Suppressing Chaos with Hysteresis in a Higher Order Neural Network

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Artificial neural networks (ANN’s) attempt to mimic various features of a most powerful computational system—the human brain. Since ANN’s consist of a large number of parallel array of simple processing elements (neurons), they are naturally suited for today’s fast-developing VLSI technology. For instance, Linares-Barranco et al. [1] designed a programmable analog neural oscillator with hysteresis appropriate for monolithic integrated circuits. Dynamic systems have many applications; however, stability is often desired. We show analytically that hysteresis at the single neuron level can provide a simple means to preserve stability in an ANN even when the nature of the system is chaotic.

Wang and Ross [2] studied static retrieval performance of a network of binary hysteretic neurons in the presence of random noise. They showed that neuronal hysteresis, which results in a tendency for each neuron to remain in its current state, helps the neurons to resist random signals and avoid random response, thereby improving the overall retrieval ability of the network. Through numerical simulations to solve an optimization problem, Takefuji and Lee [3] used binary hysteretic neurons to accelerate the convergence to the global minimum by suppressing oscillatory behaviors encountered during the convergence process. However, these oscillatory behaviors should be considered numerical artifacts, since the dynamics in neural networks used for these optimization problems are nonoscillatory and the convergence process represents a descent on a Lyapunov energy surface.

In this brief we consider $N$ binary hysteretic neurons with the following updating rule (Fig. 1) (see (1) at the top of the next page) where $S_i(t)$ represents the state of neuron $i$ at time $t$ and $\alpha$ is the half-width of the bistable region. The total input for neuron $i$ is

$$h_i(t) = \gamma_1 \sum_{j=1}^{N} T_{ij} S_j(t) + \gamma_2 \sum_{j,k=1}^{N} T_{ijk} S_j(t) S_k(t) + \eta_i \tag{2}$$

where

$$T_{ij} = C_{ij} \sum_{\mu=1}^{p} S^{\mu}_i S^{\mu}_j \quad \text{and} \quad T_{ijk} = C_{ijk} \sum_{\mu=1}^{p} S^{\mu}_i S^{\mu}_j S^{\mu}_k \tag{3}$$

are the modified first- and second-order Hebbian synaptic efficacies, $S^\mu$ is the $\mu$th stored pattern, and $p$ is the number of patterns stored. The coefficients $\gamma_1$ and $\gamma_2$ measure the relative strengths of first- and second-order interactions. We have introduced synaptic disruptions in the efficiencies $T_{ij}$ and $T_{ijk}$ by choosing random variables $C_{ij}$ and $C_{ijk}$ as follows: $C_{ij}$ is 1 with a probability $(C/N)$, $C_{ijk}$ is 1 with a probability $(2C/N^2)$, $C_{ij}$ and $C_{ijk}$ are zero otherwise. We also include in (2) a background Gaussian noise $\eta_i$ with a standard deviation $\sigma_\eta$ in order to take into account the presence of signal transmission noise.

In the absence of neuronal hysteresis ($\alpha = 0$), the above higher-order system was discussed by Wang, Pichler, and Ross [4]. The references:

\[ S_i(t+1) = \begin{cases} +1 & \text{if } S_i(t) = +1 \text{ and } h_i(t) > -\alpha; \text{ or } S_i(t) = -1 \text{ and } h_i(t) > \alpha \\ -1 & \text{if } S_i(t) = +1 \text{ and } h_i(t) \leq -\alpha; \text{ or } S_i(t) = -1 \text{ and } h_i(t) \leq \alpha \end{cases} \]

(1)

Fig. 1. Input-output response functions for a binary artificial neuron with hysteresis.

Network dynamics in the presence of neuronal hysteresis can be obtained analytically in a similar fashion:

\[
m(t+1) = \left [ \frac{1 + m(t)}{2} \right ] \text{erf} \left \{ \frac{\gamma_1 m(t) + \gamma_2 [m(t)]^2 + \alpha'}{\sqrt{2} \sigma} \right \} + \left [ \frac{1 - m(t)}{2} \right ] \text{erf} \left \{ \frac{\gamma_1 m(t) + \gamma_2 [m(t)]^2 - \alpha'}{\sqrt{2} \sigma} \right \}
\]

(4)

where \( \alpha' = \alpha / C \), \( m(t) = \langle \vec{S}(t) \cdot \vec{S}^i \rangle / N \) is a statistical average of the overlap between the state of the network and the stored pattern to which the network is initially close. \( \sigma \equiv \sqrt{\left( \gamma_1^2 + \gamma_2^2 \right)(\rho - 1)/C + (\sigma_0/C)^2} \) is a rescaled noise level that represents the combined effects of the random synaptic disruption, interference between stored patterns, and additional background noise.

Fig. 2 shows iterative solutions of the dynamic equation (4) for various widths of the bistable region in the neuronal hysteresis in the case where \( \gamma_1 = 1 \) and \( \gamma_2 = -1 \). Fig. 2(a) shows the case without neuronal hysteresis, where abundant oscillatory and chaotic activities are evident. When a small amount of neuronal hysteresis is introduced, chaos is first suppressed [Fig. 2(b)]. As the hysteretic width increases, chaos disappears and periodic oscillations also become suppressed [Fig. 2(c)]. The system becomes completely stable for \( \alpha' \geq 0.1 \).

Fig. 2. Solutions (fixed points) of \( m \) given by dynamic equation (4), the overlap between the state of the network and the initial attracting memory pattern, as a function of the rescaled noise level \( \sigma \) (a) in the absence of hysteresis, i.e., \( \alpha' = 0 \). (b) \( \alpha' = 0.01 \), incomplete bifurcation and suppressed chaotic region. (c) \( \alpha' = 0.02 \) reduction of chaos to oscillations with periods no greater than 4. (d) \( \alpha' = 0.1 \) disappearances of both chaos and periodic oscillations.

REFERENCES