Course Description

Core courses

MSM901 Fundamentals of Postgraduate Mathematics
This course aims to bring you up to speed with regard to the fundamentals of postgraduate mathematics. It involves process skills such as reading mathematics texts and writing mathematics reports, mathematical problem solving, and computational thinking via coding. It is anchored in advanced mathematics content that will allow you, as Felix Klein proposed, to view school mathematics from a higher standpoint. Content includes proof techniques, set theory and logic, and various aspects of infinity. This course is intended for educators, especially secondary and post-secondary school teachers, to help them to have an in-depth conceptual understanding of some topics in school mathematics such as number systems, calculus, and computational thinking from an advanced perspective of mathematical theory building and processes. This course will also lay a foundation for students in the Master of Science (Mathematics for Educators) programme.

Content

- 6 proofs of the infinitude of primes – including mathematical induction
- Basic set theory and logic; methods of proof
- Infinity in the secondary school curriculum
    - The Number Line
    - Cardinality
    - Probability
    - Sequences and Series

Process

- Mathematical problem solving
- Writing mathematics using Latex
- Exploring mathematics with Excel and VBA

MSM902 Selected Topics in Mathematics
This course aims to expose the candidates to some selected contemporary topics in mathematics.
(The specific content of this course is not fixed.)

As this is a topics course, the course content will be determined by the faculty teaching this course.

Elective courses

MSM903 Algebra
This course in abstract algebra aims to introduce you to rings, groups, and possibly other algebraic structures such as modules, and to present a range of examples to facilitate the understanding of the abstract theory so that you have a good grasp of the fundamental

concepts in abstract algebra. This course is intended for educators, especially secondary and post-secondary school teachers, to help them to have an in-depth conceptual understanding of some topics in school mathematics such as number systems, polynomials, from an advanced and structural perspective of abstract algebraic systems. This course will also lay a foundation for students who plan to pursue a PhD in areas related to abstract algebra.

The content consists of core topics and selected topics. The following core topics will be covered:

- Basic definitions and properties of rings, examples of rings, integral domains and fields, subrings, isomorphisms and homomorphisms of rings, ideals and quotient rings, First Isomorphism Theorem, prime and maximal ideals.
- Polynomials in one indeterminate, Division algorithm for polynomials over a field, Factor Theorem, irreducible polynomials and unique factorization, factorization of polynomials in $\mathbb{Q}[x]$, $\mathbb{R}[x]$ and $\mathbb{C}[x]$, partial fractions.
- Definitions and basic properties of groups, examples of groups, subgroups, cyclic groups, isomorphisms and homomorphisms of groups, cosets and Lagrange's Theorem, normal subgroups and quotient groups, First Isomorphism Theorem.

Selected topics include some of the following advanced topics:

- Symmetric and alternating groups, group actions, Sylow's Theorems and their applications to classification of some finite groups. Euclidean domains, principal ideal domains, unique factorization domains, field of fractions.

MSM904 Analysis

This course in real analysis aims to introduce you to the order-theoretic, algebraic and geometrical structures of the real line, and the relationships between them. In particular, you will be introduced to the concepts of sequences and convergence – first, for real number sequences, and next, for sequences of real-valued functions. This course is intended for educators, especially secondary and post-secondary school teachers, to help them gain an in-depth understanding of some topics in school mathematics such as limits of sequences, continuous functions such as polynomials, exponential function, trigonometric functions, the link between differential and integral calculi, through the lens of real analysis. This course will provide the foundation for students who reads analysis at the postgraduate level.

The content consists of core topics and selected topics. The following core topics will be covered:

- Sequences: limits of sequence, properties of convergence such as linearity property of limit operator. Completeness: upper and lower bounds, supremum and infimum, Completeness Axiom of the Real Numbers, Monotone Convergence Theorem, Cauchy sequences.
- Continuous functions via the epsilon-delta approach and the Heine's sequential approach, Three theorems involving continuity: Intermediate Value Theorem, Boundedness Theorem, and Extreme Value Theorem.
- Series, power series, radius of convergence.
- Differentiable functions: derivative by first principles, Rolle's Theorem, Mean Value Theorem, Taylor's Theorem.

Selected topics include some of the following advanced topics:

- Uniform convergence: sequence of continuous functions, differentiable functions, Dini's Theorem.
- Riemann integral: Riemann integrability, properties of Riemann integrable functions, Fundamental Theorem of Calculus, Characterization of Riemann integrability as continuity almost everywhere.
- Uniform continuity, Sufficient condition for Riemann integrability.
- Introductory metric space: basic definition of metric, open and closed sets, sequences in metric spaces, continuous functions between metric spaces, compactness and completeness, contraction mappings.
- Arzela-Ascoli's Theorem: compactness of function spaces.

MSM905 Data Science

This course is designed to introduce you the basics of data science methodology and let you be able to apply such methodology to real problems. This course is intended for educators, to empower them to perform data visualization, data preparation and prediction tasks. This course will also lay a foundation for students who plan to pursue a PhD in areas related to data science/statistics.

The content consists of core topics and selected topics. The following core topics will be covered:

- Introduction of data science.
- Introduction of Python, Anaconda, and Jupyter Notebook/Google Colab.
- Data visualization using Python.
- Data cleaning and preparation using Python.
- Supervised machine learning algorithms including Linear Regression, Logistic Regression, Decisions Trees, and Random Forests.
- Unsupervised machine learning algorithms including k-means clustering and Hierarchical clustering.

Selected topics include some of the following advanced topics:

- Basic concepts of deep learning.
- Examples of artificial neural network.

MSM906 Discrete Mathematics

This course aims to expose the candidates to counting principles which will enhance their content knowledge of teaching permutations and combinations, as well as elementary probability. Additionally, this course introduces a useful branch of discrete mathematics called graph theory which has many applications in modelling real-life contexts. This course also lays a foundation for students who plan to pursue a PhD in the area of discrete mathematics.

The content consists of core topics and selected topics. The following core topics will be covered:

- addition principle, multiplication principle, and principle of inclusion and exclusion;
- subsets and arrangements, bijection principle, and binary sequence;
- binomial expansion and Pascal's triangle;

- mathematical modelling using graphs, foundation of graph theory, and handshaking Lemma;
- isomorphism, subgraphs, and self-complementary graphs;
- ordinary generating functions, exponential generating functions, and their applications.

Selected topics include some of the following advanced topics:

- General inclusion-exclusion principle and its applications

MSM907 Geometry

Geometry is one of the foundational topics in mathematics. This course presents a complete axiomatic system for Euclidean geometry and related geometry topics. By completing this course, you will gain a clear picture of the whole hierarchical structure of geometry. You will learn the rigorous definitions of the fundamental geometry concepts, such as angles, triangles, rays, congruent/similar triangles. You will also learn the formal proofs of the fundamental results in geometry, such as the equivalence of various different triangle congruency (similarity) tests, Angle Sum Theorem and Exterior Angle Theorem as well as the Midpoint theorem. The course will also cover some advanced topics in geometry such as the non- Euclidean geometries, projective geometry or differential geometry. These advanced topics will widen and deepen students' knowledge in geometry and help those who want to pursue higher degree study.

The content consists of core topics and selected topics. The following core topics will be covered:

- General axiomatic systems
- Neutral geometry
- Euclidean parallel postulation
- Triangles, plane separation axiom
- Angle measurement
- Congruent triangles and congruency tests
- Similar triangles and similarity tests, the Fundamental Theorem on similar

triangles Selected topics include some of the following advanced topics:

- Areas in Euclidean geometry
- Hyperbolic and Elliptic geometry
- Projective geometry
- Fundamental differential geometry
- Computational geometry

MSM908 Number Theory

This course in number theory aims to introduce you to fundamental concepts in elementary number theory, including divisibility and primes, unique factorization, congruences and quadratic reciprocity. This course is intended for educators, especially secondary and post-secondary school teachers, to help them develop in-depth conceptual understanding of some topics in school mathematics such as number systems, greatest common divisor, and the Fundamental Theorem of Arithmetic. Real world applications of number theory will also be discussed. Examples include the use of check digits for error detection in our National Registration Identity Card (NRIC) numbers and the RSA encryption system for secure online transactions. This

course will also lay a foundation for students who plan to learn more advanced mathematics in areas related to algebra and number theory.

The content consists of core topics and selected topics. The following core topics will be covered:

- Axioms that define the ring of integers;
- Divisibility, Division Theorem, Greatest Common Divisor, Least Common Multiple;
- Linear Diophantine Equations in Two Variables, Extended Euclidean Algorithm, Euclid's Lemma;
- Infinitude of primes; Fundamental Theorem of Arithmetic, Applications of Unique Factorization;
- Linear Congruences, Chinese Remainder Theorem, Application to Divisibility Tests and Check Digits;
- Fermat's Little Theorem, Euler's Phi function, RSA Encryption;
- Quadratic Reciprocity;
- Representations as Sums of Squares.

Selected topics include some of the following independent/advanced topics:

- Triangular and other figurate numbers;
- Fibonacci and Lucas numbers;
- Mersenne and Fermat primes;
- Introduction to the prime number theorem;
- Gaussian integers and other quadratic integers, particularly rings without unique factorizations;
- Introduction to cryptography;
- Multiplicative functions.

MSM971 Advanced Topics in Functional Analysis
This course aims to expose students who are strong in mathematics to advanced topics in functional analysis.

The content covers topics on functional analysis and its applications.

Pre-requisites: MSM904 Analysis or equivalent

MSM972 Advanced Topics in Algebra
This course aims to expose students who are strong in mathematics to some advanced topics in algebra.

The content covers selected topics in commutative algebra.

Pre-requisites: MSM903 Algebra or equivalent