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References

[1] K. Aas. The basel II IRB approach for credit portfolios: A survey. Note No SAMBA/33/05, Norwegian Computing Center, 2005. (Cited on page 145).

[2] C. Acerbi and D. Tasche. On the coherence of expected shortfall. Preprint arXiv:cond-mat/0104295, 2001. (Cited on page 76).

[3] F. Black and J.C. Cox. Valuing corporate securities. *Journal of Finance*, 31:351–357, 1976. (Cited on page 111).

[4] D. Bosq and H.T. Nguyen. *A Course in Stochastic Processes: Stochastic Models and Statistical Inference*. Mathematical and Statistical Methods. Kluwer, 1996. (Cited on page 31).

[5] D. Brigo and K. Chourdakis. Counterparty risk for credit default swaps: impact of spread volatility and default correlation. *Int. J. Theor. Appl. Finance*, 12(7):1007–1026, 2009. doi: 10.1142/S0219024909005567. (Cited on page 148).

[6] D. Brigo and M. Masetti. Risk neutral pricing of counterparty risk. In M. Pykhtin, editor, *In Counter-party Credit Risk Modeling: Risk Management, Pricing and Regulation*, London, 2006. Risk Books. (Cited on page 148).

[7] G. Castellacci. Bootstrapping credit curves from CDS spread curves. Available at https://dx.doi.org/10.2139/ssrn.2042177, 2008. (Cited on pages 139, 148, and 234).

[8] R.-R. Chen and J.-Z. Huang. Credit spread bounds and their implications for credit risk modeling. Working paper, Rutgers University and Penn State University, 2001. (Cited on page 128).

[9] R.-R. Chen, X. Cheng, F.J. Fabozzi, and B. Liu. An explicit, multi-factor credit default swap pricing model with correlated factors. *Journal of Financial and Quantitative Analysis*, 43(1):123–160, 2008. (Cited on page 128).

[10] E. Çınlar. *Probability and stochastics*, volume 261 of *Graduate Texts in Mathematics*. Springer, New York, 2011. doi: 10.1007/978-0-387-87859-1. (Cited on page 177).

[11] R. Cont and P. Tankov. *Financial modelling with jump processes*. Chapman & Hall/CRC Financial Mathematics Series. Chapman & Hall/CRC, Boca Raton, FL, 2004. (Cited on page 41).

[12] F. De Vylder. Numerical finite-time ruin probabilities by the Picard-Lefèvre formula. *Scand. Act. J.*, 2:97–105, 1999. (Cited on page 47).

[13] C. Dellacherie, B. Maisonneuve, and P.A. Meyer. *Probabilités et Potentiel*, volume 5. Hermann, 1992. (Cited on page 125).

[14] J. L. Devore. *Probability and Statistics for Engineering and the Sciences*. Duxbury Press, sixth edition, 2003. (Cited on page 167).

[15] M. Dozzi and P. Vallois. Level crossing times for certain processes without positive jumps. *Bull. Sci. Math.*, 121(5):355–376, 1997. (Cited on page 48).

[16] D. Duffie and K.J. Singleton. *Credit risk*. Princeton Series in Finance. Princeton University Press, Princeton, NJ, 2003. (Cited on page 127).

[17] R.J. Elliott and M. Jeanblanc. Incomplete markets with jumps and informed agents. *Math. Methods Oper. Res.*, 50(3):475–492, 1999. (Cited on page 127).

[18] R.J. Elliott, M. Jeanblanc, and M. Yor. On models of default risk. *Math. Finance*, 10(2):179–195, 2000. (Cited on page 125).

[19] P. Embrechts and M. Hofert. A note on generalized inverses. *Math. Methods Oper. Res.*, 77(3):423–432, 2013. (Cited on pages 60 and 93).

[20] W. Enders. *Applied Econometric Time Series*. Wiley, 2009. (Cited on page 22).

[21] R.F. Engle and C.W.J. Granger. Co-integration and error correction: Representation, estimation, and testing. *Econometrica*, 55(2):251–276, 1987. (Cited on page 22).

[22] M. Gibson. Understanding the risk of synthetic CDOs. Federal Reserve Board Working paper, 2004. (Cited on page 116).

[23] C. Gourieroux and J. Jasiak. *Financial Econometrics: Problems, Models, and Methods*. Princeton Series in Finance. Princeton University Press, USA, 2001. (Cited on page v).

[24] M.R. Grasselli and T.-R. Hurd. Credit risk modeling. Lecture notes, McMaster University, January 3, 2010. (Cited on pages 107 and 111).

[25] X. Guo, R. Jarrow, and C. Menn. A note on Lando's formula and conditional independence. Preprint, 2007. (Cited on pages 124 and 127).

[26] G.M. Gupton, C.C. Finger, and M. Bhatia. Creditmetrics: technical document. JP Morgan & Co., 1997. (Cited on page 115).

[27] J.D. Hamilton. A new approach to the economic analysis of nonstationary time series. *Econometrica*, 57:357–384, 1989. (Cited on page 15).

[28] J.D. Hamilton. *Time Series Analysis*. Princeton University Press, 1994. (Cited on page 14).

[29] J. Hull and A. White. Valuation of a CDO and an n-th to default CDS without Monte Carlo simulation. *Journal of Derivatives*, 12:8–23, 2004. (Cited on page 116).

[30] G. Ignatova, V. Kaishev, and R. Krachunov. An improved finite-time ruin probabilities formula and its Mathematica implementation. *Insurance Math. Econom.*, 29:375–386, 2001. (Cited on page 47).

[31] J. Jacod. Grossissement initial, hypothèse (H') et théorème de Girsanov. In Th. Jeulin and M. Yor, editors, *Grossissements de filtrations: exemples et applications*, volume 1118 of *Lecture Notes in Mathematics*, pages 6–14. Springer-Verlag, Berlin, 1985. (Cited on page 127).

[32] J. Jacod and P. Protter. *Probability essentials.* Springer-Verlag, Berlin, 2000. (Cited on page 167).

[33] M. Jeanblanc, M. Yor, and M. Chesney. *Mathematical methods for financial markets.* Springer Finance. Springer-Verlag, London, 2009. doi: 10.1007/978-1-84628-737-4.

[34] Th. Jeulin. *Semi-martingales et grossissement d'une filtration*, volume 833 of *Lecture Notes in Mathematics.* Springer Verlag, 1980. (Cited on page 127).

[35] S. Jones. The formula that felled Wall Street. Financial Times Magazine, April 24, 2009. (Cited on page 115).

[36] R. Kaas, M. Goovaerts, J. Dhaene, and M. Denuit. *Modern Actuarial Risk Theory: Using R.* Springer, 2009. (Cited on page 46).

[37] O. Kallenberg. *Foundations of Modern Probability.* Probability and its Applications. Springer-Verlag, New York, second edition, 2002. (Cited on page 200).

[38] D. Lando. On Cox processes and credit risky securities. *Review of Derivative Research*, 2:99–120, 1998. (Cited on pages 124 and 127).

[39] S. Lee. Formula from hell. Forbes.com, August 8, 2009. (Cited on page 115).

[40] D.X. Li. On default correlation - a copula function approach. *The Journal of Fixed Income*, 9(4):43–54, 2000. (Cited on pages 114, 115, and 116).

[41] W. Li and T. Krehbiel. An improved approach to evaluate default probabilities and default correlations with consistency. *Int. J. Theor. Appl. Finance*, 19(5), 2016. (Cited on page 116).

[42] S. Loisel and N. Privault. Sensitivity analysis and density estimation for finite-time ruin probabilities. *J. Comput. Appl. Math.*, 230(1):107–120, 2009. (Cited on page 47).

[43] R. C. Merton. On the pricing of corporate debt: The risk structure of interest rates. *Journal of Finance*, 29:449–470, 1974. (Cited on pages 107, 116, and 144).

[44] J.S. Milne. Mathematical apocrypha. https://www.jmilne.org/math/apocrypha.html, 2005. Accessed: 2020-06-12. (Cited on page 240).

[45] L.D. Minkova. Insurance risk theory. Lecture Notes, 2010.

[46] J.R. Norris. *Markov Chains*, volume 2 of *Cambridge Series in Statistical and Probabilistic Mathematics.* Cambridge University Press, Cambridge, 1998. Reprint of 1997 original. (Cited on page 34).

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https://personal.ntu.edu.sg/nprivault/indext.html

[47] Basel Committee on Banking Supervision. An explanatory note on the basel II IRB risk weight functions. July, 2005. (Cited on page 144).
[48] P. Picard and C. Lefèvre. The probability of ruin in finite time with discrete claim size distribution. *Scand. Act. J.*, 1:58–69, 1997. (Cited on page 47).
[49] J. Pitman. *Probability*. Springer, 1999. (Cited on page 167).
[50] M. Pourahmadi. *Foundations of time series analysis and prediction theory*. Wiley Series in Probability and Statistics: Applied Probability and Statistics. Wiley-Interscience, New York, 2001. (Cited on page 10).
[51] N. Privault. *Stochastic Finance: An Introduction with Market Examples*. Financial Mathematics Series. Chapman & Hall/CRC, 2014. (Cited on pages 111, 112, 113, 123, 126, 127, 128, 136, and 227).
[52] N. Privault. *Understanding Markov Chains (Second Edition)*. Springer Undergraduate Mathematics Series. Springer, 2018. (Cited on page 35).
[53] N. Privault and X. Wei. A Malliavin calculus approach to sensitivity analysis in insurance. *Insurance Math. Econom.*, 35(3):679–690, 2004. (Cited on page 47).
[54] N. Privault and X. Wei. Integration by parts for point processes and Monte Carlo estimation. *J. Appl. Probab.*, 44:806–823, 2007. (Cited on page 47).
[55] P. Protter. *Stochastic integration and differential equations*, volume 21 of *Stochastic Modelling and Applied Probability*. Springer-Verlag, Berlin, second edition, 2004. (Cited on page 125).
[56] D. Rullière and S. Loisel. Another look at the Picard-Lefèvre formula for finite-time ruin probabilities. *Insurance Math. Econom.*, 35(2):187–203, 2004. (Cited on page 47).
[57] F. Salmon. Recipe for disaster: the formula that killed Wall Street. Wired Magazine, 23 February, 2009. (Cited on page 115).
[58] S.E. Shreve. *Stochastic calculus for finance. II*. Springer Finance. Springer-Verlag, New York, 2004. Continuous-time models. (Cited on page 111).
[59] A. Sklar. Fonctions de répartition a n dimensions et leurs marges [republication of MR0125600]. *Ann. I.S.U.P.*, 54(1-2):3–6, 2010. ISSN 1626-1607. With an introduction by Denis Bosq. (Cited on page 95).
[60] M. Sklar. Fonctions de répartition à n dimensions et leurs marges. *Publ. Inst. Statist. Univ. Paris*, 8:229–231, 1959. (Cited on page 95).
[61] D.W. Stroock. *Probability theory, an analytic view*. Cambridge University Press, Cambridge, second edition, 2011. ISBN 978-0-521-13250-3. (Cited on page 198).
[62] S. Takayanagi and K. Ishikawa. *PairTrading: classical pair trading based on cointegration in finance*, 2017. R package version 1.1. (Cited on page 25).
[63] O. Vašiček. Probability of loss on loan portfolio. KMV Corporation Technical Report, 1987. (Cited on page 116).

[64] O. Vašiček. Loan portfolio value. *Risk Magazine*, pages 160–162, 2002. (Cited on page 144).

[65] M. Yor. Grossissements de filtration et absolue continuité de noyaux. In Th. Jeulin and M. Yor, editors, *Grossissements de filtrations: exemples et applications*, volume 1118 of *Lecture Notes in Mathematics*, pages 6–14. Springer-Verlag, Berlin, 1985. (Cited on page 127).

These notes cover some aspects of financial risk and analytics. This includes classical topics such as Value at Risk and expected shortfall, as well as structures of random dependence. Credit default is treated via defaultable bonds, Credit Default Swaps (CDS) and collateralized debt obligations (CDOs), based on stochastic calculus. Basic risk Theory and credit scoring are presented with examples in R. The concepts presented are illustrated by examples and by 43 exercises with their complete solutions.

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