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NEW STOCHASTIC DOMINANCE THEORY FOR INVESTORS WITH RISK-AVERSE AND RISK-SEEKING UTILITIES WITH APPLICATIONS INCLUDING SOLUTIONS FOR THE FRIEDMAN-SAVAGE PARADOX AND THE DIVERSIFICATION PUZZLE

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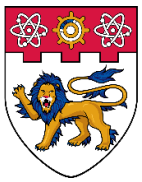

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New stochastic dominance theory for investors with risk-averse and risk-seeking utilities with applications including solutions for the Friedman-Savage paradox and the diversification puzzle *

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Abstract

In this paper, we first state some well-known problems including the Friedman-Savage paradox raised by Friedman and Savage (1948) who wonder why individuals would like to buy insurance as well as buy lottery tickets. To provide solutions to the problems, we first use the idea from Fishburn and Kochenberger (1979), Thon and Thorlund-Petersen (1988), and Chew and Tan (2005) to use two-way stochastic dominance to define the j-order risk-averse and risk-seeking utility that consists of both risk-averse and risk-seeking components and we call the utility AD utility and call investors with AD utility AD investors. Thereafter, we develop a new stochastic dominance theory for AD investors and we call the theory ADSD theory. We then develop some properties for the ADSD theory, including properties of expected-utility maximization, hierarchy, transitivity, and diversification, and properties under the additional condition of equal mean so that we can use the theory to get the solutions for all the problems and hypotheses we set in this paper. Applying the ADSD theory, we first get a new solution for the Friedman-Savage paradox. In addition, we find that AD investors could invest in both completely diversified portfolio and individual assets and, in general, buy any pair of both less-risky and more-risky assets. For example, AD investors could invest in both bonds and stocks, both bonds and futures, and both stocks and futures to get higher expected utility.

JEL Classification: D81, G11

Keywords: Stochastic Dominance; Risk Aversion; Risk Seeking; Utility Function; riskier Asset; Less Risky Asset