Chapter 9
Credit Scoring

In this chapter we review the use of discriminant analysis, with application to credit scoring.

9.1 Discriminant Analysis

A set $\Omega$ of credit applicants is decomposed into a partition

$$\Omega = G \cup B,$$

where $G$ denotes the set of “good” (or solvent) applicants and $B$ denotes the set of “bad” (or insolvent) applicants, in such a way that

$$P(G) + P(B) = 1,$$
where $\mathbb{P}(B)$ represents the probability that an applicant chosen at random will default.

In addition, each applicant $\omega \in \Omega$ is assigned a real-valued rating (or score) $X(\omega)$ via a random variable

$$X : \Omega \longrightarrow \mathbb{R}$$

$$\omega \longmapsto X(\omega).$$

**Definition 9.1.** The functions

$$\mathbb{R} \longrightarrow [0, 1]$$

$$x \longmapsto \mathbb{P}(B \mid X = x)$$

and

$$\mathbb{R} \longrightarrow [0, 1]$$

$$x \longmapsto \mathbb{P}(G \mid X = x)$$

are respectively called the probability default curve and the probability acceptance curve.

By the Bayes formula, we have

$$\mathbb{P}(B \mid X = x) = \frac{\mathbb{P}(X = x \text{ and } B)}{\mathbb{P}(X = x)} = \frac{\mathbb{P}(X = x \mid B)\mathbb{P}(B)}{\mathbb{P}(X = x)} = \frac{\mathbb{P}(B)\mathbb{P}(X = x \mid B)}{\mathbb{P}(G)\mathbb{P}(X = x \mid G) + \mathbb{P}(B)\mathbb{P}(X = x \mid B)},$$

and

$$\mathbb{P}(G \mid X = x) = \frac{\mathbb{P}(G)\mathbb{P}(X = x \mid G)}{\mathbb{P}(G)\mathbb{P}(X = x \mid G) + \mathbb{P}(B)\mathbb{P}(X = x \mid B)}.$$

**Definition 9.2.** The True Positive Rate (TPR) is the tail distribution function

$$F_G(x) := \mathbb{P}(X \geq x \mid G) = \int_x^{\infty} d\mathbb{P}(X \leq y \mid G), \quad x \in \mathbb{R}.$$

On the other hand, the False Positive Rate (FPR) is the tail distribution function

$$F_B(x) := \mathbb{P}(X \geq x \mid B) = \int_x^{\infty} d\mathbb{P}(X \leq y \mid B), \quad x \in \mathbb{R}.$$
Example. In case $X$ is Gaussian distributed given $\{G, B\}$ with the conditional densities
\[
dP(X \leq x \mid G) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/(2\sigma^2)} dx
\] (9.1)
and
\[
dP(X \leq x \mid B) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu^B)^2/(2\sigma^2)} dx,
\] (9.2)
with $\mu_B < \mu_G$, we find the probability default curve
\[
P(B \mid X = x) = \frac{\mathbb{P}(B)d\mathbb{P}(X \leq x \mid B)}{\mathbb{P}(G)d\mathbb{P}(X \leq x \mid G) + \mathbb{P}(B)d\mathbb{P}(X \leq x \mid B)}
= \frac{\mathbb{P}(B)e^{-(x-\mu^B)^2/(2\sigma^2)}}{\mathbb{P}(G)e^{-(x-\mu)^2/(2\sigma^2)} + \mathbb{P}(B)e^{-(x-\mu^B)^2/(2\sigma^2)}}
= \frac{1}{1 + \exp(\alpha + \beta x)}, \quad x \in \mathbb{R},
\] (9.3)
with
\[
\beta := \frac{\mu_G - \mu_B}{\sigma^2} > 0
\]
and
\[
\alpha := -\beta \frac{\mu_G + \mu_B}{2} + \log \left( \frac{\mathbb{P}(G)}{\mathbb{P}(B)} \right),
\]
Fig. 9.1: Probability default curve $x \mapsto \mathbb{P}(B \mid X = x) = 1/(1 + e^{\alpha + \beta x})$.

On the other hand, the probability acceptance function is given by
\[
P(G \mid X = x) = \frac{\mathbb{P}(G)d\mathbb{P}(X \leq x \mid G)}{\mathbb{P}(G)d\mathbb{P}(X \leq x \mid G) + (1 - \mathbb{P}(G))d\mathbb{P}(X \leq x \mid B)}
= \frac{\mathbb{P}(G)e^{-(x-\mu)^2/(2\sigma^2)}}{\mathbb{P}(G)e^{-(x-\mu)^2/(2\sigma^2)} + \mathbb{P}(B)e^{-(x-\mu^B)^2/(2\sigma^2)}}
= \frac{1}{1 + \exp(-\alpha - \beta x)}, \quad x \in \mathbb{R}.
\] (9.4)
9.2 Decision rule

We decide to accept the applicants whose rating $X(\omega)$ belongs to a decision set $\mathcal{A}$, and to reject those whose rating $X(\omega)$ belongs to the complement $\mathcal{A}^c = X \setminus \mathcal{A}$ of $\mathcal{A}$ in $X$. Let

- $L(x)$ represents the loss (or missed earnings) incurred by the rejection of an applicant with rating $x \in \mathcal{A}^c$, and

- $D(x)$ represents the loss incurred by the default of an applicant with rating $x \in \mathcal{A}$.

The cost associated to this decision rule becomes

$$\frac{D(X)1_{\{X \in \mathcal{A}\} \cap B}}{\text{Cost of accepting a 'bad' applicant}} + \frac{L(X)1_{\{X \in \mathcal{A}^c\} \cap G}}{\text{Loss from rejecting a 'good' applicant}}$$

**Proposition 9.3.** The acceptance optimal set $\mathcal{A}^*$ that minimizes the expected cost

$$\mathbb{E} \left[ D(X)1_{\{X \in \mathcal{A}\} \cap B} + L(X)1_{\{X \in \mathcal{A}^c\} \cap G} \right]$$

is given by

$$\mathcal{A}^* = \left\{ x \in \mathbb{R} : \lambda(x) \geq \frac{D(x) \mathbb{P}(B)}{L(x) \mathbb{P}(G)} \right\},$$

where $\lambda(x)$ is the likelihood ratio

$$\lambda(x) := \frac{d\mathbb{P}(X \leq x \mid G)}{d\mathbb{P}(X \leq x \mid B)} = \frac{\mathbb{P}(G \mid X = x) \mathbb{P}(G)}{\mathbb{P}(B \mid X = x) \mathbb{P}(B)}, \quad x \in \mathbb{R}.$$ 

**Proof.** The expected cost corresponding to an acceptance set $\mathcal{A}$ can be written as

$$\mathbb{E} \left[ D(X)1_{\{X \in \mathcal{A}\} \cap B} + L(X)1_{\{X \in \mathcal{A}^c\} \cap G} \right]$$

$$= \int_{\mathcal{A}} D(x)d\mathbb{P}(\{X \leq x\} \cap B) + \int_{\mathcal{A}^c} L(x)d\mathbb{P}(\{X \leq x\} \cap G)$$
In terms of the likelihood ratio
\[
\lambda(x) := \frac{d\mathbb{P}(X \leq x \mid G)}{d\mathbb{P}(X \leq x \mid B)} = \frac{\mathbb{P}(G \mid X = x)}{\mathbb{P}(B \mid X = x)}, \quad x \in \mathbb{R},
\]
we have
\[
\mathcal{A}^* = \left\{ x \in \mathbb{R} : \lambda(x) \geq \frac{D(x) \mathbb{P}(B)}{L(x) \mathbb{P}(G)} \right\}.
\]

For simplicity we can assume that \( D = D(x) \) and \( L = L(x) \) are constant in \( x \in \mathbb{R} \), in which case we have
\[
\mathcal{A}^* = \left\{ x \in \mathbb{R} : \lambda(x) \geq \frac{D \mathbb{P}(B)}{L \mathbb{P}(G)} \right\}.
\]

In the Gaussian example (9.1)-(9.2) the likelihood ratio is given by
\[ \lambda(x) = \frac{d\mathbb{P}(X \leq x \mid G)}{d\mathbb{P}(X \leq x \mid B)} = e^{-(x-\mu_G)^2/(2\sigma^2)+(x-\mu_B)^2/(2\sigma^2)} = e^{-(\mu_G^2-\mu_B^2-2x(\mu_G-\mu_B))/(2\sigma^2)} = e^{\beta x-(\mu_G^2-\mu_B^2)/(2\sigma^2)}, \quad x \in \mathbb{R}, \]

hence the condition

\[ \lambda(x) = e^{\beta x-(\mu_G^2-\mu_B^2)/(2\sigma^2)} \geq \frac{D}{L} \frac{\mathbb{P}(B)}{\mathbb{P}(G)} \]

is equivalent to

\[
x \geq \frac{\mu_G^2-\mu_B^2}{2\beta\sigma^2} + \frac{1}{\beta} \log \left( \frac{D}{L} \frac{\mathbb{P}(B)}{\mathbb{P}(G)} \right) \\
\geq \frac{\mu_G^2-\mu_B^2}{2(\mu_G-\mu_B)} + \frac{1}{\beta} \log \left( \frac{D}{L} \frac{\mathbb{P}(B)}{\mathbb{P}(G)} \right) \\
\geq \frac{\mu_G + \mu_B}{2} + \frac{1}{\beta} \log \left( \frac{D}{L} \frac{\mathbb{P}(B)}{\mathbb{P}(G)} \right),
\]

provided that

\[ \beta := \frac{\mu_G - \mu_B}{\sigma^2} > 0. \]

Therefore we have

\[ \mathcal{A}^* = [x^*, \infty) = \left[ \frac{\mu_G + \mu_B}{2} + \frac{1}{\beta} \log \left( \frac{D}{L} \frac{\mathbb{P}(B)}{\mathbb{P}(G)} \right), \infty \right), \]

where

\[ x^* := \frac{\mu_G + \mu_B}{2} + \frac{1}{\beta} \log \left( \frac{D}{L} \frac{\mathbb{P}(B)}{\mathbb{P}(G)} \right), \]

under the condition

\[ \mathbb{E}[X \mid B] = \mu_B < \mu_G = \mathbb{E}[X \mid G]. \]

We note that the optimal boundary point \( x^* \) satisfies the relation

\[ \lambda(x^*) = \frac{\mathbb{P}(X = x^* \mid G)}{\mathbb{P}(X = x^* \mid B)} = \frac{\mathbb{P}(G \mid X = x^*) \mathbb{P}(G)}{\mathbb{P}(B \mid X = x^*) \mathbb{P}(B)} = \frac{D}{L} \frac{\mathbb{P}(B)}{\mathbb{P}(G)}, \]

i.e.

\[ \frac{\mathbb{P}(G \mid X = x^*)}{\mathbb{P}(B \mid X = x^*)} = \frac{D}{L}, \quad (9.5) \]

The next animated Figure 9.3 illustrates the optimal decision rule by taking \( L = D = \$1 \) and using the default and acceptance curves (9.3)-(9.4). The
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The purple curve corresponds to the conditional expected cost function

\[ x \mapsto D(x) \mathbb{P}(B \mid X = x) \mathbb{1}_{A}(x) + L(x) \mathbb{P}(G \mid X = x) \mathbb{1}_{A^c}(x) \]

for a given set \( A \), and its uniform minimum is obtained for \( A^* \) of the form \( A^* = [x^*, \infty) \) with \( x^* \) at the intersection of the curves \( x \mapsto D(x) \mathbb{P}(B \mid X = x) \) and \( x \mapsto L(x) \mathbb{P}(G \mid X = x) \) as in (9.5).

Fig. 9.3: Animated graph of optimal decision rule.*

The acceptance rate, or probability that an applicant is accepted according to the rule \( A \), is given by

\[
\mathbb{P}(X \in A) = \mathbb{P}(\{X \in A\} \cap G) + \mathbb{P}(\{X \in A\} \cap B) \\
= \mathbb{P}(X \in A \mid G) \mathbb{P}(G) + \mathbb{P}(X \in A \mid B) \mathbb{P}(B),
\]

where

\[ \mathbb{P}(\{X \in A\} \cap B) = \mathbb{P}(X \in A \mid B) \mathbb{P}(B) \]

is the default rate, or probability that an applicant accepted according to the rule \( A \) will default.

Our next goal is to minimize the default rate \( \mathbb{P}(\{X \in A\} \cap B) \) subject to a given acceptance rate \( \mathbb{P}(X \in A) = a \).

9.3 Logistic Regression

We consider a set of \( p \) financial criteria or indicators \((x_{i,j})_{j=1,2,\ldots,p}\) applying to each of \( n \) credit applicants \( i = 1, 2, \ldots, n \). The credit rating class (“good” or “bad”) of applicant \( n^i \) is denoted by \( c_i \in \{0, 1\} \) depending on his status.

* The animation works in Acrobat Reader on the entire pdf file.
install.packages("caret")
library(caret)
data(GermanCredit)
head(GermanCredit)

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<th>Property.RealEstate</th>
<th>Housing.Own</th>
<th>CreditHistory</th>
<th>Class</th>
</tr>
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<tbody>
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<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

The score of a given credit applicant \(n^o\) \(i\) is written as

\[ z_i = \sum_{j=1}^{p} \beta_j x_{i,j}, \quad i = 1, 2, \ldots, n, \]

where \((\beta_j)_{j=1,2,\ldots,p}\) is a family of linear coefficients, and the probability \(p_i\) that applicant \(n^o\) \(i\) will default is modeled as

\[ p_i = f(z_i) = f\left(\sum_{j=1}^{p} \beta_j x_{i,j}\right), \quad i = 1, 2, \ldots, n. \]

When applying a linear regression model we would take \(f(x) := x\), hence the equation

\[ c_i = \sum_{j=1}^{p} \beta_j x_{i,j}, \]

would be used to estimate the coefficients \((\beta_j)_{j=1,2,\ldots,p}\).

**Binomial logistic regression**

The shortcoming of linear models is that \(f(z_i)\), which is assumed to represent a probability value, may exit the interval \([0, 1]\). Logistic regression models address this issue by replacing \(f(x) = x\) with the logistic function

\[ f(x) := \frac{e^x}{1 + e^x}, \quad x \in \mathbb{R}. \]
knowning that $f \left( -\sum_{i=1}^{n} \beta_j x_{i,j} \right)$, resp. $1 - f \left( -\sum_{i=1}^{n} \beta_j x_{i,j} \right)$ represents the probability that applicant $n^o j$ is rated "good", resp. "bad", the objective is to minimize the log-likelihood ratio

$$
\log L(\beta|x) = \log \prod_{j=1}^{p} \left( 1 - f \left( -\sum_{i=1}^{n} \beta_j x_{i,j} \right) \right)^{c_j} \left( f \left( -\sum_{i=1}^{n} \beta_j x_{i,j} \right) \right)^{1-c_j}
$$

$$
= \log \prod_{j=1}^{p} \left( \frac{1}{1 + e^{-\sum_{i=1}^{n} \beta_j x_{i,j}}} \right)^{c_j} \left( \frac{e^{-\sum_{i=1}^{n} \beta_j x_{i,j}}}{1 + e^{-\sum_{i=1}^{n} \beta_j x_{i,j}}} \right)^{1-c_j}
$$

$$
= \sum_{j=1}^{p} c_j \log \frac{1}{1 + e^{-\sum_{i=1}^{n} \beta_j x_{i,j}}} + \sum_{j=1}^{p} (1 - c_j) \log \frac{e^{-\sum_{i=1}^{n} \beta_j x_{i,j}}}{1 + e^{-\sum_{i=1}^{n} \beta_j x_{i,j}}}
$$

over $\beta = (\beta_j)_{j=1,2,...,p}$. The default probabilities $p_i$ are then given by

$$
p_i = f \left( -\sum_{j=1}^{p} \beta_j x_{i,j} \right),
$$

which corresponds to the logit

$$
\log \frac{p_i}{1-p_i} = -\sum_{j=1}^{p} \beta_j x_{i,j}
$$
or log-odds, which are probabilities on a logit scale.

The data is randomly split into a training set and a testing set using the createDataPartition command in R. The training set is used to fit the data in a generalized linear model using the glm() command. The testing set is then used to estimate the corresponding default probabilities.
Based on the above 100 samples, the next code identifies the True Positive Rate TPR = \( \frac{34}{77} = 44.16\% \), and the False Positive Rate FPR = \( \frac{8}{23} = 34.78\% \), at the level \( p = 0.75 \).

```r
cat("TPR5=",length(testsamp$Score5[testsamp$Score5>0.75 &
        testsamp$Class=="Good"])/length(testsamp$Score5[testsamp$Class=="Good"]),\'\n\n")
cat("FPR5=",length(testsamp$Score5[testsamp$Score5>0.75 &
          testsamp$Class=="Bad"])/length(testsamp$Score5[testsamp$Class=="Bad"]),\'\n\n")
```
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mod.glm<-glm(Class ~ Duration + Amount + InstallmentRatePercentage +
ResidenceDuration + Age + NumberExistingCredits + NumberPeopleMaintenance +
Telephone + ForeignWorker + CheckingAccountStatus.1.0 +
CheckingAccountStatus.0.to.200 + CheckingAccountStatus.gt.200 +
CheckingAccountStatus.none + CreditHistory.NoCredit.AllPaid +
CreditHistory.ThisBank.AllPaid + CreditHistory.PaidDuly + CreditHistory.Delay +
CreditHistory.Critical + Purpose.NewCar + Purpose.UsedCar +
Purpose.DomesticAppliance + Purpose.Repair + Purpose.Education +
Purpose.Vacation + Purpose.Retraining + Purpose.Business + Purpose.Other +
SavingsAccountBonds.lt.100 + SavingsAccountBonds.100.to.500 +
SavingsAccountBonds.500.to.1000 + SavingsAccountBonds.gt.1000 +
EmploymentDuration.1.to.4 + EmploymentDuration.4.to.7 +
EmploymentDuration.gt.7 + EmploymentDuration.Unemployed +
OtherDebtorsGuarantors.None + OtherDebtorsGuarantors.CoApplicant +
OtherDebtorsGuarantors.Guarantor + Property.RealEstate + Property.Insurance +
Property.Car.Other + Property.Unknown + OtherInstallmentPlans.Bank +
OtherInstallmentPlans.Other + OtherInstallmentPlans.None +
Housing.Rent + Housing.Own + Housing.ForFree +
Job.UnemployedUnskilled +
Job.UnskilledResident + Job.SkilledEmployee +
Job.Management.SelfEmp.HighlyQualified, data=training, family="binomial")

testing$Score61<-predict(mod.glm, newdata=testing, type="response")
testsamp <- head(testing,100)

ggplot(testsamp, aes(x=ID, y=Score61, fill=Class)) + geom_bar(stat="identity") +
scale_fill_manual(values=c( "red","darkgreen")) +
scale_y_discrete(limits=c(0,1),expand = c(0,0)) + theme_bw() + xlab(NULL) +
theme(axis.text.x = element_blank()) + geom_hline(yintercept = 0.75)

Fig. 9.6: Logistic regression output with 61 criteria.

In comparison with Figure 9.5, the 100 samples in Figure 9.6 above yield a
higher True Positive Rate TPR = 51/77 = 66.23% and a lower False Positive
Rate FPR = 6/23 = 26.09% at the level \( p = 0.75 \). In other words, the count
of true positive samples has increased from 34 to 51, and the count of false
positive samples has decreased from 8 to 6.
9.4 Receiver Operating Characteristic (ROC) Curve

The ROC curve is a plot of the True Positive Rate function

\[ x \mapsto \bar{F}_B(x) \]

against the False Positive Rate function

\[ x \mapsto \bar{F}_G(x). \]

In other word, the ROC curve is a plot of the curve

\[ [0, 1] \rightarrow [0, 1] \]
\[ p \mapsto \bar{F}_G(\bar{F}_B^{-1}(p)) \]

as a function of the threshold \( p \in [0, 1] \). The slope of the ROC curve at the point \( p \) is given by

\[
\frac{d}{dp} \bar{F}_G(\bar{F}_B^{-1}(p)) = \bar{F}_G'(\bar{F}_B^{-1}(p)) \frac{d}{dp} \bar{F}_B^{-1}(p)
\]
\[
= \frac{\bar{F}_G'(\bar{F}_B^{-1}(p))}{\bar{F}_B'(\bar{F}_B^{-1}(p))}
\]
\[
= \frac{\bar{F}_G'(x)}{\bar{F}_B'(x)}
\]
\[
= \lambda(x)
\]

with \( x := \bar{F}_B^{-1}(p) \), where \( \lambda(x) \) is the likelihood ratio

\[ \lambda(x) = \frac{d\mathbb{P}(X \leq x \mid G)}{d\mathbb{P}(X \leq x \mid B)}, \]

hence the ROC curve can be rewritten as the integral

\[
\bar{F}_G(\bar{F}_B^{-1}(p)) = \int_0^p \lambda(\bar{F}_B^{-1}(q)) dq, \quad 0 \leq p \leq 1.
\]

In the Gaussian example where \( X \) is Gaussian distributed given \( \{G, B\} \) with the conditional densities

\[
d\mathbb{P}(X \leq x \mid G) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(x-\mu_G)^2/(2\sigma^2)} dx
\]
and
\[ dP(X \leq x \mid B) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(x-\mu_B)^2/(2\sigma^2)} dx, \]
we have
\[ \lambda(x) = \frac{dP(X \leq x \mid G)}{dP(X \leq x \mid B)} = e^{\beta x - (\mu_G^2 - \mu_B^2)/(2\sigma^2)}, \quad x \in \mathbb{R}, \]
with \( \beta := (\mu_G - \mu_B)/\sigma^2 \). The next Figure 9.7 plots two ROC curves in the Gaussian example with successively \((\mu_B, \mu_G) = (1, 4)\), \((\mu_B, \mu_G) = (1, 2)\), and \((\mu_B, \mu_G) = (1, 1)\).

Fig. 9.7: Gaussian ROC curves.

We check that the classification is better when \( \mu_B << \mu_G \). The ROC curve represents the performance of the classification procedure. A perfect classification would correspond to a single point with coordinates \((0, 1)\), which corresponds to FPR=0% false negatives and TPR=100% true positives.

On the other hand, a completely random guess would correspond to a point on the diagonal line. Points above the diagonal represent good classification results (better than random), while points below the line represent poor results (worse than random) (Wikipedia).
The above True Positive Rates (TPR) and False Positive Rates (FPR) based on 5 and 61 criteria at the level \( p = 0.75 \) are recomputed in the above R code on the whole 400 samples, and plotted on the next ROC graphs of Figure 9.8.

The ROC graphs in the next Figure 9.8 confirm the improvement in classification reached when switching from 5 to 61 criteria.
Exercises

Exercise 9.1  Consider a set $\Omega$ of applicants decomposed as the partition $\Omega = G \cup B$, where each applicant $\omega$ is assigned a rating $X(\omega)$ which is exponentially distributed given $\{G, B\}$ with the conditional densities

$$d\mathbb{P}(X \leq x \mid G) = \lambda_G e^{-\lambda_G x}1_{[0, \infty)}(x)dx$$

and

$$d\mathbb{P}(X \leq x \mid B) = \lambda_B e^{-\lambda_B x}1_{[0, \infty)}(x)dx,$$

where $\lambda_B > \lambda_G > 0$.

a) Compute the probability default curve

$$x \mapsto \mathbb{P}(B \mid X = x) = \frac{\mathbb{P}(B)d\mathbb{P}(X \leq x \mid B)}{\mathbb{P}(G)d\mathbb{P}(X \leq x \mid G) + \mathbb{P}(B)d\mathbb{P}(X \leq x \mid B)}.$$

b) Determine the acceptance set

$$A := \{x \in \mathbb{R} : D\mathbb{P}(B \mid X = x) \leq L\mathbb{P}(G \mid X = x)\},$$

where

- $L$ represents the loss incurred by the rejection of an applicant, and
- $D$ represents the loss incurred by the default of an applicant.

Exercise 9.2  Consider a set $\Omega$ of applicants decomposed as the partition $\Omega = G \cup B$, where each applicant $\omega$ is assigned a uniformly distributed rating $X(\omega)$ given $\{G, B\}$, with the conditional densities

$$d\mathbb{P}(X \leq x \mid G) = \frac{1}{\lambda_G}1_{[0, \lambda_G]}(x)dx$$

and

$$d\mathbb{P}(X \leq x \mid B) = \frac{1}{\lambda_B}1_{[0, \lambda_B]}(x)dx,$$

where $0 < \lambda_B < \lambda_G$.

a) Compute the probability default curve

$$x \mapsto \mathbb{P}(B \mid X = x) = \frac{\mathbb{P}(B)d\mathbb{P}(X \leq x \mid B)}{\mathbb{P}(G)d\mathbb{P}(X \leq x \mid G) + \mathbb{P}(B)d\mathbb{P}(X \leq x \mid B)}.$$

b) Determine the acceptance set

$$A := \{x \in \mathbb{R}_+ : D\mathbb{P}(B \mid X = x) \leq L\mathbb{P}(G \mid X = x)\},$$
where

- $L$ represents the missed earnings incurred by the rejection of applicant,

- $D$ represents the loss incurred by the default of an applicant.