A.1 Proof of Proposition 1

**Proof.** We extend the example in Figure 1 to a more general form. That is, there are \(|O| = n\) different paths from the adversary’s initial location to the external world through \(n\) different exit nodes with two time steps. Specifically, any two paths do not intersect. On the other hand, there are \(n\) different paths starting from the defender resource’s initial location \(v_0\) to different exit nodes by one time step. Similar to the toy example in Figure 1, the probability of catching the adversary and the defender strategy under the NE, while the probability of catching the adversary is \(U_d = \frac{1}{n}\) by the optimal non-real-time strategy under the NE. Then, \(\frac{n}{n} = n\), can be made arbitrarily large by increasing \(n\).

A.2 Proof of Theorem 1

**Proof.** We reduce the Set Cover problem to computing the NE of NEST, which is described as follows: given a set \(U\) of elements, a collection \(S \subseteq 2^U\) of subsets of \(U\), and an integer \(m\), determine whether there exists a set \(C \subseteq S\) of size \(m\) or less, such that \(\cup C \subseteq C\).

Reduction: The network structure in NEST is demonstrated by the right figure, where \(v_0^U\) and \(v_0^I\) are starting points for the adversary and the defender respectively. Between them, there are three layers of nodes. The \(S\) layer is fully connected to \(v_0^U\) where each node \(v_C\) corresponds to the set \(C \in S\). U layers I and II are two identical layers representing all the elements in the ground set \(U\). These two layers are connected with each other in an element-wise manner. Each node \(v_I^i\) in layer I is linked with \(v_C^i\) in the \(S\) layer if \(i \in C\). On the other hand, the adversary can move from \(v_0^U\) to any node in the layer II. Moreover, the defender has \(m\) resources, all located at \(v_0^I\) initially. The time horizon \(t_{max}\) is set to 2, and the nodes in \(U\) layer I are characterized as exit nodes. It is easy to verify that this is a polynomial-time reduction.

With a horizon of two time steps, suppose that the adversary reaches \(v_I^i\) at the first time step, and the defender is aware of exit node \(v_C\) chosen by the adversary. If any one of the defender’s resources reaches \(v_C\) in the \(S\) layer with \(i \in C\) at the first time step, the adversary will be captured for sure. On the other hand, if there is no set cover, then it is easy to show that there is a positive probability that the adversary will move to a \(v_I^i\) such that the corresponding \(v_C\) is not protected by any defender resource and thus, the adversary can escape. Thus, the NE of NEST answers the Set Cover problem: there exists a set \(C \subseteq S\) of size \(m\) or less covering the ground set \(U \Leftrightarrow \text{NE}, \text{defender captures the adversary with probability 1, which requires the defender to move the m resources to nodes at S layer which fully protect U layer I.}

A.3 Proof of Theorem 2

**Proof.** Given \(x\), we first show that conditions (3a)-(3c) are satisfied. By Eq.(4), we have:

\[
\sum_{d \in A_d} f_{s,t} = \sum_{d \in A_d} P_x(s) x_{s,t} = \sum_{d \in A_d} x_{s,t} = 1,
\]

which is Eq.(3a). By Eq.(2c), \(P_x(s) = \sum_{d \in A_d} x_s, \forall s \in S\). Moreover, by Eq.(1), \(\forall s \in S \cap (\{s_0\} \cup S_l)\),

\[
\sum_{d \in A_d} P_x(s) x_{s,t} = \sum_{d \in A_d} x_{s,t}.
\]

Further, by Eq.(4), \(\forall s \in S \cap (\{s_0\} \cup S_l)\),

\[
\sum_{d \in A_d} x_{s,t} = \sum_{d \in A_d} f_{s,t}.
\]

which is Eq.(3b). Obviously, Eq.(3c) is obtained from Eqs.(1), (2d) and (4). For each \(o \in O\),

\[
U_d(x, o) = \sum_{s \in S_A, h_o \subseteq o} P_x(s) = \sum_{s \in S_A, h_o \subseteq o} \sum_{d \in A_d} P_x(s)x_{s,t} = \sum_{s \in S_A, h_o \subseteq o} f_{s,t} = U_d(f, o).
\]

Therefore, \(\forall x, \exists f\) defined by Eq.(4) such that \(U_d(x, o) = U_d(f, o) \forall o \in O\).

Given \(f\), we define \(x\) in Eq. (5). Obviously, \(P_x(s) x_{s,t} = f_{s,t}\), and then \(U_d(f, o) = U_d(x, o)\). Therefore, \(\forall f, \exists x\) defined by Eq. (5) such that \(U_d(f, o) = U_d(x, o) \forall o \in O\).

A.4 Proof of Theorem 3

**Proof.** After calling our BR algorithm, we can obtain the best response policy starting from \(s_0\) against \(y\) in \(G(S, A)\). Note that \(V(s_0)\) is the defender utility by the best response against \(y\) in \(G(S, A)\) by our BR algorithm. Then \(V(s_0) \geq U_d(x, y)\). If \(V(s_0) > U_d(x, y)\), \(x\), i.e., the output at Line 4, must contain some states \(s\) with \(V(s) > 0\) or their actions that are not in \(G(S', A')\). Consequently, new states and actions will be added to \(G(S', A')\), and then \(G(S', A')\) is expanded. In the worst case, \(G(S', A') = G(S, A)\), where IGRS will stop and \(V(s_0) = U_d(x, y)\). Therefore, IGRS will converge with \(V(s_0) = U_d(x, y)\) with a finite number of iterations because the number of states and actions in \(G(S, A)\) is finite. Therefore, \(U_d(x, y) \geq U_d(x', y)(\forall x')\). Note that \(U_d(x, y) \geq U_0(x, y)(\forall y)\). Then, \((x, y)\) is an NE.

A.5 Proof of Theorem 4

**Proof.** Because \(O_{h_2}\) in \(G_{s_2}\) is similar to \(O_{h_2}\) in \(G_{s_2}\), we have

\[
V_{x_1 \rightarrow x_2}(s_2) = \sum P_{x_1 \rightarrow x_2}(s_1) y_0 = \sum P_{x_1 \rightarrow x_2}(s_1) y_0 = \sum P_{x_1}(s_1) y_0 = V_{x_1}(s_1).
\]
Suppose $\pi_{s_1 \rightarrow s_2}$ is not the best response in $G_{s_2}$ against $y'$. Then, there is a best response strategy $\pi'_{s_2}$ against $y'$ such that $V^{\pi'_{s_2}}(s_2) > V^{\pi_{s_1 \rightarrow s_2}}(s_2)$. Let $O_{y'}$ be the support set of $y'$. If $O_y \cap O_{s_2} = \emptyset$, then $V^{\pi_{s_1 \rightarrow s_2}}(s_2) = V^{\pi_{s_1 \rightarrow s_2}}(s_2) = 0$, which leads to a contradiction. If $O_y \cap O_{s_2} = O^* \neq \emptyset$, then $V^{\pi_{s_1 \rightarrow s_2}}(s_1) = V^{\pi_{s_1 \rightarrow s_2}}(s_2) > V^{\pi_{s_1 \rightarrow s_2}}(s_2) = V^{\pi_{s_1 \rightarrow s_2}}(s_1)$, i.e., $\pi_{s_1 \rightarrow s_2}$ is not the best response in $G_{s_1}$ against $y$, which causes a contradiction. Therefore, $\pi_{s_1 \rightarrow s_2}$ is the best response in $G_{s_2}$ against $y'$.

A.6 Proof of Lemma 1

Proof. If there is a strategy $y$ such that $V^{\pi^*_s}(s) = \sum_{o \in O_y} y_o \sum_{s_i \in S_i \cap S_{G_2}} h_{s_i} \leq P_{\pi^*_s}(s_c) = \sum_{o \in O_y} y_o$ under $\pi^*_s$, then there is a path $o^* \in O_h$ such that $\sum_{s_i \in S_i \cap S_{G_2}} h_{s_i} \leq P_{\pi^*_s}(s_c) = 0$. Therefore, if $\pi^*_s$ is played against $y$ with $y_o > 0$ ($\forall o \in O_h$), $V^{\pi^*_s}(s) = \sum_{o \in O_y} y_o \sum_{s_i \in S_i \cap S_{G_2}} h_{s_i} \leq P_{\pi^*_s}(s_c) = \sum_{o \in O_y} y_o \sum_{s_i \in S_i \cap S_{G_2}} h_{s_i} \leq P_{\pi^*_s}(s_c)$, which contradicts the definition of $\pi^*_s$ in Eq.(9).

A.7 Proof of Theorem 5

Proof. Suppose $\pi^*_s$ is not optimal in NEST. Then, there is a strategy $y$, and $\pi_s$ is the best response in $G_{s_2}$ such that with $V^{\pi^*_s}(s) > V^{\pi^*_s}(s)$. Let $O_y$ be $y$'s support set. If $O_y \cap O_{h_2} = \emptyset$, then $V^{\pi^*_s}(s) = V^{\pi^*_s}(s)$, which leads to a contradiction. If $O_y \cap O_{h_2} = O^* \neq \emptyset$, then, by Lemma 1, $V^{\pi^*_s}(s) = \sum_{o \in O_y} y_o \geq V^{\pi^*_s}(s)$, which also causes a contradiction.

A.8 Proof of Theorem 6

Proof. Suppose $\pi^*_1 \rightarrow s_2$ is not optimal in $G_{s_2}$. Then, there is a strategy $y$, and $\pi_{s_2}$ is the best response in $G_{s_2}$ such that with $V^{\pi^*_1 \rightarrow s_2}(s_2) > V^{\pi^*_1 \rightarrow s_2}(s_2)$. Let $O_y$ be $y$'s support set. If $O_y \cap O_{h_2} = \emptyset$, then $V^{\pi^*_1 \rightarrow s_2}(s_2) = V^{\pi^*_1 \rightarrow s_2}(s_2) = 0$, which leads to a contradiction. If $O_y \cap O_{h_2} = O^* \neq \emptyset$, then $V^{\pi^*_1 \rightarrow s_2}(s_2) > V^{\pi^*_1 \rightarrow s_2}(s_2)$, then there is a path $o^* \in O_{h_2}$ which is not predicted by $\pi^*_1 \rightarrow s_2$ i.e., $P_{\pi^*_1 \rightarrow s_2}(s_c) = 0$ ($\forall s_c \in S_c \cap S_{G_2}$ with $h_{s_1} \leq o^*$). Note that, given a deterministic policy $\pi_s$ in $G_s$, for each path $o \in O_{h_2}$, there is at least one capture state $s_c$ such that $P_{\pi^*_1 \rightarrow s_2}(s_c) = 1$ and $h_{s_1} \leq o^*$. For each $o \in h_{s_1}$, if there is a capture state $h_{s_1} \leq o^*$ because resources at the key locations take the same actions in semi-similar games in both subgames by Eq.(10). Therefore, $o^* \in O_{h_1}$ with $q(h_{s_1} \leq o^*) = q(h_{s_1} \leq o^*)$ does not generate the history in any capture state reached from $s_1$ by $\pi^*_1 \rightarrow s_2$, i.e., $P_{\pi^*_1 \rightarrow s_2}(s_c) = 0$ ($\forall s_c \in S_c \cap S_{G_2}$ with $h_{s_1} \leq o^*$). It means that, for $y'$ with $y'_o > 0$ ($\forall o \in O_{h_2}$), $V^{\pi^*_1 \rightarrow s_2}(s_1) = \sum_{o \in O_{h_2}} y_o' \sum_{s_i \in S_i \cap S_{G_2}} h_{s_i} \leq P_{\pi^*_1 \rightarrow s_2}(s_c) = \sum_{o \in O_{h_2}} y_o' \sum_{s_i \in S_i \cap S_{G_2}} h_{s_i} \leq P_{\pi^*_1 \rightarrow s_2}(s_c) \leq \sum_{o \in O_{h_2}} y_o'$, which contradicts Lemma 1 for $\pi^*_1 \rightarrow s_2$. Therefore, $V^{\pi^*_1 \rightarrow s_2}(s_2) = \sum_{o \in O_{h_2}} y_o$ for $y$ with $y_o > 0$ ($\forall o \in O_{h_2}$), and $\pi^*_1 \rightarrow s_2$ is optimal in NEST by Theorem 5.

A.9 Proof of Theorem 7

Proof. This theorem is implied by Theorems 3–6.

B Illustration for Two Techniques

B.1 Mapping Between Subgames

As shown in Figure 3(a), in the full game, the defender’s strategy space $(S, A)$ includes all states and actions, while the adversary’s strategy space $O$ includes all escaping paths. Subgames of $G_{s_1}, G_{s_2}, G_{s_3}, G_{s_4}$, and $G_{s_5}$ are part of the full game starting from states $s_1, s_2, s_3, s_4$, and $s_5$, respectively.

To illustrate the characteristic of subgames, we analyze the scenario shown in the right figure. The adversary arrives at $v_{12}$ through two different directions generating history $h_1 (\langle y_{0}^*, y_{13}, v_{12} \rangle)$ and history $h_2 (\langle y_{0}^*, y_{14}, y_{12} \rangle)$, respectively. Exit nodes are $v_1$ and $v_2$.

We consider three initial locations for two defender resources: $l_1 = \langle v_{11}, v_{5} \rangle$, $l_2 = \langle v_{3}, v_{4} \rangle$, and $l_3 = \langle v_{3}, v_{5} \rangle$. By combining the observed adversary history with the locations of two defender resources, we consider five states $s_1 = \langle v_{11}, v_{5} \rangle, s_1 = \langle v_{11}, v_{5} \rangle, s_2 = \langle v_{3}, v_{4} \rangle, s_3 = \langle v_{3}, v_{4} \rangle, s_4 = \langle v_{3}, v_{4} \rangle, s_5 = \langle v_{3}, v_{4} \rangle$, and $s_5 = \langle v_{3}, v_{4} \rangle$, which are the initial states of five subgames: $G_{s_1}, G_{s_2}, G_{s_3}, G_{s_4}$, and $G_{s_5}$.

We consider time horizon $t_{max} = 4$. Then, the set of paths generating $h_1$ is $O_{h_1} = \{1, 2 \}$ with $o_1 = h_1 \cup \langle v_{6}, v_{11} \rangle = h_{132}$ and $o_2 = h_1 \cup \langle v_{7}, v_{12} \rangle = h_{1424}$, while $CH_{h_1} = \{h_{132}, h_{124} \}$ with $h_{132} = h_{124} \cup \langle v_{6}, v_{11} \rangle = h_{23}$ and $o_3 = h_2 \cup \langle v_{7}, v_{12} \rangle = h_{24}$, while $CH_{h_2} = \{h_{21}, h_{22} \}$ with $h_{21} = h_{22} \cup v_{6}$ and $h_{22} = \ldots$.
that is reached from \( v_3 \) in \( s_5 \), moves to \( v_1 \) while the second resource in \( s_{51} \) stays at the current node by Eq.(10). This \( \pi_{s_5 \rightarrow s_5} \) defined by Eq.(10) in \( G_{s_5} \) is optimal in the full game. By \( \pi_{s_5 \rightarrow s_5} \), as shown in Figure 3(b), we define the payoff 1 for the defender in state \( s_5 \), i.e., the defender’s expected utility in subgame \( G_{s_5} \) is \( y_{o_5} + y_{o_5} \). Similarly, \( G_{s_5} \) and \( G_{s_5} \) are semi-similar and \( \pi_{s_3 \rightarrow s_4} \) in \( G_{s_4} \) by Eq.(10) is optimal in NEST.

### B.2 Adding Multiple Strategies at One Iteration

For our technique to add multiple best response strategies at one iteration, the procedure of IGRS is shown in Figure 5. Specifically, before the convergence, we sample the uniform adversary strategies based on the states that are part of the best response and then call \( BR(s_0, 0) \) at Line 4 in IGRS to compute the best response strategies against them. Here, \( S^* \) is the set of the states involved in the best response, \( A^* \) is the set of actions involved in the best response, and a uniform strategy over \( O_h \) means \( y_o = 1/|O_h| \) (\( \forall o \in O_h \)). Especially, we sample a strategy with \( y_o = 1/|O| \) (\( \forall o \in O \)) at the initial step in IGRS (i.e., after \( s_0 \) is added to \( S^* \)) because we can compute the best response efficiently by using our effective pruning techniques including the mapping technique. Each strategy will be sampled at most once.

### C The Reason to Consider Similar Subgames

We consider the scenario shown in Figure 4 again. For subgame \( G_2 \), we know that \( \pi_2 (s_2, s_1) = (6, 5) \) transiting to a capture state \( s_{11} = (6, 5, h_{11}) \) such that \( V^{\pi_2} (s_1) = 0.2 \) is the best response in \( G_2 \) against the adversary strategy with \( y_{o_5} = 0.2 \) and \( y_{o_5} = 0.1 \). Here, only the first resource contributes to the interdiction, i.e., interdicting the adversary in \( s_{11} \) due to \( \eta(h_{11}) = 6 \). Now we consider subgame \( G_{s_6} \) with initial state \( s_6 = ((11, 8), h_2) \). Obviously, \( G_{s_6} \) and \( G_{s_6} \) are semi-similar on \( l = (11) \) that is also the key location of \( G_{s_7} \). Let us define the mapping strategy of \( \pi_{s_7 \rightarrow s_6} \) by Eq.(10). That is, \( \pi_{s_7 \rightarrow s_6} (s_6) = (6, 8) \). However, \( \pi_{s_1 \rightarrow s_6} \) is not the best response in \( G_{s_6} \) against the adversary strategy with \( y_{o_5} = 0.2 \) and \( y_{o_5} = 0.1 \) because taking action \( (6, 7) \) in \( s_6 \) will result in \( V(s_6) = 0.3 \) (larger than

![Figure 5: Adding multiple best response strategies at one iteration.](image-url)
$V^{π_1→π_6}(s_6) = 0.2)$. That is, given a best response (non-optimal) strategy in $G_{s_1}$ with the key location set $l_{s_1}^*$, resources in a semi-similar subgame $G_{s_6}$ of $G_{s_1}$, who are not initially at nodes in $l_{s_1}^*$ may contribute to the interdiction, which results in a better strategy than its mapping strategy. Therefore, we cannot have the property similar to Theorem 4 between semi-similar subgames. However, Theorem 4 holds between similar subgames.