Cognitive Demand of Pre-University Mathematical Assessment Items in Singapore Schools

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Background

- Students are motivated to learn what is emphasised and valued in assessment (Smith et al., 1996). Thus, the cognitive demands, or cognitive processes that examiners place on students to perform the tasks set out in the assessment items (Pollitt, Ahmed, and Crisp, 2007), play a significant role in the function and purpose of the assessment.
- In the context of Singapore, Pre-University or Advanced Level (A-level) Mathematics teachers use national A-level Mathematics examinations items as tutorial problems for students to work on. However, little is known about A-level Mathematics teachers' perceptions of the cognitive demand of the items that they use. Additionally, while cognitive demand taxonomies are available (e.g., Bloom's taxonomy), these taxonomies are often not well suited for A-level Mathematics teachers to use to gauge the cognitive demand of assessment items.

Objectives

- To examine teachers' perceptions of the cognitive demand of A-level mathematical assessment items through the development of a cognitive demand instrument (CD Instrument) specifically for examining A-level mathematical assessment items.
- To explore the relationships between teachers' perceptions of cognitive demand and students' learning.

Conceptual Framework

This study draws mainly from the work of Hughes, Pollitt, and Ahmed (1998) on the CRAS (Complexity-Resources-Abstractness-Strategy) framework, which was constructed for studying examination items.

Mathematical Assessment Items — Questions used in tests or examinations that are intended to assess specific learning and assessment objectives in the mathematics curriculum.

Cognitive Demand Dimensions

1. **Complexity:** Number of subgoals or steps, including the links between them, that are specified (or implied) in a question to guide students.
2. **Abstractness:** Working with abstract elements rather than concrete elements, which are completely specified.
3. **Strategy:** Extent to which students are required to decide for themselves how to tackle a question or task.

Teachers' Perceptions of Cognitive Demand — *Influenced by teachers' characteristics and professional knowledge* (e.g. gender, academic qualifications, and number of years of teaching experience)

Method

SAMPLE

14 A-level mathematics teachers and 66 first-year pre-university (or Grade 11) students from a junior college and an Integrated Programme school in Singapore.

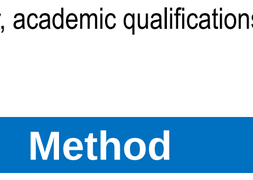
INSTRUMENTS AND MEASURES

- **Cognitive Demand:** Teachers' perceptions of cognitive demand of the assessment items were measured at two levels: (i) *Overall Cognitive Demand*, and (ii) *Complexity*, *Abstractness*, and *Strategy*, using the CD Instrument (see Figure 1).
- **Teacher questionnaires:** Part 1 – demographic information, how teachers generally judge if an item is easy or challenging; Part 2 – usefulness and usability of the CD Instrument.
- **Mathematical Assessment Items:** 12 items on Pure Mathematics topics (e.g., Functions and Graphs, Equations and Inequalities, Calculus) taken from past national A-level mathematics examinations.

Dimension	1	2	3	4	5
COMPLEXITY Refers to the demand placed on students in terms of the subgoals or set of steps, specified (or implied) in a question to guide them. The levels 1 to 5 describe the extent to which these steps are very few or many in number, and the extent to which these steps are independent or interconnected.	a single step or very few steps, with no linkages between steps	a few steps, with no linkages between steps	a few steps, with simple linkages between steps	several steps, with fairly simple linkages between steps	many steps, with complex linkages between steps
ABSTRACTNESS Refers to the demand placed on students in terms of working with abstract elements, which are mathematical objects and ideas that require considerable mental construction or some imagination (e.g., variable coefficients, unknown functions, asymptotes and limits), rather than concrete elements, which are completely specified (e.g., numerical coefficients, visible intersections). The levels 1 to 5 describe the relative proportions of abstract versus concrete elements in a question.	All, or almost all, concrete elements	Mostly concrete, and a few abstract elements	Some abstract, and some concrete elements	Mostly abstract, and a few concrete elements	All, or almost all, abstract elements
STRATEGY Refers to the demand placed on students in terms of how they decide to tackle a question. The levels 1 to 5 describe the extent to which the students are told what to do (either by the question itself or because the procedure is a standard one which they have memorised) or they are left to decide for themselves what to do.	strategy completely specified, with step by step instructions given, or involving standard procedure which students have memorised	strategy mostly specified, with much guidance given on how to proceed, leaving very few choices to be made as regards strategy	strategy somewhat specified, with some guidance given on how to proceed, leaving a few choices to be made as regards strategy	strategy mostly unspecified, with little guidance given on how to proceed, leaving some choices to be made as regards strategy	strategy completely unspecified, with no guidance given on how to proceed, leaving many choices to be made as regards strategy

Figure 1. **Cognitive Demand Instrument**

PROCEDURES AND DATA COLLECTION



Develop CD Instrument

- Adapted from CRAS framework (Hughes et al., 1998).
- Pilot trial of instrument (on another set of items) followed by minor modifications.

Actual Study with Teachers & Students

- *Stage 1:* Teachers rate overall demand of the 12 assessment items and complete Questionnaire Part 1.
- *Stage 2:* Teachers trained on how to use CD Instrument. Next, they rate the 12 items using the CD Instrument and complete Questionnaire Part 2.
- Students solve the 12 assessment items as take-home assignment.

Research supervisors: Dr Ng Wee Leng (main supervisor) and Dr Paul Shutler (co-supervisor), Mathematics and Mathematics Education Academic Group, NTU/NIE.

Results

RELIABILITY OF COGNITIVE DEMAND INSTRUMENT

- All items were successfully classified according to the levels of cognitive demand for the three dimensions.

Cognitive Demand	Krippendorff's alpha	Cohen's kappa
Complexity	.442 [0.380, 0.504]	.171***
Abstractness	.547 [0.491, 0.601]	.213***
Strategy	.531 [0.473, 0.587]	.245***
Overall	.445 [0.335, 0.549]	.292***

Note. *** $p < .001$.

Table 1. **Interrater Reliability Estimates**

Interrater reliability *below recommended threshold for Krippendorff's alpha but fair agreement for Cohen's kappa*

Cognitive Demand	1	2	3	4
1. Complexity	—	—	—	—
2. Abstractness	.753**	—	—	—
3. Strategy	.858**	.781**	—	—
4. Overall	.872**	.781**	.846**	—

Note. **$p < .01$.

Table 2. **Intercorrelations among Cognitive Demand Dimensions and Overall Demand (for 12 items)**

TEACHERS' CHARACTERISTICS AND COGNITIVE DEMAND PERCEPTIONS

- Teachers related cognitive demand to the **complexity of the mathematical processes and strategies**, **how easy or difficult the mathematical concepts** required for solving the items are, and **students' familiarity with the content** of the items.
- Teachers' characteristics and professional knowledge have implications on their perceptions of demand.

Years of Teaching Experience — **Significant differences for strategy dimension**, $\chi^2(2) = 15.95$, $p < .001$. Teachers with at least 20 years of experience rated the items as more demanding compared to those with between 11 to 19 years of teaching experience. No significant differences for complexity, for abstractness, and for overall demand.

Academic Qualifications — **Significant differences for complexity dimension**, $\chi^2(2) = 6.71$, $p = .035$, but post-hoc tests did not show any significant pairwise differences. No significant differences for abstractness, for strategy, and for overall demand.

Gender — **Significant differences for complexity dimension**, $\chi^2(2) = 8.33$, $p = .004$, **for abstractness**, $\chi^2(2) = 5.33$, $p = .021$, **and for strategy**, $\chi^2(2) = 4.46$, $p = .035$. Female teacher participants tended to perceive the assessment items to be more demanding in these three dimensions than male teachers. No significant differences for overall demand.

TEACHERS' COGNITIVE DEMAND RATINGS AND STUDENT SCORES

Comparing median ratings of teachers' ratings and mean student scores

Abstractness ($\beta = -.681$, $p = .050$) **and overall demand** ($\beta = -.791$, $p = .001$) **were significant predictors** of student scores, whereas complexity ($\beta = -.791$, $p = .225$) and strategy ($\beta = -.791$, $p = .142$) were not significant predictors.

Comparing individual teachers' ratings of cognitive demand with mean scores of their classes

Complexity ($\beta = -.576$, $p < .001$), **strategy** ($\beta = -.546$, $p < .001$), **and overall demand** ($\beta = -.356$, $p = .040$) **were significant predictors** of student scores, while abstractness ($\beta = -.411$, $p = .105$) was not a significant predictor.

Cognitive Demand	1	2	3	4	5
1. Complexity	—	—	—	—	—
2. Abstractness	.464	—	—	—	—
3. Strategy	.847*	.847*	—	—	—
4. Overall	.955**	.595	.891**	—	—
5. Scores	–.286	–.714	–.559	–.378	—

Note. *$p < .05$. ** $p < .01$.

Table 3. **Intercorrelations among Cognitive Demand Dimensions and Student Scores (for 7 items)**

USEFULNESS AND USABILITY OF COGNITIVE DEMAND INSTRUMENT

Item		SD	D	A	SA
(a)	The rubrics are useful in helping me to gauge the cognitive demand of questions.	0	0	7	5
(b)	I am more aware of the cognitive demand of questions after using the rubrics.	0	0	2	10
(c)	The rubrics cover important dimensions of cognitive demand.	0	0	9	3
(d)	The rubrics can be applied to any topic in the A-level mathematics syllabus.	0	0	9	3
(e)	The rubric descriptors provide a common language for me to discuss the demands of mathematics questions with my colleagues.	0	0	9	3
(f)	I can use the rubrics as a guide when I set questions for tests/exams in future.	0	0	10	2

Note. SD = strong disagree; D = disagree; A = agree; SA = strongly agree.

Overall, teacher participants found the CD Instrument to be useful in helping them gauge the cognitive demand of A-level mathematical assessment items.

Table 4. **Summary of Teachers' Feedback on the Usefulness and Usability of the Cognitive Demand Instrument (*n* = 12)**

Conclusion

- CD instrument provided insights into the dimensions that constitute cognitive demand for A-level mathematical assessment items on different topics; has potential to be used by teachers to develop a common understanding of cognitive demand for A-level mathematics.
- Further research: Examine how the validity and reliability of the CD instrument could be improved; examine students' perceptions of cognitive demand of assessment items.

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