Geotechnical Insights from Reliability-Based Design to Improve Partial Factor Design Methods

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ABSTRACT

This paper presents insights obtained from reliability analysis and reliability-based design (RBD) for three geotechnical engineering examples, namely an underwater slope in San Francisco Bay mud that failed during excavation, a spread footing sustaining both vertical and horizontal loads, and laterally loaded piles in soil modelled with nonlinear and depth-dependent Matlock strain-softening p-y curves. The aim is to show how RBD via the first-order reliability method (FORM) can overcome some limitations of the load and resistance factor design (LRFD) method and the Eurocode 7 (EC7) design method. The differences and similarities between the design point in RBD and those in LRFD and EC7 are discussed. The ability of RBD-via-FORM to provide valuable information at its design point and to automatically reflect parametric uncertainties, correlations, and case-specific sensitivities and to resolve subtle load-resistance duality is demonstrated. It is suggested that RBD-via-FORM can be conducted in tandem with partial factor design methods in order to overcome some limitations which sometimes arise in the latter.

INTRODUCTION

The design approach based on the overall factor of safety has long been used by geotechnical engineers. More recent alternatives are the partial factor design approach, including the load and resistance factor design (LRFD) method in North America, and the Eurocode 7 (EC7). Yet another approach can play at least a useful complementary role to LRFD and EC7, namely the design based on a target reliability index that explicitly reflects the uncertainty of the parameters and their correlation structure. Among the various versions of reliability indices, that based on the first-order reliability method (FORM) is most consistent.

The similarities and differences between the design point of the FORM and that of the partial factor design methods are summarized in Fig. 1. RBD-via-FORM is used in this paper. Examples of RBD-via-FORM are presented to show that RBD can offer interesting insights to complement the evolving LRFD and EC7. Specifically, this study investigates (i) FORM analysis of an underwater slope in San Francisco Bay mud that failed during excavation, (ii) RBD of a spread footing sustaining both vertical and horizontal loads, and (iii) RBD of laterally loaded piles in soil modelled with nonlinear and depth-dependent Matlock strain-softening p-y curves. It is shown that RBD can automatically reveal critical design scenarios and obtain design values of loads and resistance which reflect different parametric sensitivities across different scenarios, and resolves stabilizing-destabilizing duality. Focus is on insights and how RBD can complement LRFD.

The classical u-space approach for the FORM and the Hasofer-Lind method is described in Ang and Tang (1984), Haldar and Mahadevan (1999), Melchers (1999), Baecher and Christian
(2003), for example. Low and Tang (2007) presented an intuitive perspective and practical procedure for FORM reliability analysis and RBD, which obtains the same results as the classical u-space approach but is much more transparent than the latter. Response surface method can be used to bridge standalone numerical software with FORM analysis, e.g. Chan and Low (2012).

The useful insights and information from FORM-based RBD may not be obtainable in other probabilistic approaches like the first-order second-moment (FOSM) method and the Monte Carlo simulation method.

![Figure 1. FORM design points reflect parametric correlations and context-sensitivities; design points in LRFD and Eurocode 7 are based on nominal (or characteristic) values and partial factors.](image)

Design point in FORM:

\[ \beta = \frac{R}{r} \]

Limit state surface: boundary between safe and unsafe domain

1. \( \beta \)-ellipses
2. 1-\( \sigma \) equivalent dispersion ellipse
3. Equivalent mean-value point
4. Design point
5. Safe domain
6. Unsafe domain

Design point in LRFD:

\[ \sum (LF) Q_{ni} = (RF) R_n \]

Load factor
Resistance factor

Design point in Eurocode 7:

"Conservative", for example, 20 percentile for strength parameters, 80 percentile for loading parameters

Nominal loads
Nominal resistance

The design point in LRFD is obtained when the sum of amplified (factored) loads \( Q_1, Q_2, Q_3 \ldots \) is equal to the diminished (factored) resistance \( R \).
Figure 2. (a) An underwater slope failure in San Francisco Bay mud, (b) the average cutrend, and the $\mu \pm 1$ std. dev. lines, and (c) the spread of probability of failure reflects the spread of factor of safety.
Load and Resistance Factor Design (LRFD) approach and Eurocode 7 design approach

The basic design criterion for LRFD is given as (e.g., Salgado, 2014):

\[ \sum (LF)_i Q_{ni} \leq (RF) R_n \text{ or } \sum \gamma_i Q_{ni} \leq \phi R_n \]  

(1a)

where \( LF = \gamma \) = load factor, \( RF = \phi_R \) = resistance factor, \( Q_{ni} \) = nominal loads, and \( R_n \) = nominal resistance.

Economical design is achieved when the above inequality sign becomes the equal sign:

\[ \sum (LF)_i Q_{ni} = (RF) R_n \text{ or } \sum \gamma_i Q_{ni} = \phi R_n \]  

(1b)

which means:

\[ S_{\text{Design (factored)}} \text{ values of } Q_i = \text{Design (factored) value of resistance } R \]  

(1c)

Eq. (1c) is analogous to the FORM design point on the limit state surface in Fig. 1(a), in the two dimensional space of loads \( Q_1 \) and \( Q_2 \), and \( c' \) and \( \phi' \). Visualizing the image of a FORM design point and a dispersion ellipsoid tangential to a limit state surface in the higher dimensional space of the examples in this study can be done in the mind’s eye.

An underwater slope in San Francisco Bay mud is examined next to address some general concerns about probabilistic analysis.

INSIGHTS FROM A SLOPE FAILURE IN SAN FRANCISCO BAY MUD

The failure of a slope excavated underwater in San Francisco Bay has been described in Duncan and Wright (2005). The slope was part of a temporary excavation and was designed with an unusually low factor of safety to minimize construction costs. During construction a portion of the excavated slope failed.

Low and Duncan (2013) analyzed the same underwater slope that failed, first deterministically using data from field vane shear and laboratory triaxial tests, Fig. 2, then probabilistically, accounting for parametric uncertainty and positive correlation of the undrained shear strength and soil unit weight. In the deterministic analysis, the factors of safety were computed with search for critical noncircular slip surface based on a reformulated Spencer method. The \( F_S \) values obtained were 1.20, 1.16 and 1.00, for three types of undrained shear strength data, namely field vane tests, UU triaxial tests on trimmed 35 mm specimens, and UU triaxial tests on untrimmed 70 mm specimens, respectively. For comparison, the corresponding \( F_S \) values for critical circular slip surfaces were 1.23, 1.19 and 1.03, respectively, slightly higher than those of the critical noncircular slip surfaces. It was noted that measured strength values were affected by disturbance and rate of loading effects. Subtle errors were also caused by extrapolation of the undrained shear strength (in situ and lab tests data, available only for the upper 21 m of the Bay mud, from depth 6 m to depth 27 m, Fig. 2) to the full depth of underwater excavation. Since the midpoint of a slip circular arc is at about the two-third depth, this means that in limit equilibrium slope stability analysis, half the slip surface was based on extrapolated strength. In the probabilistic analyses, FORM analyses based on lognormal distributions and critical circular slip surfaces produced failure probability values (\( P_f \approx \Phi(-\beta) \)) of 9.7\%, 19.4\% and 45.6\%, for undrained shear strength from field vane tests, UU triaxial tests on trimmed 35 mm specimens, and UU triaxial tests on untrimmed 70 mm specimens, respectively. These \( P_f \) values are much higher than the \( P_f \) of about 0.6\% implied by the commonly required \( \beta \) of 2.5, or \( P_f \) of 0.14\% for a target \( \beta \) of 3.0. Hence a failure was not unlikely, and did happen.

FORM analyses can be performed easily with search for the reliability-based critical
noncircular slip surfaces, producing probabilities of failure \( (P_f \approx \Phi(-\beta)) \) a few % higher than the above mentioned 9.7\% to 45.6\% range (circular slip surfaces) for the three types of undrained shear strength data. However, obtaining probability of failure from Monte Carlo simulations for critical noncircular slip surfaces (for comparison with FORM \( P_f \)) would be very time-consuming because a search for critical noncircular slip surface is required for each set of random numbers generated in Monte Carlo simulations. In contrast, it is much simpler to obtain Monte Carlo \( P_f \) values with search for critical circular slip surface, and these (9.8\%, 19.5\%, 45.8\%) are practically identical to the FORM \( P_f \) values of 9.7\%, 19.4\% and 45.6\% mentioned above.

The following may allay some concerns of practitioners regarding probabilistic approach:

(i) The probability of failure in the San Francisco underwater slope is not unique, but depends on the type of test data (whether in situ field vane, trimmed or untrimmed triaxial specimens). The overall factor of safety also varies with the data used. However, the reliability analysis revealed unacceptably high probability of failure regardless of data type. The FORM \( P_f \) values of about 10\%, 19\% and 46\% (Fig. 2(c)) are much higher than that corresponding to the commonly required \( \beta \) of 2.5. Hence it is not surprising that a section of the slope failed.

(ii) The same limitations to probabilistic approaches with respect to approximate inputs, idealized formulations and non-exhaustive factors also apply to the outputs of deterministic analysis (e.g. displacement prediction). One is reminded of Terzaghi’s pragmatic approach of aiming at designs such that unsatisfactory performance is not likely, instead of aiming at designs which would behave precisely (e.g. footing settlement of exactly 25mm). It is in the same spirit that RBD aims to achieve sufficiently safe design, not at a precise probability of failure. For example, in a RBD for a target reliability index of \( \beta = 2.5 \), the resulting design is not to be regarded as having exactly a probability of failure equal to \( \Phi(-\beta) = 0.6\% \), but as a design aiming at a sufficiently small probability of failure (e.g. <1\%). One may note that a LRFD design or EC7 design, via conservative nominal (characteristic) values and code-specified partial factors, also aims at a sufficiently safe design by implicit considerations of parametric uncertainties and sensitivities. In comparison, the statistical data and correlations in RBD are open to view.

Instead of shunning probabilistic approaches, case-specific scrutiny and counter-suggestions for more reasonable statistical inputs and related issues in RBD are more likely to result in advancements and improvements.

The next example demonstrates that the design point in RBD-via-FORM contains valuable information, and deftly reflects different parametric sensitivities across different scenarios.

LOAD-RESISTANCE DUALITY AND CONTEXT-SENSITIVITY AUTOMATICALLY RESOLVED IN RBD-VIA-FORM

The bearing capacity example problem in this section aims to demonstrate that the design point of RBD contains sensitivity information, and also allows load and resistance factors (LF and RF) to be back-calculated from the RBD results, if desired. However, such back-calculated LF and RF are case-specific and non-intrinsic, because they depend on (i) the assumed nominal values of LRFD, (ii) the target reliability index in RBD, and (iii) subtleties and varying sensitivities arising from the stabilizing-destabilizing duality of vertical loads when a horizontal load is also acting.

The strip footing in Figure 3 is subjected to a horizontal load \( Q_h \) and centrally applied vertical live load \( Q_L \) and dead load \( Q_D \). The base of the foundation is at a depth of 1.0 m in a silty sand
with mean $\phi' = 25^\circ$, $c' = 15$ kN/m$^2$, and unit weight $\gamma = 18$ kN/m$^3$. A negative correlation coefficient of $-0.7$ between $c'$ and $\phi'$ is modelled in the correlation matrix. The coefficients of variation (COV, i.e. StDev/Mean) for $c'$, $\phi'$, $Q_h$, $Q_l$, and $Q_D$ are 0.20, 0.10, 0.15, 0.20 and 0.10, respectively. The water table is far below the foundation. This case is similar to a case in Low (2017), but with additional plots here for values of $\beta$ from 2.5 to 3.5 in Fig. 3 and Fig. 4(a), and the revelatory Fig. 4(b) signifying the load-resistance duality of the vertical loads $Q_h$ and $Q_l$ when the mean value of the horizontal load $Q_h$ increases from 0 to 200 kN. The y axis of Fig. 4(b) is the normalized equivalent design value, $n_i = \left( x_i - \mu_i^N \right) / \sigma_i^N$, in which the superscript $N$ denotes equivalent normal mean and equivalent normal standard deviation of the lognormal distribution.

![Figure 3: RBD reflects case-specific sensitivities, target $\beta$, and input uncertainties](image)

With respect to bearing capacity failure, the performance function ($\text{PerFunc}$) is:

$$g(x) = q_u - q$$

(2a)

where

$$q_u = c'N_c d_i d_e + p_o N_q d_i d_e + \frac{B'}{2} \gamma N_\gamma d_i d_e q = (Q_L + Q_D)/B'$$

(2b,c)

in which $q_u$ is the ultimate bearing capacity, $q$ the applied bearing pressure, $p_o$ the effective overburden pressure at foundation level, $B'$ the effective width of foundation = $B - 2e_B$ where $e_B$ is the eccentricity of the resultant inclined load, and $N_c$, $N_q$, and $N_\gamma$ are bearing capacity factors, which are functions of the friction angle ($\phi'$) of soil. The six factors $d_i$ and $i_j$ in Eq. (2b) account for the depth effects of foundation and the inclination effect of the applied load. The formulas for $N_c$, $N_q$, and $N_\gamma$ and the six depth and load inclination factors are based on Bowles (1996). One should be aware that there are other bearing capacity equations for $N_\gamma$ and correction factors in
the literature, which could yield numerical outcomes different to some extent from those discussed below, but without changing the trends and manifestations of the results observed. Because this study focuses on insights from RBD and its complementary role to LRFD, the findings and conclusions are not likely to be sensitive to the version of equations used.

For the case without horizontal load, a base width B of 1.42 m is required to achieve a target reliability index of $\beta = 2.5$ against bearing capacity failure. The design value of $c'$, 15.40 kPa in Fig. 3, is slightly above the $c'$ mean value of 15 kPa, due to negative correlation coefficient of -0.7 between $c'$ and $\phi'$. One may note that the ultimate bearing capacity $q_u$ consists of three components, with $c'$ affecting the first component only, but with $\phi'$ (via $N_c$, $N_q$, $N_\gamma$, $i_q$, $i_c$, $d_q$, $d_c$) affecting all the three components. Hence it is not surprising that for this case the sensitivity to $\phi'$ is much more than the sensitivity to $c'$, with n values -1.61 versus 0.23 in the column labelled n. RBD-via-FORM is able to reflect case-specific sensitivity (i.e. context-sensitivity).

![Figure 4](image.png)

Figure 4. (a) Back-calculated LF and RF are scenario-dependent, (b) Load-resistance duality is revealed in RBD-via-FORM.

When $Q_h = 0$, the vertical load $Q_v$ is an unfavorable action without ambiguity. However, when $Q_h$ is acting and of significant magnitude relative to $Q_v$, the latter possesses load-resistance duality, because load inclination and eccentricity decreases with increasing $Q_v$. RBD automatically takes this load-resistance duality into account. Interestingly, RBD reveals that the design values of $Q_L$ and $Q_D$ are even lower than their mean values, when the mean value of the horizontal load $Q_h$ is 200 kN, thereby revealing the load-resistance duality of $Q_v$ when significant $Q_h$ is acting. It might be difficult for partial factor design approaches to deal with a parameter that possesses load-resistance (unfavorable-favorable, destabilizing-stabilizing) duality, such as the vertical load $Q_v$ in the presence of horizontal load $Q_h$.

Note that the bearing capacity equation is approximate, even for idealized conditions. The six factors $d_i$ and $i_j$ account for the depth effects of the strip foundation and the inclination effect of the applied loads. The formulas for these factors are based on Bowles (1996), which differ from those in EC7.
RBD can be done for cases with multiple failure modes (ultimate limit states, or ULS) and serviceability limit states (SLS), as illustrated next for laterally loaded piles.

<table>
<thead>
<tr>
<th></th>
<th>LF</th>
<th>mRF</th>
</tr>
</thead>
<tbody>
<tr>
<td>SLS</td>
<td>1.38</td>
<td>0.97</td>
</tr>
<tr>
<td>ULS</td>
<td>1.40</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Figure 5. RBD-via-FORM reveals context-sensitivity of loads and resistance for laterally loaded pile (a) with 26 m cantilever length and (b) fully embedded pile with zero cantilever.

**CASE-SPECIFIC SENSITIVITIES IN RBD OF LATERALLY LOADED PILES**

Figure 5(a) shows a steel tubular pile in a breasting dolphin, which was analysed deterministically in Tomlinson (1994). This laterally loaded pile with embedment depth of 23 m below seabed and a cantilever length $e$ of 26 m above seabed is analysed probabilistically here with RBD-via-FORM, including investigation on the effect of cantilever length. Soil-pile interaction was based on the nonlinear and strain-softening Matlock $p$-$y$ curves. At the mean input values of $P_h$ and undrained shear strength $c_u$, the pile deflection $y$ is 0.06 m at seabed, and 1 m at pile head. For reliability analysis, the $P_h$ was assumed to be normally distributed, with mean value 421 kN and a coefficient of variation of 25%. The mean $c_u$ trend is $\mu_{c_u} = 150 + 2z$, kPa, with a coefficient of variation of 30%. Spatial autocorrelation was modelled for the $c_u$ values below seabed by $\rho_{ij} = \exp\left(-\frac{|z_i - z_j|}{\delta}\right)$, with an autocorrelation distance $\delta$ of 2 m. For $D = 1.3$ m and $t = 30$ mm, the $\beta$ index obtained was 1.514 with respect to yielding at the outer edge.
of the annular steel cross section. The sensitivities of $P_H$ and $c_u$ change with the cantilever length $e$. The different sensitivities from case to case are automatically revealed in reliability analysis and RBD, but will be difficult to consider in codes based on partial factors.

A target $\beta$ of 3.0 can be achieved in RBD for both ULS (bending) and SLS (assuming $y_{\text{limit}} = 1.4 \text{ m}$) using steel wall thickness $t = 32 \text{ mm}$ and external diameter $d = 1.42 \text{ m}$, Fig. 5(a).

The laterally loaded pile example of Fig. 5(a) is one of a group of piles in a breasting dolphin, with 23 m embedment length below seabed and 26 m cantilever length in sea water. For both the pile bending failure ULS and the pile head deflection SLS, the design point in RBD shows decreasing sensitivity of $c_u$ with depth, i.e., decreasing $(c_u^* - \mu_{c_u})/\sigma_{c_u}$ with depth, where $c_u^*$ are the design undrained shear strength values at various depths obtained in RBD.

RBD-via-FORM does not require estimated conservative nominal (characteristic) values and partial factors. Nevertheless, it is possible to back-calculate the partial factors (or load and resistance factors) from the design point of FORM. This was done in for a laterally loaded pile in sea with 26 m cantilever length (Fig. 5(a)), and a laterally loaded pile on land with zero cantilever length (Fig. 5(b)). The spatially autocorrelated soil properties and statistical inputs are the same for both cases. The back-calculated load factors (LF) and resistance factors (mRF) are tabulated in Fig. 5, for the case with 26 m cantilever length (Fig. 5(a)), and for the case with zero cantilever length (i.e. fully embedded), Fig. 5(b), for pile head displacement SLS and pile bending ULS. The case-specific and non-intrinsic sensitivities of the loads and resistance are revealed. Deeper discussions for this case are in Low (2017). Other examples are presented in Low et al. (2017), which is Chapter 4 of the TC205/TC305 joint report.

SUMMARY AND CONCLUSIONS

Reliability analyses and reliability-based designs were conducted for three soil engineering problems, namely an underwater slope failure in San Francisco Bay mud, a spread footing with vertical and horizontal loads, and laterally loaded piles with different cantilever lengths. The aim is to show how RBD via the first-order reliability method (FORM) can overcome some limitations and difficulties of LRFD and EC7. Among the merits of FORM is the information contained in its design point (the most probable point of failure) where an expanding dispersion ellipsoid (or equivalent ellipsoid if non-Gaussian distributions are involved) just grazes the limit state surface. The differences and similarities between the design point in RBD and those in LRFD and EC7 are discussed. The ability of RBD-via-FORM to provide interesting and useful information at its design point and to automatically reflect parametric uncertainties, correlations, and case-specific sensitivities are emphasized. It may be concluded that reliability analysis and RBD-via-FORM can provide insights and guidance to the evolving partial factor design approaches when (i) Partial factors for the myriad soil engineering parameters are not yet fully covered in EC7; (ii) The sensitivities of parameters vary from case to case; (iii) Physical considerations justify modelling of parametric correlations; (iv) A resistance or load parameter possesses stabilizing-stabilizing duality; (v) different target reliability index values are aimed at to reflect different consequence of failure.

RBD-via-FORM can be used in the following two ways to unveil potentially tricky and intricate cases like those presented in this study, and hence to complement LRFD and EC7:

(a) Estimate statistical inputs, and conduct FORM analysis on a design that derives from LRFD or EC7, to estimate the reliability index and the probability of failure, and to compare the design point of FORM with that from the partial factor design approach. Note that RBD, like LRFD and EC7, aims at a sufficiently safe design, not a design with

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a precise probability of failure.

(b) Estimate statistical inputs, then obtain a design based on a FORM target $\beta$ (e.g. 2.5), for comparison with the design from LRFD (or EC7) which is based on applying specified partial factors to conservative nominal (characteristic) values. Parametric correlations should be modelled in RBD-via-FORM if justified by physical considerations.

REFERENCES


