Index

σ-algebra, 713
Markov property, 160
complement rule, 715

absence of arbitrage, 25, 541
abstract Bayes formula, 499
accreting swap, 579
adapted process, 145
adjusted close price, 198, 660
admissible portfolio strategy, 177
affine model, 534
affine PDE, 294, 535, 977
American
binary option
finite expiration, 490
perpetual, 490
forward contract, 492
option
call, 459, 469
dividend, 485, 487
finite expiration, 473
perpetual, 459
put, 459
amortizing swap, 579
annuity numéraire, 579, 613, 628
approximation
gamma, 555
lognormal, 423
arbitrage, 21
absence of, 25
continuous time, 177
discrete time, 56
opportunity, 23
price, 13, 38, 68, 78, 247
triangular, 21
arithmetic average, 414
Asian option, 413, 414, 416
basket, 429
call, 413
dividends, 442
hedging, 442
asset pricing
first theorem, 27
continuous time, 179, 583
discrete time, 67
second theorem, 32
continuous time, 182
discrete time, 68
at the money, 57, 260
attainable, 31, 37, 77, 182
Bachelier model, 188, 225, 230, 261
backward induction, 86, 88
backward stochastic differential equation, 266
Barone-Adesi & Whaley model, 484
barrier forward contract, 384
down-and-in
long, 384, 901
down-and-out
long, 384, 902
up-and-in
long, 384, 897
up-and-out
long, 384, 899
barrier level, 359
barrier option, 58, 355, 359
down-and-in
call, 360, 373
put, 360, 375
down-and-out
call, 360, 369, 383
put, 360, 371
in-out parity, 360
up-and-in  
call, 360, 375  
put, 360, 376  
up-and-out  
call, 360, 361  
put, 360, 366  
basis point, 621  
basket option, 428  
BDT model, 564  
bear spread option, 259, 849  
Bermudan swaption, 620  
Bernoulli distribution, 726  
Bessel function, 532, 554  
BGM model, 595, 610  
binary option, 57, 116, 229, 262, 525, 771  
barrier, 385  
tree, 69  
binary option  
American  
finite expiration, 490  
perpetual, 490  
binomial distribution, 726  
bisection method, 228, 309  
bizdays (R package), 216  
Black (1976) formula, 611  
caplet formula, 610  
Black-Derman-Toy model, 564  
Black-Scholes calibration, 313  
formula, 223, 227, 249, 512, 596  
call option, 202, 611  
put option, 210, 612  
PDE, 200, 222, 228, 380, 383, 704  
with jumps, 687  
bond convertible, 560  
convexity, 565  
duration, 561, 565  
immunization, 980  
ladder, 613  
option, 523, 607, 966  
pricing PDE, 543, 593  
yield, 551  
zero-coupon, 540  
Borel-Cantelli, 171  
boundary condition, 705  
brake-even rate, 578, 580  
strike price, 45  
underlying asset price, 86, 215  
Bretton Woods, 509  
bridge model, 598  
Brownian bridge, 166, 557  
extrema, 347  
motion, 129  
geometric, 238  
Lévy’s construction, 136, 168, 826  
series construction, 134, 137  
BSDE, 266  
bull spread option, 259, 849  
business time, 216  
buy back guarantee, 7  
calendar time, 216  
call price, 360  
call level, 359  
call option, 8  
call spread collar option, 118  
call-put parity, 211, 251, 265, 322, 512, 908  
callable  
bear contract, 59, 359, 360  
bull contract, 360  
Cantor function, 333  
cap pricing, 612  
Capital Asset Pricing Model (CAPM), 268, 871  
caplet pricing, 609  
cash settlement, 13, 56, 209  
cash-or-nothing option, 57, 525  
cattle futures, 202  
Cauchy distribution, 723  
sequence, 812  
CBBC, 59, 359, 360  
CBOE, 531  
Volatility Index®, 323  
CEV model, 534  
change of measure, 243  
change of numéraire, 498, 512  
characteristic function, 743  
Chasles relation, 151  
Chi square distribution, 261, 532, 858  
Chicago Board Options Exchange, 531  
chooser option, 264, 864  
CIR model, 167, 225, 261, 532, 559  
CKLS, 558  
Clark-Ocone formula, 99, 406  
collar option, 9  
call spread, 118  
costless, 11  
put spread, 117

This version: January 15, 2020
https://www.ntu.edu.sg/home/nprivault/index.html
elasticity, 215
enewal processes, 645
entitlement ratio, 8, 208, 212, 315, 316
equivalent probability measure, 27, 34, 67, 179, 245
Esscher transform, 698
ETF, 268
Euclidean path integral, 556
Euler discretization, 708
EURIBOR, 576
European option
  knock-in, 386
  knock-out, 386
event, 713
ex-dividend, 226, 442
exchange option, 488, 515
exchange-traded fund, 268
exercise price, 6
exotic option, 58, 83, 253, 355
  Asian, 413
  continuous time, 355, 387, 413
  discrete time, 97
  lookback option, 387
expectation, 728
  conditional, 731, 738
exponential
  model, 689
  distribution, 643, 722
  Vasicek model, 166, 534, 800
  extrinsic value, 85, 214
face value, 540, 564
Fano factor, 555
Fatou’s lemma, 237, 454, 716
Feller condition, 532
filtration, 61, 131, 445
finite differences
  explicit scheme, 702, 705
  implicit scheme, 703, 706
first theorem of asset pricing, 27, 67, 179, 583
fixed
  income, 529
  derivatives, 603
  leg, 578
  rate, 609
floating
  leg, 578
  rate, 609
  strike, 59
floorlet, 612, 623
fOptions (R package), 895
foreign exchange, 508
  option, 511
foreign exchange option, 195
formula
  Lévy-Khintchine, 650
  smoothing, 650
  Tanaka, 167, 808
  Taylor, 842
forward
  contract, 118, 201, 228, 249, 523, 567, 837, 963
  American, 492, 955
  non-deliverable, 202
  measure, 561, 604
  price, 498
  rate, 567
  agreement, 567
  spot, 567–569, 609
  swap, 577
  start option, 260
  swap rate, 577
four-way zero-collor option, 9
Fourier synthesis, 136
Fourier transform, 294
  inversion, 294
FRA, 567
Fubini theorem, 653
fugazi (the), 308
future contract, 202, 776
FX option, 195
gains process, 82
Galton board, 106
Gamma, 213
gamma
  approximation, 555
  Greek, 207
  process, 658
gamma distribution, 723
gamma function, 723
gap, 679
Garman-Kohlagen formula, 511
Gaussian
  cumulative distribution function, 110, 608
  distribution, 203, 722
  random variable, 743
gearing, 85, 214
effective, 92, 214
Geman-Yor method, 422
generating function, 167, 743
geometric
  average, 416, 440
  Brownian motion, 188, 238
distribution, 726
Girsanov Theorem, 243, 244, 267, 504

This version: January 15, 2020
https://www.ntu.edu.sg/home/nprivault/index.html
jump processes, 666, 684  
Greeks, 213  
Delta, 200, 205–207, 213, 227, 231, 256  
Gamma, 207, 213  
Rho, 213  
Theta, 213, 231, 264, 863  
Vega, 213, 231, 385, 905  
gross world product, 5, 16  
guarantee  
buy back, 7  
price lock, 8  
GWP, 5  

Hartman-Watson distribution, 421  
Hawaiian option, 415  
heat  
equation, 216, 701  
map, 340  
hedge and forget, 201, 775, 964, 1028  
hedge ratio, 92, 215  
hedging, 32, 87, 89, 97, 252  
change of numéraire, 517  
static, 201, 775, 964, 1028  
strategy, 253  
with jumps, 693  
Heston model, 280, 303  
Hexanomial model, 792  
HIBOR, 576  
historical  
probability measure, 241  
volatility, 281, 305  
hitting  
probability, 455  
time, 449  
HJM  
condition, 583  
model, 581  
Ho-Lee model, 535  
Hull-White model, 535, 582  
immunization, 980  
implied  
probability, 15  
volatility, 308  
in the money, 57, 315, 765  
in-out parity, 360, 908  
independence, 717, 718, 721, 725, 727, 732, 741, 743, 744  
independent increments, 236, 668  
indicator function, 719  
infinum, 727  
infinitesimal, 153  
information flow, 62  
instantaneous forward rate, 570  

interest rate  
differential, 267  
model  
affine, 534  
Constant Elasticity of Variance, 534  
Courtadon, 533  
Cox-Ingersoll-Ross, 261, 532  
Dothan, 534, 552  
exponential Vasicek, 166, 534, 800  
Ho-Lee, 535  
Hull-White, 535  
Marsh-Rosenfeld, 534, 559  
Vasicek, 529, 535  

interest rate model  
Courtadon, 559  
Cox-Ingersoll-Ross, 225  
intrinsic value, 40, 85, 214  
invariant distribution, 292, 300, 531, 533  
inverse Gaussian process, 659  
IPython notebook, 13, 69, 83, 87, 89, 96, 126, 136, 137, 204, 228, 309, 484, 588, 790  
Itô  
formula, 155, 262  
pathwise, 655  
with jumps, 656  
isometry, 139, 143, 148, 652  
process, 155, 157, 199, 839  
stochastic integral, 139, 147, 148, 235  
table, 158  
with jumps, 661  

Jamshidian’s trick, 623  
Jensen’s inequality, 118, 237, 418, 448, 775, 848, 926  
joint  
cumulative distribution function, 357, 724  
probability density function, 724  
jump-diffusion process, 679  
knock-in option, 386  
knock-out option, 58, 360, 386  
Kullback-Leibler entropy, 686  

Lévy  
construction of Brownian motion, 136, 168, 826  
process, 657  
Lévy-Khintchine formula, 650  
Lagrangian, 556  
Laplace transform, 422, 482  

law  
of total expectation, 734
numéraire invariance, 518

OLS, 535
opening jump, 679
optimal stopping, 474

option
Asian, 413
basket, 429
call, 413
at the money, 260
barrier, 58, 355
basket, 428
bear spread, 259, 849
binary, 57, 116, 525, 771
bull spread, 259, 849
cash-or-nothing, 57, 525
chooser, 264, 864
digital, 57, 116, 525, 771
drawdown, 410
effective gearing, 92, 214
exotic, 58, 83, 97, 253, 355, 387, 413
extrinsic value, 85, 214
foreign exchange, 195
forward start, 260
gearing, 85, 214
Hawaiian, 415
intrinsic value, 85, 214
issuer, 13, 32
knock-out, 58, 360
lookback, 387
on average, 57, 258, 413, 441
on extrema, 356
out of the money, 263
path-dependent, 97, 253
power, 119, 188, 227, 260, 261, 776
premium, 32, 86, 215
straddle, 867
tunnel, 111, 113
vanilla, 200
variance call, 284
variance swap, 282
volatility swap, 291
writer, 13, 32
zero-collars, 9

optional
sampling, 451
stopping, 451
order book, 829

Ornstein-Uhlenbeck process, 529
out of the money, 57, 263

Paley-Wiener series, 136
par value, 540, 564
parity

call-put, 211, 251, 265, 322, 512, 908
in-out, 360, 908
Partial integro-differential equation, 689
partition, 718, 738
Pascal distribution, 727
path freezing, 623
path integral, 85, 331, 355, 500, 555
Euclidean, 556
path-dependent option, 97, 253
pathwise Itô formula, 655
payable date, 226
payer swap, 578
payoff function, 7, 8, 355
PDE
affine, 294, 535, 977
Black-Scholes, 200, 222
Heston, 292
integro-differential, 689
variational, 476
pentanomial model, 792
perfect
correlation, 592
physical delivery, 13, 56, 209
PIDE, 687, 689
Planck constant, 556
Poisson
compound martingale, 645, 684
distribution, 727
process, 637
compound, 674
portfolio, 20
process, 82
replicating, 89, 94
strategy, 31, 50, 77, 180, 183
admissible, 177, 182
update, 180, 183
value, 54, 78
power option, 119, 188, 227, 260, 261, 776
predictable process, 64, 81, 651
premium
early exercise, 466
negative, 27
option, 86, 215
risk, 27, 178, 239
price
critical, 484
graph, 6, 8, 9, 117, 772, 773
price lock guarantee, 8
pricing, 77, 82
with jumps, 685
principal amount, 621
probability
conditional, 717
density function, 721

This version: January 15, 2020
https://www.ntu.edu.sg/home/nprivault/index.html
integral decomposition, 99, 164, 252, 255
process, 50
stop-loss start-gain strategy, 195
stopped process, 450
stopping time, 448
theorem, 451
straddle option, 867
Stratonovich integral, 816
strike price, 6, 31
floating, 59
string model, 600
strong Markov property, 643
submartingale, 446
super-hedging, 32, 68
supermartingale, 446
Svensson parametrization, 587
swap, 577
amortizing, 579
measure, 498, 613, 630, 632
payer, 578
seller, 578
variance, 282
swaption, 615
Bermudan, 620
Tanaka formula, 167, 808
Taylor’s formula, 153, 842
telescoping sum, 580
tenor structure, 497, 577, 603
terms and data, 85, 213
ternary tree, 73, 111, 124
theorem
asset pricing, 27, 32, 67, 68, 179, 182, 583
dominated convergence, 462, 471
Fubini, 653
Girsanov, 243, 244, 267, 504, 666, 684
stopping time, 451
Theta, 213, 231, 264, 863
TIBOR, 576
time
business, 216
time splitting, 194, 262, 825
tower property, 63, 65, 81, 82, 86, 149, 235, 237, 255, 257, 499, 543, 733, 737, 741, 757, 790
transform
Esscher, 698
Fourier, 294
Laplace, 422, 482
martingale, 64, 82, 451
treasury note, 531

Notes on Stochastic Finance

This version: January 15, 2020
https://www.ntu.edu.sg/home/nprivault/indext.html
Notes on Stochastic Finance

Author index

Achdou, Y. 321
Albanese, C. 532
Albrecher, H. 294
Applebaum, D. 657
Aristotle 5
Attari, M. 294
Bachelier, L. 2, 137
Barone-Adesi, G. 484
Barrieu, P. 421
Benth, F.E. 429
Bermin, H. 406
Björk, T. 41, 589
Black, F. 3, 182, 197, 564, 610, 611
Bosq, D. 639
Boulding, K.E. 183, 496
Brace, A. 4, 595
Breeden, D.T. 318
Brémaud, P. 650
Brigo, D. 546, 594, 977
Brown, R. 1
Burdzy, K. 332
Carr, P. 287, 303, 422
Chan, C.M. 59, 360
Chan, K.C. 558
Charpentier, A. 574
Çinlar, E. 721
Cont, R. 650, 657, 665, 671, 679
Courtadon, G. 533, 559
Cox, J.C. 68, 225, 261, 532
Crépey, S. 415
Curran, M. 427
Da Fonseca, J. 302
Dahl, L. O. 429
Dana, R.A. 398
Dash, J. 500
Dassios, A. 410
Deelstra, G. 429
Demeterfi, K. 304
Denson, N. 523
Derman, E. 304, 319, 564
Devore, J.L. 711
Di Nunno, G. 97, 252
Diallo, I. 429
Doob, J.L. 446, 451, 483
Dothan, L.U. 534, 552
Downes, A. 523
Dudley, R.M. 143
Dufresne, D. 422
Dupire, B. 319
Dvoretzky, A. 332
Einstein, A. 2
El Karoui, N. 498, 518
El Khatib, Y. 406, 408
Elliott, R.J. 466
Erdos, P. 332
Eriksson, J. 59, 360, 380
Ewald, C.-O. 442
Faff, R. 973
Feller, W. 532, 858
Folland, G.B. 133
Föllmer, H 28, 32, 67, 68, 109
Fouque, J.-P. 280, 298
Friz, P. 287, 323
Galton, F. 106
Gao, M. 302
Garman, M.B. 511
Gatarek, D. 4, 595
Gatheral, J. 287, 298, 303, 323
Geman, H. 420, 422, 498, 518, 923
Gerber, H.U. 488, 698
Glasserman, P. 708
Gray, P. 973
Guirreri, S. 574

This version: January 15, 2020
https://www.ntu.edu.sg/home/nprivault/index.html
Notes on Stochastic Finance

Pliska, S.R. 179, 182
Poisson, S.D. 637
Prayoga, A. 290, 555
Profeta, C. 344
Proske, F. 97, 252
Protter, P. 155, 160, 243, 254, 255, 504, 515, 518, 543, 545, 711

Radon, J. 243
Rebonato, R. 281
Revuz, D. 133
Rochet, J.-C. 498, 518
Rogers, C. 434
Rosenfeld, E.R. 534, 559
Ross, S.A. 68, 225, 261, 532
Rouah, F.D. 294
Rouault, A. 421
Royerette, B. 344
Rubinstein, M. 68
Rudin, W. 141, 142
Ruiz de Chávez, J. 97

Samuelson, P.A. 3
Sanders, A.B. 558
Santa-Clara, P. 600
Sato, K. 672
Schied, A. 28, 32, 67, 68, 97, 109
Schoenmakers, J. 620
Scholes, M. 3, 4, 182, 197
Schoutens, W. 294
Schröder, M. 422
Schwartz, E.S. 478, 481
Scorsese, M. 308
She, Q.H. 302
Shi, Z. 434
Shiryaeve, A.N. 179, 182
Shiu, E.S.W. 488, 698
Shreve, S. 343, 351, 366, 438, 464, 485, 526, 896

Sircar, K.R. 280, 298, 323
Sircar, R. 232, 298
Solna, K. 280, 298
Sornette, D. 600
Steele, J.M. 475
Stroock, D.W. 740

Tankov, P. 650, 657, 665, 671, 679
Teng, T.-R. 517, 611, 620
Thales 5
Tistaert, J. 294
Toy, B. 564
Turnbull, S.M. 423

Uy, W.I. 554

Vanmaele, M. 429
Vašiček, O. 529, 535, 546
Vorst, A.C.F. 418

Wakeman, L. 423
Watanabe, S. 147, 237
Wei, X. 620
Whaley, R.E. 484
White, A. 535
Widder, D.V. 218
Wiener, N. 2, 136
Williams, D. 97
Wilmott, P. 488
Wong, H.Y. 59, 360
Woodward, D.E. 281, 328, 884
Wu, X. 565

Yang, Z. 442
Yor, M. 133, 280, 344, 416, 420–422, 553, 923
Yu, J.D. 426, 555

Zhang, Q. 302
Zou, J. 304

This version: January 15, 2020
https://www.ntu.edu.sg/home/nprivault/index.html
References


Notes on Stochastic Finance


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Notes on Stochastic Finance


This book is an introduction to the pricing and hedging of financial derivatives, including vanilla and exotic options, by stochastic calculus and partial differential equation methods. The presentation is done both in discrete and continuous-time financial models, with an emphasis on the complementarity between algebraic and probabilistic methods. In particular it covers the pricing of some interest rate derivatives, of American options, of exotic options such as barrier, lookback and Asian options, and stochastic models with compound Poisson jumps. The text is accompanied with a number of figures and simulations, and includes numerous examples based on actual market data. The concepts presented are also illustrated by 209 exercises and 12 problems with complete solutions.