\Sigma_{1,c} = \{11, 13, 15\}$, $\Sigma_{1,o} = \{11, 15\}$
- $\Sigma_2 = \{21, 22, 23, 25\}$, $\Sigma_{2,c} = \{21, 23, 25\}$, $\Sigma_{2,o} = \{21, 25\}$
Specification

• To avoid collision, $C_1$ and $C_2$ can’t reach the same state together
  – States (1,1), (2,2), (3,3) should be avoided in $C_1 \times C_2$
Synthesis Procedure in TCT

• Create the plant
  \[ G = \text{Sync}(C_1, C_2) \]  
  \( (25 ; 40) \)

• Create the specification
  \[ E = \text{mutex}(C_1, C_2, [(1,1), (2,2), (3,3)]) \]  
  \( (20 ; 24) \)

• Supervisor Synthesis
  \[ K = \text{Supscop}(E, G, [12, 13, 22, 23]) \]  
  \( (16 ; 16) \)
Transition Structure of $K$
A Proper Supervisor $S$ under Partial Observation

\[ K = L_m(S/G) \]
Some Fact

- Perform the following TCT operation

\[ W = \text{Condat}(G,K) \]

- Only events 11 and 21 are required to be disabled.
- Therefore, we only need one traffic light at Track 1.
A Slight Modification

- $\Sigma_{1,o} = \{11, 15\}$
- $\Sigma_{2,o} = \{21, 25\}$
- $\Sigma_{1,o} = \{11, 13\}$
- $\Sigma_{2,o} = \{21, 23\}$
Synthesis Result

- Create the plant
  \[ G = \text{Sync}(C_1, C_2) \quad (25; 40) \]

- Create the specification
  \[ E = \text{Mutex}(C_1, C_2, [(1,1),(2,2),(3,3)]) \quad (20; 24) \]

- Supervisor Synthesis
  \[ K = \text{Supscop}(E, G, [12, 15, 22, 25]) \quad \text{(empty)} \]
  - Explain intuitively why this can happen (homework)
Conclusions

• Partial observation is important for implementation.
  – A supervisor can make a move only based on observations.

• The current observability is not closed under set union.
  – Thus, there is no supremal observable sublanguage (unfortunately).

• Normality is closed under set union.
  – Thus, the supremal normal sublanguage exists.
  – But the concept of normality is too conservative.