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Abstract

Under incomplete contracts, a worker faces a ratchet effect of innovating when closely monitored: if he uncovers a process innovation, the firm raises the future performance requirement; anticipating this, the worker never innovates. With reduced oversight, the worker accrues private information about his innovation, which generates information rent that feeds back as the worker's ex-ante innovation incentive. The firm's strategic ignorance is complemented with an incentive scheme that simultaneously induces effort from the worker and endogenously generates asymmetric information *against* the firm. The resulting mechanism provides a novel rationale to weak incentives and low-scale production at early stages of relationships.

Keywords: Freedom at work, incomplete contracts, ratchet effect, strategic ignorance, innovation, asymmetric information

JEL Classification: D21, D82, D86, M52, M54, O31

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"If you give people freedom, they will amaze you. All you need to do is give them a little infrastructure and a lot of room." — Laszlo Bock (Google's SVP, People Operations).

1 Introduction

The provision of more freedom to employees has increasingly become an important element of successful management practices. For example, a report released by the LRN at the World Economic Forum in 2014 concluded that companies with more freedom in their employment relationships are associated with more innovations and better long-term successes.[1] There is also a plethora of empirical evidence suggesting that autonomy provision to individuals often leads to improved performances.[2]

Freedom provision necessarily implies less oversight over the workers, which raises the question: why would firms benefit from less information about their workers' actions? This does not resonate well with the tenet of the canonical principal-agent model which predicts that the principal benefits from more information about the agent's actions (Holmström, 1979). Relatedly, why does a worker become more efficient when given freedom at work? There is an extensive work in the psychology and management literature studying this question, and they generally attribute this positive effect to the worker's intrinsic motivation.[3]

[1]The report is available online at “http://pages.lrn.com/the-freedom-report”.

[2]For example, Andrews and Farris (1967) show that scientists are more likely to innovate under higher degree of freedom; Amabile et al. (1990) find that workers exhibit lower levels of creativity when being watched; Shalley et al. (2000) find that autonomy increases employees' commitment to the organization and fosters innovation. Falk and Kosfeld (2006) find experimental evidence that agents react negatively to actions deemed controlling by reducing performances; Bloom et al. (2015) find that call center operators experience a 13% increase in overall performance when working from home.

[3]Intrinsic motivation are incentives that are derived through non-monetary rewards. In the psychology literature, Deci and Ryan (1987) suggest that autonomy is associated with greater interest and creativity (among other positive behaviors). In the management literature, intrinsic motivation from freedom takes the form of an empowering leadership style that encourages employees to develop self-control and initiative. Since autonomy leads to higher job satisfaction, employees under less controlling work environments have a stronger sense of ownership in the success of the organization. For example, Elloy (2005) finds that employees in self-managed teams exhibit higher levels of organizational commitment; and Amabile et al. (1996) identify

But if the worker's intrinsic motivation is the only explanation, why do we observe heterogeneity in firms' adopted monitoring environments? For example, why is there usually less monitoring in startups and "tech firms" than in factory assembly lines?

This paper focuses on the (reduced) monitoring aspect of freedom at work and illustrates an extrinsic motivation (i.e. monetary incentives) that it provides to the workers for process innovation.[4] The intuition is the following. When a worker is closely watched by his firm, any new knowledge gained from his innovation attempt is simultaneously learned by the firm. If the worker uncovers a more efficient work method, the firm, being aware of it, will raise the worker's future performance requirement; anticipating this, the worker never tries to innovate. If the worker is given freedom at work instead – for example, he is allowed to work from home – he accrues private information about any discovery made. This private information generates information rent for the worker which feeds back as his ex-ante incentive to innovate. But the resulting asymmetric information also creates ex-post inefficiency, which has to be weighed against the expected innovation benefit. This in turn explains why, for example, startups and "tech firms" give their employees a lot of freedom at the workplace whereas a factory assembly line is often associated with tight monitoring.

While this notion that a principal can benefit from "strategic ignorance" has been noted in the literature in other contexts,[5] the mechanism behind how this feature arises here is different, and the novelty of this paper lies in understanding how a firm's strategic ignorance influences its incentive scheme, which has to simultaneously induce effort from the worker and endogenously encourage asymmetric information *against* the firm. The contract design under strategic ignorance involves an adverse selection problem with moral hazard, and there is an endogenously created mass point in an otherwise atom-less distribution over the agent's

freedom provision as an important managerial practice to foster creativity at work.

[4]For papers that study how extrinsic incentives affect intrinsic motivation, see for example Bénabou and Tirole (2003) and Sliwka (2007).

[5]See, for example, Riordan (1990), Crémer (1995), Aghion and Tirole (1997), Prat (2005) and Shin and Strausz (2014).

continuous private type space. This in turn leads to interesting features in the optimal menu of contracts. The evolution of the optimal incentive scheme also provides a novel rationale for why workers' incentives are sometimes weak in the early stages of their career and why some economic relationships have to start off with low-scale production.

I consider a two-period principal-agent model with an embedded two-arm bandit problem.[6] In each period, the agent chooses between producing with an old technology (o) or a new technology (n). Output is binary and the probability of getting the high output with either technology is the agent's effort. The two technologies differ in their effort costs. The cost efficiency of o, denoted by θ^o, is commonly known – technology o represents the currently available work method inside the firm. On the other hand, the cost efficiency of n, denoted by θ^n, is ex-ante unknown to both players – technology n represents an untested work method. A higher cost efficiency implies a lower cost of effort,[7] and the common prior on θ^n is represented by a continuous distribution. Under the prior, o is ex-ante more cost-efficient than n in expectation (i.e. $\theta^o > E[\theta^n]$), but n is possibly more cost-efficient than o (i.e. $\Pr(\theta^n > \theta^o) > 0$). The agent can perfectly learn about θ^n by producing with n in period 1, which then creates an option value for period 2: if n turns out to be more cost-efficient than o (i.e. $\theta^n > \theta^o$), the agent can continue to use n in period 2; otherwise he can revert to using o. Hence, using n in period 1 is interpreted as an innovation attempt to uncover a more cost-efficient technology for period 2, but it comes at a cost of using the (in expectation) myopically less cost-efficient technology n in period 1. The contracting environment is highly incomplete – the principal can only enforce spot contract on the output. Hence the principal offers a bonus-pay contract to the agent at the start of each period.

Under a "*tight-monitoring*" environment (hereafter TM), the principal perfectly observes all of the agent's actions. This also implies that if n is used, the principal also learns about

[6] I use the female pronoun for the principal and the male pronoun for the agent throughout.

[7] Formally, the cost of effort e on technology $\tau \in \{o, n\}$ is $\psi\left(\frac{e}{\theta^\tau}\right)$ where ψ is strictly increasing and convex. Hence a higher θ^τ implies a lower cost of effort and hence, a higher cost efficiency.

θ^n together with the agent. The agent thus faces a *ratchet effect* of innovating. If the agent produces with n in period 1 and uncovers that $\theta^n > \theta^o$, the principal will offer the agent a lower period-2 bonus since she knows the agent's cost of effort has decreased, thus extracting away all the innovation surplus. Anticipating no gain in period 2, the agent always uses o in period 1 since $\theta^o > E[\theta^n]$. Hence innovation is never possible under TM.

On the other hand, under a "*freedom-at-work*" environment (hereafter FW), the agent *privately* learns θ^n after using technology n in period 1. With asymmetric information in period 2, the principal solves a contracting problem that consists of both adverse selection and moral hazard – the principal has to elicit the agent's private information on θ^n with a menu of bonus-pay contracts (adverse selection), while each contract in the menu must provide incentives for the agent to make the appropriate technology and effort choices (moral hazard). The optimal period-2 menu will have a bonus schedule that is increasing in the agent's innovation success and a fixed wage schedule that is decreasing in it. This is because an agent with a more cost-efficient technology incurs a lower cost to increase the probability of getting the bonus. Hence he is willing to accept a "riskier" contract with a lower fixed wage in exchange for a higher bonus. The increasing bonus schedule implies that the more successful innovations are utilized more intensively and efficiently.

An interesting feature of the optimal menu is the pooling of contracts for types with low cost efficiency, indicating that the principal optimally chooses not to distinguish an agent with a mildly successful innovation (i.e. θ^n that is slightly above θ^o) from an agent who failed in his innovation attempt (i.e. $\theta^n \leq \theta^o$). This feature arises due to the *mass point* of failed-innovator agents who revert to using o in period 2; these failed-innovator agents are essentially of the same type whose cost efficiency in period 2 is θ^o. If the principal wants to also distinguish the mildly successful innovators from these failed innovators (i.e. type θ^o), she has to further lower the bonus of the contract meant for θ^o. The loss in surplus from doing so is very costly to the principal because the atom-less distribution of any one type of successful innovator is incommensurate with the mass point at θ^o. Hence the possibility of innovation failure implies that innovations have to be sufficiently successful before they are "recognized" by the principal with increasingly powerful bonuses.

Under the optimal period-2 menu of contracts, only an agent who uses n in period 1 and discovers that $\theta^n > \theta^o$ will get information rent in period 2. While the potential for rent in period 2 creates incentives for the agent to use the ex-ante less cost-efficient n in period 1, the principal must also complement it with a low period-1 bonus. This is because under any period-1 bonus, the agent's optimal effort exerted on o is always higher than his optimal effort exerted on n because of $\theta^o > E[\theta^n]$. A high period-1 bonus would then encourage the agent to use o to maximize his likelihood of getting the period-1 bonus and forgo the potential period-2 rent that comes from experimenting on n.

Since the bonus represents the agent's opportunity cost of getting a low output, the requirement for a low period-1 bonus can be interpreted as the firm having to show tolerance to early failures by its worker to encourage him to innovate. This feature of low period-1 bonus in equilibrium also provides an alternative mechanism to explain why a worker's incentives tend to be weak in the early stages of his career, which differs from the explanation from the career concern literature (Holmström, 1999); and why some economic relationships have to start off with low stakes, which contrasts with the rationale in the reputation-building literature (e.g. Watson 1999, 2002; Halac 2012).

Finally, the contracts in equilibrium illustrate the evolution of the incentive scheme under the two different monitoring environments. Under TM, the technology level remains at the cost efficiency of o in both periods and hence, the incentive scheme stays constant over time. On the other hand, the incentive scheme under FW starts off with weak incentive in period 1, followed by idiosyncratic changes in incentive in period 2 through the agent's choice from the menu of contracts, which is in turn dependent on the outcome of the agent's innovation attempt. This evolution under FW can be interpreted as changes in job scopes and wage structures for the workers over time, which is akin to career progression or promotion opportunities inside the firm, which is otherwise absent under TM.

The rest of the paper proceeds as follows. The next section discusses the related literature and Section 3 outlines the main model. Next, Sections 4 and 5 solve for the equilibrium under the TM and FW environments respectively, and Section 6 compares the two monitoring environment via a comparative statics exercise and provides the conditions under which a firm would prefer FW over TM or vice versa. Section 7 discusses some related issues and Section 8 concludes.

2 Related Literature

Freixas et al. (1985), Lazear (1986), Gibbons (1987), Laffont and Tirole (1987, 1988) and Kanemoto and MacLeod (1992) are some early contributions on the ratchet effect which shows that an agent rationally restricts output as he expects the principal to respond to higher performances with more demanding requirements later. In contrast, the ratchet effect here concerns the worker not choosing the principal's desired technology instead of output restriction. More substantially, the ratchet effect literature assumes that the agent has ex-ante private information and studies the principal's problem due to short-term contracting; in contrast, information is ex-ante symmetric here. Two notable exceptions are Bhaskar (2014) and Bhaskar and Mailath (2015) who identify a ratchet effect under ex-ante symmetric uncertainty. However, the ratchet effect in these two papers arises from the agent shirking and gaining rent from the principal holding wrong (in particular, more pessimistic) beliefs about the underlying state. Moreover, while the principal suffers from the agent's ability to create differential beliefs there, the principal can benefit here from the agent having private information over the underlying state.

The ratchet effect of innovation here is reminiscent of the hold-up problem and the literature has noted that asymmetric information about the investment outcome can potentially protect the investing party from being held up, thus restoring ex-ante investment incentives. Gul (2001), González (2004), Lau (2008) and Hermalin (2013b) make this point by studying deterministic investment outcomes, and asymmetric information is created endogenously by the investor randomizing over the investment decision. In contrast, the (analog of) investment outcome here is stochastic. Hermalin and Katz (2009) and Halac (2015) consider stochastic investment outcome but their models have no room for screening and have different focus from here.[8] More substantially, this paper is not concerned with the intensive margin on the investment level, but it embeds a moral hazard problem in production at the investment stage. In turn, a key focus here is also on the incentive contract which has to be appropriately designed to induce the asymmetric information.

This paper is also related to the strategic experimentation literature. Bergemann and Hege (1998, 2005), Hörner and Samuelson (2013), and Halac et al. (2016) (among others) study optimal contracting on the experimentation outcome to solve the moral hazard problem. The way in which the tension between experimentation and forgoing current productivity is incorporated here is similar to Manso (2011). In this literature, the uncertainty is modeled on the productivity; since the productivity directly affects the contractible output, the experimentation outcome is at least partially contractible. In contrast, the uncertainty here is modeled on the cost which has no direct effect on the output, thus implying that the experimentation outcome is completely uncontractible.

More broadly, this paper also concerns the effect of information in screening problems and the agent's incentives to acquire it – see Crémer and Khalil (1992), Lewis and Sappington (1997), Crémer et al. (1998a,b), Szalay (2005, 2009) and Iossa and Martimort (2015). This literature takes the agent's cost of information acquisition as given and considers writing a menu of contracts that affects the agent's incentives to gather information.[9] In contrast, the

[8] Only González (2004) has screening among these papers.

[9] Relatedly, but with a different focus, Condorelli and Szentes (2017) study how the agent would choose

menu of contracts here *cannot* affect the agent's information acquisition incentives since the menu is offered only after the information acquisition process in period 1; instead the principal influences the agent's information acquisition incentives via the monitoring environment, which affects the agent's expected gain from information acquisition, and via the period-1 contract, which determines his opportunity cost of information acquisition.

3 Model

Consider a two-period principal-agent model. To abstract away from other frictions, I assume that both parties are risk neutral, do not discount the future, have no limited liability and have a zero per-period outside option.[10] In each period, the agent can exert effort on one of two available technologies $\tau \in \{o, n\}$; the agent can use different technology across periods. The effort choice set is always the continuum $[0, 1]$. Both technologies generate a stochastic per-period revenue of $y \in \{0, Y\}$ for the principal with $Y > 0$, and the probability of getting Y with either technology is the agent's effort. Hence the two technologies have the same productivity. To save on time subscript, period-1 effort is denoted by e and period-2 effort by its Greek letter counterpart ε; subscripts will exclusively denote partial derivative.

The cost of effort depends on the technology chosen. An effort level e (respectively ε) exerted on technology τ incurs a private cost of $\psi\left(\frac{e}{\theta^\tau}\right)$ (respectively $\psi\left(\frac{\varepsilon}{\theta^\tau}\right)$) to the agent, where the cost function $\psi(\cdot)$ satisfies Assumption 1 below. $\theta^\tau \in (0, 1]$ is the effort cost efficiency for technology τ which is constant across periods; a higher value of θ^τ indicates a more cost-efficient technology.

Assumption 1. *For all $x \in [0, 1)$, $\psi(x)$ is continuously differentiable, strictly increasing and strictly convex. Moreover, $\psi(0) = \psi'(0) = 0$, $\lim_{x\to 1} \psi(x) = \infty$, and $0 \leq \psi'''(x) \leq$*

the distribution of his private information before the contracting stage to maximize his rent.

[10] An earlier draft with limited liability for the agent is available from the author.

$\frac{2\left(\psi''(x)\right)^2}{\psi'(x)}$.[11]

In terms of information, θ^o is commonly known. On the other hand, θ^n is ex-ante unknown to both players and their common prior on θ^n is represented by a distribution function $F(\cdot)$ on $\left[\underline{\theta}, \bar{\theta}\right]$, with $0 < \underline{\theta} < \bar{\theta} \leq 1$. If n is used in period 1 with $e > 0$, the value of θ^n will be realized at the end of period 1. Hence using n in period 1 is interpreted as an innovation attempt to uncover a possibly more cost-efficient technology.[12]

Assumption 2. *F has an atom-less density f which is differentiable and strictly positive in its support. Moreover $E[\theta^n] \leq \theta^o < \bar{\theta}$, and $\frac{d}{d\theta^n}\left(\frac{1-F(\theta^n)}{f(\theta^n)}\right) \leq 0$.*

The assumption $E[\theta^n] \leq \theta^o$ implies that o is ex-ante more cost-efficient than the innovative activity n. The non-increasing inverse hazard rate property is a standard assumption in the screening literature that ensures that the screening problem is well-behaved, although as shown later, the screening problem in period 2 will still exhibit "bunching".

3.1 Contracts

The revenue y is the only contractible term throughout; neither the effort choice, technology choice nor the experimentation outcome can be contracted upon. Effort is not contractible for the usual non-verifiability issues. τ is not contractible as n represents an innovative attempt which is in general difficult to identify and describe ex-ante. Innovative activities typically involve coming out with new ideas or exploring previously untested approaches, both of which often end up with no tangible outcomes to show for. It is thus difficult to distinguish between a worker who is carrying out an honest experimentation from a worker

[11] The Inada condition for $\psi(\cdot)$ ensures that the agent's optimal effort choice is always interior. The bounds on the third derivative of $\psi(\cdot)$ is a technical condition that ensures that the marginal effect of incentives on the agent's optimal effort choice is increasing in θ^τ. Examples of functions that satisfy Assumption 1 include $\psi(x) = -\log(1-x) - x$, and $\psi(x) = \frac{x^{m_2}}{(1-x)^{m_1}}$ for any $m_1 \geq 1$ and $m_2 > m_1$.

[12] There is no intensive margin on the innovative attempt here; the Supplementary Appendix extends the model to incorporate this by allowing the agent to also choose an innovative effort level.

who is merely shirking. In the same vein, the experimentation outcome θ^n is also assumed to be non-contractible because of the difficulties in describing the innovation before it is realized and also in verifying the value of an innovation by a third party.[13]

The principal cannot enforce long-term contracts. Hence, at the start of each period, the principal offers the agent a take-it-or-leave-it contract that specifies a fixed transfer and a bonus for output Y. Like the notation for effort, period-1 contract is denoted by Roman alphabets and period-2 contract by their Greek letter counterparts. The contracts are thus (a, b) and (α, β) respectively for periods 1 and 2, where a and α are the fixed wages, and b and β are the bonuses. Henceforth, whenever unspecified, all payoffs refer to expected payoffs, and negative payments are transfers from the agent to the principal.

3.2 Monitoring Environments

I consider the principal's problem under two types of monitoring environment – *tight-monitoring* (TM) or *freedom-at-work* (FW).[14] The principal's revenue y each period is publicly observed under both environments. Moreover, under TM, the principal observes all the actions of the agent which include his technology and effort choices. Importantly, if the agent uses n, both parties would observe θ^n. On the other hand, under FW, the principal does not observe any of this information. This implies that if the agent uses n in period 1, the θ^n realization will be *privately* observed by the agent which then results in asymmetric information in period 2.[15]

[13] Holmström (1989) provides an excellent discussion on why innovation attempts are often difficult-to-measure activities, citing reasons that innovation projects are often multi-staged and unpredictable, and it is often not clear ex-ante what is the correct course of action. Aghion and Tirole (1994) also argue how innovations are always unpredictable and hence difficult to contract on before they are realized. Moreover, innovations are often complex such that even if parties can agree on the innovation ex-ante, courts can still lack the requisite technical knowledge to enforce the contract, or that the innovation can take a long time for its full value to be realized such that an agreement on the immediate value is impossible.

[14] The Supplementary Appendix discusses more general monitoring environments.

[15] The observability of effort and/or technology choices under FW will be outcome-irrelevant because they are not contractible (see Remark 2). The only substantial difference between the two environments is the observability of θ^n which determines if asymmetry in period-2 payoff-relevant information exists.

3.3 Exploitation vs Exploration

As Schumpeter (1934) points out, a central concern for development in firms is the relation between exploiting its current capabilities and exploring for new possibilities. Following March (1991), this is termed as the tension between exploitation and exploration. This tension arises here via its two-armed bandit problem setup in which o is a known arm (exploitation) while n is an unknown arm (exploration). The assumptions made on the costs reflect the nature of conventional work methods versus innovative activities. Innovative activities, as represented by n, tend to be more costly to engage in initially. On the other hand, the gains from exploration is learning – engaging in innovative activities might lead to the uncovering of a more efficient technology for future use.

The equilibrium concept used throughout is the perfect Bayesian Equilibrium (henceforth equilibrium). An equilibrium that features the agent choosing o in period 1 is termed an *exploitation equilibrium*, and an equilibrium that features the agent choosing n in period 1 is termed an *exploration equilibrium*.

4 Tight-Monitoring (TM)

This section illustrates the ratchet effect of innovating and shows that the only equilibrium under TM is an exploitation equilibrium. Under TM, information is always symmetric in period 2. Hence the principal can efficiently "sell the firm to the agent". Since the agent's period-2 payoff is always zero, he myopically maximizes only his period-1 payoff at the start of period 1. Consider a period-1 bonus b and let e^n be the agent's corresponding optimal period-1 effort. Since:

$$\begin{aligned} e^n b - E\left[\psi\left(\frac{e^n}{\theta^n}\right)\right] &< e^n b - \psi\left(\frac{e^n}{E\left[\theta^n\right]}\right) \\ &\leq e^n b - \psi\left(\frac{e^n}{\theta^o}\right) \\ &\leq \max_e \left\{eb - \psi\left(\frac{e}{\theta^o}\right)\right\}, \end{aligned}$$

the agent will use o in period 1. This illustrates the ratchet effect: when the employer cannot commit not to expropriate all the efficiency gains, the worker responds by not trying to improve efficiency at all. This thus provides an economic rationale for the "conventional wisdom" that a more intrusive working environment stiffens the worker's creativity.

Proposition 1. *Under TM, the equilibrium is unique. On the equilibrium path, the optimal contracts are time-stationary:* $a^* = \alpha^* = -\max_e \left\{eY - \psi\left(\frac{e}{\theta^o}\right)\right\}$*;* $b^* = \beta^* = Y$*.*
(Ratchet effect) The agent uses o *every period.*

Remark 1. The ratchet effect remains even if effort is contractible. To see this, suppose that the principal can enforce payments that are contingent on the agent's effort, but the technology that the agent chooses to exert the effort on remains uncontractible. In period 2, since information is symmetric, the principal will enforce the agent to exert first-best (conditional on the available information in period 2) effort and then exactly compensates the agent for his effort based on the agent's more cost-efficient technology. Hence the agent always earns zero payoff in period 2. In turn, under any effort obligation in period 1, the agent always chooses to fulfill the effort obligation using o since $\theta^o \geq E\left[\theta^n\right]$.

5 Freedom-at-Work (FW)

This section considers the FW environment, focusing on deriving the principal-optimal exploration equilibrium.[16] The analysis begins at period 2 (Section 5.1) with the agent privately knowing θ^n; the principal thus screens the agent with a menu of bonus contracts. Note that since the agent's period-1 effort is chosen *before* he learns the value of θ^n, the realization of y in period 1 has no information about the agent's period-1 technology choice nor the agent's privately observed θ^n. Hence, any deviation from the equilibrium actions by the agent in period 1 is never detectable. This implies that the principal's choice of the period-2 menu is independent of the period-1 realization of y. The optimal on-path menu thus also determines the agent's off-path period-2 payoffs. Next, the optimal period-1 contract is derived in Section 5.2 while taking into account these period-2 payoffs. Section 5.3 discusses the implications of the optimal contracts.

5.1 Period 2

The principal's period-2 problem is one of adverse selection with moral hazard – the menu as a whole must elicit information about θ^n from the agent (adverse selection), while each individual contract has to incentivize him to make the appropriate technology and effort choices (moral hazard). The analysis begins by defining the appropriate type space of the agent which has a mass point of failed innovators. The moral hazard problem is then subsumed by working with the agent's indirect utility. The optimal menu of contracts,

[16] One can also view this as the principal's cost-minimization problem to induce the agent to explore with probability one. The Supplementary Appendix considers other types of equilibrium, including exploitation equilibrium, and mixed equilibrium in which the agent plays a mixed strategy for his period-1 technology choice. It is shown there that if an exploitation equilibrium exists under FW, the principal's total expected payoffs in such an equilibrium cannot be higher than her payoffs in the unique (exploitation) equilibrium under TM in Proposition 1. Moreover, the principal's total payoffs under the exploration equilibrium derived in this section is strictly higher than her payoffs under any mixed equilibrium. Hence, it suffices to focus attention on the exploration equilibrium under FW when one is concern with the principal's choice between TM or FW.

which features "bunching" because of the mass point, is characterized in Proposition 2.

5.1.1 Type-space and Subsuming the Moral Hazard Problem

In period 2, if $\theta^n < \theta^o$, its exact value is irrelevant since the agent will use o. Hence his payoff-relevant private information (i.e. type) is $\theta := \max\{\theta^n, \theta^o\}$, which has support $\left[\theta^o, \bar{\theta}\right]$ and probability distribution $\Pr(\theta \leq x) = F(\theta^o) + \int_{\theta^o}^{x} f(t)dt$. Type-$\theta^o$ agent is a failed innovator who reverts to o in period 2; this happens with probability $F(\theta^o) \in (0,1)$ on the equilibrium path. A successful innovator has a type $\theta > \theta^o$ and a higher type has a more successful innovation.

After accepting a period-2 contract (α, β), the type-θ agent's effort choice problem is $\max_{\varepsilon}\left\{\alpha + \varepsilon\beta - \psi\left(\frac{\varepsilon}{\theta}\right)\right\}$. Let $\varepsilon(\beta,\theta)$ denote the solution which is implicitly characterized by the first-order condition:

$$\beta = \frac{1}{\theta}\psi'\left(\frac{\varepsilon(\beta,\theta)}{\theta}\right). \tag{5.1}$$

By the revelation principle, attention can be restricted to truth-telling direct mechanisms with an obedience constraint given by condition (5.1). This is equivalent to considering the agent's indirect utility. Let:

$$u(\beta,\theta) \;:=\; \max_{\varepsilon}\left\{\varepsilon\beta - \psi\left(\frac{\varepsilon}{\theta}\right)\right\} \;=\; \varepsilon(\beta,\theta)\beta - \psi\left(\frac{\varepsilon(\beta,\theta)}{\theta}\right). \tag{5.2}$$

The agent's indirect utility under a contract (α, β) is thus $\alpha + u(\beta,\theta)$. By viewing α as a "transfer" and β as an "allocation", the problem becomes a pure screening problem with the agent having quasi-linear preferences. The properties of the agent's indirect utility are stated in Lemma 1. Henceforth subscript i denotes the partial derivative with argument i.

Lemma 1. *Both $\varepsilon(\beta,\theta)$ and $u(\beta,\theta)$ are continuously differentiable and strictly increasing in both their arguments. $\varepsilon(\beta,\theta)$ is strictly concave in β and strictly convex in θ. $u(\beta,\theta)$ is strictly convex in both its arguments. The agent's indirect utility satisfies the single-crossing* *condition:* $u_{\theta\beta}(\beta,\theta) > 0\ \forall\beta > 0$.

5.1.2 The Optimal Contracting Problem

Consider a menu of contracts denoted by $\{\alpha(\theta),\beta(\theta)\}_{\theta\in\left[\theta^o,\bar{\theta}\right]}$,[17] and let $\bar{U}(\theta) = \alpha(\theta) + u(\beta(\theta),\theta)$. The menu is feasible if:

$$\bar{U}(\theta) \geq 0 \quad , \quad \forall\theta \in \left[\theta^o,\bar{\theta}\right] \tag{IR}$$

$$\bar{U}(\theta) \geq \alpha\left(\tilde{\theta}\right) + u\left(\beta\left(\tilde{\theta}\right),\theta\right) \quad , \quad \forall\tilde{\theta},\theta \in \left[\theta^o,\bar{\theta}\right] \tag{IC}$$

(IR) is the agent's participation constraint and (IC) is his truth-telling constraint. The problem of the principal is program $\mathcal{P}$:

$$\max_{\alpha(\cdot),\beta(\cdot)} \int_{\theta^o}^{\bar{\theta}} \Bigg(\varepsilon(\beta(\theta),\theta)\left[Y - \beta(\theta)\right] - \alpha(\theta)\Bigg) dF(\theta) \tag{5.3}$$

subject to (IR) and (IC).

By standard arguments (eg. Salanié (2005) Chapter 2.3.1):

Lemma 2. *A menu of contracts $\{\alpha(\theta),\beta(\theta)\}_{\theta\in\left[\theta^o,\bar{\theta}\right]}$ satisfies constraint (IC) if and only if $\forall\theta \in \left[\theta^o,\bar{\theta}\right]$, the following two conditions are satisfied:*

1. $\bar{U}(\theta) = \bar{U}(\theta^o) + \int_{\theta^o}^{\theta} u_\theta(\beta(t),t)\,dt$.

2. $\beta(\theta)$ *is non-decreasing.*

Lemma 2.1 implies that $\bar{U}(\theta^o) = 0$ under optimality. Substituting in $\alpha(\theta)$ by $\alpha(\theta) =$

[17] The convexity of $u(\beta,\theta)$ in β (see Lemma 1) is non-standard relative to textbook treatments of screening problems (for example, chapter 7 of Fudenberg and Tirole (1991)). This convexity leaves open the possibility that the principal might benefit from offering stochastic bonuses to the agent. However this turns out not to be the case under optimality (see footnote 27). For ease of exposition, I restrict attention to only deterministic contracts but note that doing so is without loss.

$\bar{U}(\theta) - u(\beta(\theta), \theta)$ and Lemma 2.1, and after some rearrangement, program $\mathcal{P}$ can be reformulated as program $\mathcal{P}'$:

$$\max_{\beta(\cdot)\in\mathcal{B}} \int_{\theta^o}^{\bar{\theta}} v(\beta(\cdot), \theta)\, dF(\theta), \tag{5.4}$$

where $\mathcal{B}$ is the set of non-decreasing functions with domain $\left[\theta^o, \bar{\theta}\right]$, and:

$$v(\beta(\cdot), \theta) := \underbrace{\varepsilon(\beta(\theta), \theta)\, Y - \psi\left(\frac{\varepsilon(\beta(\theta), \theta)}{\theta}\right)}_{s(\beta(\theta),\theta)} - \underbrace{\int_{\theta^o}^{\theta} u_\theta(\beta(t), t)\, dt}_{r(\beta(\cdot),\theta)}. \tag{5.5}$$

$v(\beta(\cdot), \theta)$ is the principal's period-2 payoff from type θ under a bonus schedule $\beta(\cdot)$. It consists of the social surplus $s(\beta(\theta), \theta)$ less the agent's information rent $r(\beta(\cdot), \theta)$. Program $\mathcal{P}'$ is thus maximizing the expectation of this, subject to a monotonicity constraint on the bonus schedule $\beta(\cdot)$. Doing integration by parts, (5.4) becomes:

$$\max_{\beta(\cdot)\in\mathcal{B}} \int_{\theta^o}^{\bar{\theta}} \phi(\beta(\theta), \theta)\, f(\theta) d\theta + s(\beta(\theta^o), \theta^o)\, F(\theta^o), \tag{5.6}$$

where $\phi(\beta, \theta) := s(\beta, \theta) - u_\theta(\beta, \theta)\, H(\theta)$, and $H(\theta) := \frac{1-F(\theta)}{f(\theta)}$. ϕ is typically called the *virtual surplus* in the screening literature. In the context here, the term $\int_{\theta^o}^{\bar{\theta}} \phi(\beta(\theta), \theta)\, f(\theta) d\theta$ is the principal's expected payoff from the successful innovators, while the term $s(\beta(\theta^o), \theta^o)\, F(\theta^o)$ is the expected payoff from the failed innovators.[18]

[18] The mass point at θ^o violates the non-increasing inverse hazard ratio property. This implies that the standard procedure of "optimizing point-wise" for program $\mathcal{P}'$ in (5.6) will not give a monotonic schedule. When this happens, the literature typically uses some form of control theoretic techniques to solve the screening problem. However, the mass point at type θ^o precludes a direct application of optimal control theory. Hellwig (2010) provides a technique to handle screening problems with mass points in the support. The basic idea there is to develop a (as he terms it) "pseudo-type" $\tilde{\theta}$ of the original type θ where the distribution of $\tilde{\theta}$ has no mass point. Non-smooth analysis techniques in optimal control theory can then be applied on the "pseudo" problem, and Hellwig proves that the solution to the "pseudo" problem is also the solution to the original problem. The "bunching" of contracts for the mass point type θ^o with types slightly higher than θ^o, as shown in Proposition 2, is a general phenomenon which is proved in Theorem 2.3 in Hellwig (2010). However, because the problem here is more specific (in particular, the mass point exists only at the lowest type of the support here) than the one considered by Hellwig, the optimal bonus schedule can be solved here via a more straightforward method. This method is more in line with developing the intuition behind the optimal menu here, and it also circumvents the use of optimal control theory techniques,

Proposition 2. *Under FW, the optimal period-2 bonus schedule $\beta^*(\cdot)$ is:*

$$\beta^*(\theta) = \begin{cases} \hat{\beta}(\theta) & , \ \forall \theta \in \left[\ddot{\theta}, \bar{\theta}\right] \\ \hat{\beta}(\ddot{\theta}) & , \ \forall \theta \in \left[\theta^o, \ddot{\theta}\right] \end{cases},$$

where $\hat{\beta}(\cdot)$ is characterized by:

$$s_\beta \left(\hat{\beta}(\theta), \theta\right) = \varepsilon_\theta \left(\hat{\beta}(\theta), \theta\right) H(\theta) \ , \quad \forall \theta \in \left[\theta^o, \bar{\theta}\right], \tag{5.7}$$

and $\ddot{\theta}$ is characterized by:

$$\int_{\theta^o}^{\ddot{\theta}} \phi_\beta \left(\hat{\beta}(\ddot{\theta}), \theta\right) f(\theta) d\theta + s_\beta \left(\hat{\beta}(\ddot{\theta}), \theta^o\right) F\left(\theta^o\right) = 0. \tag{5.8}$$

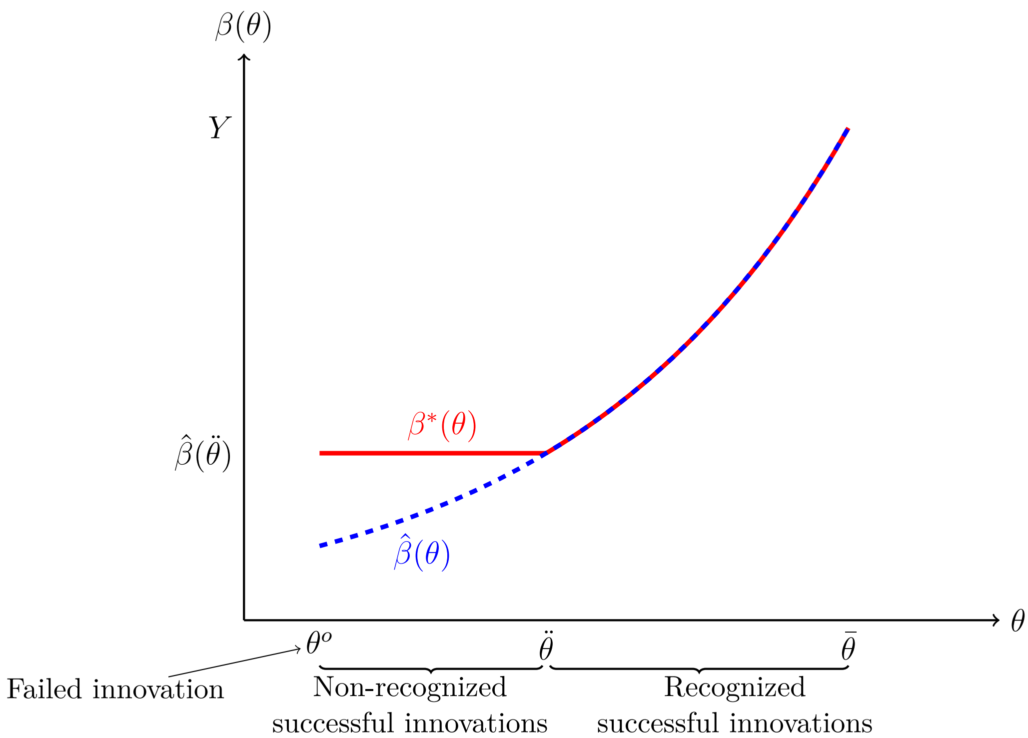
The optimal fixed wage schedule $\alpha^*(\cdot)$ is:

$$\alpha^*(\theta) = \int_{\theta^o}^{\theta} u_\theta \left(\beta^*(t), t\right) dt - u\left(\beta^*(\theta), \theta\right) \ , \quad \forall \theta \in \left[\theta^o, \bar{\theta}\right]. \tag{5.9}$$

- *$\ddot{\theta}$ is strictly in the interior of $\left[\theta^o, \bar{\theta}\right]$.*
- *$\beta^*(\cdot)$ is continuous and strictly increasing for $\theta > \ddot{\theta}$; and $\beta^*(\bar{\theta}) = Y$.*
- *$\alpha^*(\cdot)$ is continuous and strictly decreasing for $\theta > \ddot{\theta}$.*

Figure 5.1 illustrates the optimal bonus schedule $\beta^*(\cdot)$. To see why the bonus schedule is increasing while the fixed wage schedule is decreasing, first recall that the agent's optimal effort is positively correlated to his type. Since a low θ gets the bonus with a lower probability, he needs a higher fixed wage to satisfy his participation constraint. This creates incentives for the agent to under-report his type. The principal counters this by decreasing the bonus

which implies that there is no need to *a priori* assume that the optimal bonus schedule is differentiable.



The dotted graph represents $\hat{\beta}(\cdot)$. The bold graph represents the optimal bonus schedule $\beta^*(\cdot)$; the schedule is flat for $\theta \in \left[\theta^o, \ddot{\theta}\right]$ and strictly increasing for $\theta > \ddot{\theta}$.

Figure 5.1: Optimal bonus schedule $\beta^*(\cdot)$

for lower types to make their contracts less attractive to the higher types, thus decreasing the information rent to the high-type agents. But a lower bonus also decreases the surplus generated. Hence the principal faces a trade-off between efficient surplus production and information rent minimization.

This trade-off is reflected in (5.7): $s_\beta\left(\hat{\beta}(\theta), \theta\right)$ is the marginal gain in surplus from type θ when his bonus is increased; $\varepsilon_\theta\left(\hat{\beta}(\theta), \theta\right) H(\theta)$ is the resulting marginal increase in (virtual) information rent provision. As in standard screening results, such a tradeoff results in inefficiency everywhere other than the highest type.[19]

However, the trade-off in (5.7) is a local trade-off which is not valid at θ^o, because the

[19]The first-best incentive for any θ is $\beta = Y$ since it induces the first-best effort level.

zero-measure types just above θ^o is incommensurate with the mass point at θ^o. Instead, the principal has to pool some types above θ^o to do this trade-off, thus creating a "bunching" region, which is optimally determined in (5.8). The term $s_\beta\left(\hat{\beta}(\ddot{\theta}),\theta^o\right)F(\theta^o)$ represents the marginal increase in surplus generated by θ^o when his bonus is increased; this surplus is significant due to the mass $F(\theta^o)$. On the other hand, $\int_{\theta^o}^{\ddot{\theta}}\phi_\beta\left(\hat{\beta}(\ddot{\theta}),\theta\right)f(\theta)d\theta$ represents the marginal increase in the information rent to higher types when increasing the bonus for type θ^o. Contract pooling at the low types implies that only sufficiently successful innovations $\theta\in(\ddot{\theta},\bar{\theta}]$ are "recognized" and distinguished with increasingly powerful incentive contracts; the mildly successful innovations $\theta\in(\theta^o,\ddot{\theta}]$ are treated as if their innovation was a failure.

5.1.3 On-path and Off-path Payoffs of the Agent.

Let R be the agent's on-path expected period-2 information rent:

$$R \;:=\; E\left[\bar{U}(\theta)\right] \;=\; \int_{\theta^o}^{\bar{\theta}}\left(\int_{\theta^o}^{\theta}u_\theta\left(\beta^*(t),t\right)dt\right)dF(\theta)\;>0 \quad .$$

If the agent deviates to o in period 1, he will not know θ^n coming into period 2. Since $\theta^o\geq E\left[\theta^n\right]$, the deviating agent will then use o in period 2, which means that he is of type θ^o, and $\bar{U}(\theta^o)=0$.

Corollary 1. *If the agent uses* n *in period 1, his expected period-2 payoff is* R*. If he deviates to using* o *in period 1, his expected period-2 payoff is* 0*.*

Remark 2. Corollary 1 continues to hold even if the agent's choice of effort and/or technology are also observed by the principal but are not contractible. The only difference is that when the agent deviates to choosing o in period 1 now, the principal will know about it. Instead of offering the menu of contracts in Proposition 2 in period 2 in this case, the principal will offer the contract $\beta=Y$ and $\alpha=-s\left(Y,\theta^o\right)$ which still gives the agent a zero period-2 payoff.

5.2 Period 1

This subsection completes the equilibrium characterization by deriving the optimal period-1 contract. Under a period-1 contract (a, b), the agent's total expected payoffs for using n with period-1 effort e is $a + W^n(e, b)$, where:

$$W^n(e,b) := eb - E\left[\psi\left(\frac{e}{\theta^n}\right)\right] + R. \tag{5.10}$$

His total expected payoffs for deviating to o is $a + W^o(e, b)$ where:

$$W^o(e,b) := eb - \psi\left(\frac{e}{\theta^o}\right). \tag{5.11}$$

For $\tau \in \{o, n\}$, let $\bar{W}^\tau(b) := \max_e W^\tau(e, b)$ and $e^\tau(b) := \arg\max_e W^\tau(e, b)$. The following provides the definition of an exploration equilibrium:

Definition 1. A period-1 contract (a, b), together with the agent using technology n with effort e^* in period 1, forms an equilibrium if: (i) $a + \bar{W}^n(b) \geq 0$; (ii) $\bar{W}^n(b) \geq \bar{W}^o(b)$; and (iii) $e^n(b) = e^*$.

(i) is the agent's participation constraint; (ii) ensures he uses n in period 1; and (iii) is his period-1 effort incentive compatibility constraint.

Lemma 3. *$e^\tau(b)$ is strictly increasing for both $\tau \in \{o, n\}$, and $e^o(b) > e^n(b)$ $\forall b > 0$. There exists a unique bonus threshold $\bar{b} > 0$ such that $\bar{W}^n(b) \geq \bar{W}^o(b)$ if and only if $b \leq \bar{b}$. The threshold $\bar{b}$ is increasing in R.*

The first property of $e^\tau(\cdot)$ is intuitive: higher bonus induces higher effort. Next, for any b, the agent chooses higher effort under o than under n since o is ex-ante more cost-efficient in expectation. As for the property of $\bar{W}^\tau(\cdot)$, observe that in deciding which technology to use in period 1, the agent trades off between the payoffs across the two periods. $R > 0$ implies that n provides an additional period-2 rent, whereas o increases his probability of getting the period-1 bonus. Hence, if the period-1 bonus is too high, the agent's total payoffs from using o in period 1 will be higher than that from using n.

Proposition 3. *Under FW, the optimal period-1 contract that induces the agent to choose technology n in period 1 is unique: $a^* = -\bar{W}^n(b^*)$ and $b^* = \min\left\{Y, \bar{b}\right\}$.*

If the agent always uses n in period 1, the principal's maximizes period-1 payoff by making the agent the residual claimant of the output (i.e. set b at Y) and then extracting the surplus via the fixed wage. From Lemma 3, however, there is an upper threshold on the bonus that will induce the agent to use n in period 1. By verifying that the principal's payoff is concave in b, the optimal bonus is thus the minimum of Y and the threshold.

5.3 Discussion on the Optimal Contracts

5.3.1 Low Incentive in Period 1

Proposition 3 illustrates that low early incentive is needed to motivate exploration and this feature has a number of interesting interpretations. First, the bonus in the model reflects the agent's opportunity cost of getting a low output. The requirement for low early incentive thus implies that firms must show tolerance to early failures by their workers to encourage them to innovate. This complements Manso's (2011) point that *long-term* contracts that encourages exploration must both tolerate early failures and reward long-term successes.

Second, it also explains why workers' incentives are weak in the early stages of their career. The prevalent explanation for this observation in the literature is career concern (Holmström 1999) where low incentive is sufficient to induce effort from workers in their early career due to the reputation concern regarding their ability. In contrast, low early incentive is necessary here as innovation incentives to induce the workers to take on risky experimentation. The optimality of weak incentives in certain situations has also been noted in other contexts.

For example, in the multi-tasking literature (Holmström and Milgrom, 1991), the principal lowers the incentive for one task to prevent negligence on other uncontractible tasks. The parallel here is not drawn directly on the multi-tasking aspect of the model; rather, the two periods here can be viewed as different "tasks", and low incentive in period 1 induces the agent to put emphasis on the period-2 rent, which then encourages him to experiment on n in period 1.

Finally, the requirement for a low period-1 bonus also implies that the expected surplus from the principal-agent relationship is small in period 1. That relationship sometimes has to "start small" is reminiscent of the reputation building literature (e.g. Ghosh and Ray 1996; Watson 1999, 2002; Halac 2012).[20] In this literature, there is ex-ante asymmetric information about the players' type and hence, the early stages of relationships are characterized by low stakes to allow the asymmetric information among players to be gradually resolved. In contrast, information is ex-ante symmetric here, and the players "start small" here to create asymmetric information rather than to resolve it.

5.3.2 Costs for Innovation

There are three types of costs incurred to innovate. First it involves the use of the ex-ante inefficient technology n; second there is a limit to the period-1 surplus due to the upper bound $\bar{b}$ on the period-1 bonus; third there is a cost due to the asymmetric information created in period 2. The first type of cost is an inherent cost to innovate. The second type is a friction which has been discussed above. I discuss the third type of cost now.

Without experimentation, the period-2 social surplus is $s(Y, \theta^o)$. With experimentation, under the optimal menu of contracts, the social surplus generated from innovation θ is $s(\beta^*(\theta), \theta)$. Define the innovation surplus of θ as $Innov(\theta) := s(\beta^*(\theta), \theta) - s(Y, \theta^o)$.

[20]The term "start small" is taken from Watson (1999, 2002).

Proposition 4. *$Innov(\theta)$ is continuous and strictly increasing, with $Innov(\theta^o) < 0$ and $E[Innov(\theta)] > 0$.*

Although innovation is socially beneficial in expectation ex-ante (i.e. $E[Innov(\theta)] > 0$), innovation failure is always costly (i.e. $Innov(\theta^o) < 0$). Moreover, the agency costs ex-post due to the asymmetric information implies that an innovation is socially beneficial ex-post *only* if it is sufficiently successful. This is seen by noticing that $Innov(\theta)$ is negative even when θ is slightly above θ^o.

6 Tight-Monitoring Versus Freedom-at-Work

This section considers the comparative statics with respect to the distribution of θ^n. The results here speak to why and when would different firms adopt different monitoring environments in practice. Throughout this section, the contracts and the payoffs under FW will refer to the contracts and the resulting payoffs of the principal-optimal exploration equilibrium under FW as characterized in Propositions 2 and 3. Likewise, the contracts and the payoffs under TM will refer to the contracts and the resulting payoffs of the unique exploitation equilibrium under TM in Proposition 1. All the proofs for this section are found in the Supplementary Appendix.

Under TM, the contracts and payoffs of both players are independent of the distribution of θ^n. Hence, I only have to consider the effects of the distribution of θ^n under FW. To do so, I first parametrize the set of distributions of θ^n by a parameter k. Let $F(\cdot;k)$ be a distribution on $\left[\underline{\theta}, \bar{\theta}(k)\right]$, and let $\mathcal{F} := \{F(\cdot;k)\}$ denote a family of such distributions.

Assumption 3. *$\mathcal{F}$ satisfies the following assumptions:*

1. *For all $F(\cdot;k) \in \mathcal{F}$, the distribution $F(\cdot;k)$ has a density function $f(\cdot;k)$ that satisfies Assumption 2.*

2. $k'' > k'$ *implies that* $\bar{\theta}(k'') \geq \bar{\theta}(k')$ *and* $H(\theta^n; k'') > H(\theta^n; k')$ $\forall \theta^n \in \Big[\underline{\theta}, \bar{\theta}(k')\Big]$.

3. *For any* $\theta^n \in \Big[\underline{\theta}, \bar{\theta}(k)\Big)$, $f(\theta^n; k)$ *is absolutely continuous in* k.

Assumption 3.1 implies that all the results from Section 5 apply. Assumption 3.2 implies that the parameter k orders the distributions in terms of both the upper bound and the inverse hazard rate. Assumption 3.3 is a regularity condition. An example of $\mathcal{F}$ is the *family of uniform distributions* in which θ^n is uniformly distributed over $[\underline{\theta}, k]$:

$$f(\theta^n; k) = \begin{cases} \frac{1}{k-\underline{\theta}} & , \ \theta^n \in [\underline{\theta}, k] \\ 0 & , \ \theta^n \notin [\underline{\theta}, k] \end{cases} \tag{6.1}$$

Remark 3. Under Assumption 3, $k'' > k'$ implies that $F(\theta^n; k'') < F(\theta^n; k')$ $\forall \theta^n \in \Big[\underline{\theta}, \bar{\theta}(k'')\Big)$.

Under a family of distributions $\mathcal{F}$ that satisfies Assumption 3, a higher k implies that the distribution (i) has a higher upper bound, (ii) is more concentrated at more successful innovations (first-order stochastic dominance from Remark 3), and (iii) has less risk (second-order stochastic dominance). k thus provides a measure of the ex-ante innovation potential of technology n.

Let $V\Big(\beta(\cdot); k\Big) := \int_{\theta^o}^{\bar{\theta}(k)} v\Big(\beta(\cdot), \theta\Big) dF(\theta; k)$; let the optimal bonus schedule associated with distribution $F(\cdot; k) \in \mathcal{F}$ be $\beta^*(\cdot; k) := \underset{\beta(\cdot) \in \mathcal{B}(k)}{\arg\max}\ V\Big(\beta(\cdot); k\Big)$, where $\mathcal{B}(k)$ is the set of non-decreasing functions with domain $\Big[\theta^o, \bar{\theta}(k)\Big]$; let the principal's maximized period-2 payoff be $\mathcal{V}(k) := V\Big(\beta^*(\cdot; k); k\Big)$.

Proposition 5. *Suppose that* $k'' > k'$. *Under FW:*

1. *(Period-2 Bonus Schedule.)* $\beta^*(\theta; k'') < \beta^*(\theta; k')$ $\forall \theta \in \Big[\theta^o, \bar{\theta}(k')\Big]$.[21]

2. *(Period-2 Payoff.)* $\mathcal{V}(k'') > \mathcal{V}(k')$.

[21] $\beta^*(\cdot; k')$ is not defined for $\theta \in \Big(\bar{\theta}(k'), \bar{\theta}(k'')\Big]$.

The period-2 bonuses fall for every type as k increases (Proposition 5.1). To see why, recall from Lemma 2.1 that the information rent for type-θ agent is the integral of $u_\theta\Big(\beta(t), t\Big)$ over all types t below θ. When the principal further distorts the bonus schedule downward for all types below θ, the rents given to all agents with types above θ are reduced. The downside of doing so is that it lowers the surplus generated by the lower types. However, when k is higher, the density is more concentrated at the higher types. Hence the savings from rent-provision to the higher types outweighs the loss in surplus generated from the lower types, thus making such a downward distortion of the bonus schedule profitable for the principal.[22]

With regards to the principal's period-2 payoff (Proposition 5.2), the effect from the change in the bonus schedule due to a change in k (Proposition 5.1) is of only a second-order magnitude at the optimum. Instead, the first-order effect comes from the change in distribution of θ. It is verified that the surplus generated by a higher θ increases faster than the rent provided (i.e. $v\left(\beta(\cdot), \theta\right)$ increases in θ). Hence a higher k, which implies higher density at more successful innovations, results in a higher period-2 payoff for the principal.

Since the agent's period-2 rent is captured by the principal via the period-1 fixed transfer (Proposition 3), the principal's total payoff is ultimately the total surplus generated over the two periods. From Proposition 5.2, if the agent's period-2 information rent is increasing with k, then the period-2 surplus must be increasing in k. Moreover, a higher period-2 rent also increases the period-1 bonus threshold $\bar{b}$ which in turn decreases the distortion in period-1 surplus; in addition, a higher k implies an increase in period-1 expected efficiency of technology n. Hence, a sufficient condition for the principal's total payoff to increase in k is that the agent's rent is also increasing in k.

Unfortunately, due to a lack of closed-form solution under general parametrizations, it

[22]From Lemma 2.1, $\frac{d}{d\theta}\bar{U}(\theta) = u_\theta\big(\beta^*(\theta; k), \theta\big)$ whenever the derivative exists, and this is increasing in the bonus. Proposition 5.1 thus also implies that if the innovative activity has more potential ex-ante, the agent's marginal gain from having a better innovation decreases.

is not clear how the agent's information rent changes with k – a higher k lowers the bonus (Proposition 5.1) and hence the rent for each type; on the other hand, a higher k increases the probability of being a higher type who receives a higher rent. Proposition 6 provides a particular parametrization of the model in which the agent's period-2 information rent can be shown to be always increasing in k.

Proposition 6. *Suppose that $\mathcal{F}$ is the family of uniform distributions in (6.1) and $\psi(x) = \xi x^2$ where $\xi > 0$ is sufficiently large such that $\arg\max_e \left\{eY - \psi\left(e/\bar{\theta}(k)\right)\right\} < 1 \ \forall k$.*[23] *Under this parametrization, the agent's period-2 information rent is strictly increasing in k; hence the principal's total expected payoffs under FW is strictly increasing in k.*

Corollary 2. *Suppose that the model follows the parametrization in Proposition 6. Then there exists a threshold potential k^* for the innovative activity n such that the principal prefers FW to TM if and only if $k > k^*$.*

Another comparative statics that one might consider is on θ^o (i.e. the firm's current efficiency).[24] While the expected surplus in period 2 is always higher under FW than TM (see Proposition 4), it is readily verified that this difference converges to zero as θ^o approaches $\bar{\theta}$. On the other hand, the period-1 surplus under TM is always higher than under FW since $E[\theta^n] \leq \theta^o$. Hence, when θ^o is sufficiently high, the loss in the period-2 surplus under TM relative to under FW will be offset by the gain in period-1 surplus:

Corollary 3. *There exists $\delta > 0$ such that the principal always prefers TM to FW if $\theta^o > \bar{\theta} - \delta$.*

Implications on the choice of monitoring environment. The results in this section suggest that one is more likely to observe a FW type of environment when the firm's expected

[23] This cost function violates the Inada condition that $\lim_{x \to 1} \psi(x) = \infty$, but since the principal will never offer a bonus greater than Y, the agent's effort choice is still always interior, which in turn ensures that all the results in Section 5 hold.

[24] An earlier draft, which is available from the author, contains formal results for this comparative statics.

gains from process innovation attempts are high. Within industries, young firms should be more likely than old firms to adopt a FW type of environment since they are still learning about their own operational needs and improving upon the current operating procedures. Across industries, new-market firms (e.g. "tech firms" and start-ups) are unlikely to have efficient operating procedures from others that they can mimic, and they thus have higher gains from their employees constantly trying to find better operating strategies. Similarly, firms in volatile market conditions face a constant need to adapt their strategies to the changing market conditions; firms in such industries should then be expected to adopt a FW type of environment which promotes process innovations by their employees. On the other hand, when expected improvements from the status quo method are unlikely or the expected gains are small, the jobs are expected to be carried out under a TM type of environment. These could include mechanical jobs and relatively homogenous works (e.g. factory assembly lines) where the experience of others can also be readily adapted and used.

7 Discussions

7.1 Persistence of the Monitoring Environment

While not strictly needed for the results, the model implicitly assumes that the monitoring environment is unchanged over time. In practice, the monitoring technology in a firm depends on features such as the workspace arrangement, and the availability and effectiveness of surveillance facilities. Many of these features are costly to change, thus justifying the persistence in the monitoring environment in the model.

More importantly, the essence of "freedom-at-work" (or the lack of it) here is about the *trust* that a worker has on his managers not secretly watching over him and trying to learn what he has uncovered. Such trust takes time to build, evolves from the experiences of both the worker himself and his co-workers, and it should thus be part of the identity and *culture* of the firm. As Hermalin (2013a, pp. 458) writes: "*[...] corporate culture is not a precisely defined concept. It can be seen as encompassing the norms and customs of a firm, its informal and unwritten rules of behavior. [...] Consequently, given normal personnel turnover, the culture will persist over time, evolving slowly if at all.*" Given the persistence of culture, the essence of the monitoring environment (TM or FW) should remain fixed at least in the short run.

7.2 Outside Options for Innovation

Since the agent's period-2 outside option is independent of θ^n, the model assumes that the worker cannot take his innovation elsewhere to reap its benefits. For example, the innovation could be firm-specific, or it could be difficult for other firms to verify the innovation. More generally, this assumption is in line with a firm's need to protect its own innovations from being stolen by its workers. Hence, through "assignment of inventions" and "non-compete" clauses in employment contracts, firms in practice often have in place a default arrangement that prevents its workers from taking *any* innovation to another firm.

An "assignment of inventions" clause cedes intellectual property rights of any invention created during a worker's tenure in the firm to his employer. Cherensky (1993) observes that "*virtually all technical employees agree, as a condition of employment, to assign to the employer all rights to inventions conceived by the employee while at work, or in subject matters related to work, or while using any resources of the employer*". Howell (2012) notes that such agreements "*have become important and commonplace facets of employment agreements*".

A "non-compete" clause forbids employees from leaving the firm to work for a rival or setting up a competing business. The *Economist* reports that about 90% of managerial and technical employees in the United States have signed such "non-compete" agreements, and the classifications of "rival" or "competing" businesses are defined very broadly.[25] These

[25] See "*Ties that bind*", Dec 14th 2013, The *Economist*.

agreements typically have a lifespan of up to two but can go as many as five years.

7.3 Innovation Incentives in Long-lived Relationships

Employment relationships are fairly long-lived, and the literature on relational contracting beginning from MacLeod and Malcomson (1989) has noted that reputational concerns can potentially discipline parties and align incentives for actions that are non-contractible.[26] However, many small process innovations lack the possibility of an objective indicator of innovation (such as a patent), and parties can often disagree on the value (or even existence) of an innovation. For example, the firm might have private information about the value of the worker's innovation in the long run or on how it affects the other production processes in the firm; or the worker might simply over-estimate the value of his innovation. Levin (2003) and MacLeod (2003) (among others) have shown that inefficiency necessarily arises in relational contracts when parties have potentially different information on the outcome (i.e. subjective evaluations); MacLeod and Tan (2017) also show that the timing of information revelation has important implications on efficiency, and a contract that induces the desired action in such a setting might not exist even in very simple information environments. Hence, providing incentives for innovation through long-lived relationships is not always viable. This paper does not argue against the presence and use of such reputational incentives for innovations; rather, the idea here serves as a complement on how firms manage incentives for different kinds of innovations.

7.4 Other Applications

While being framed as an agency problem in organizations, the intuitions here can be applied more generally to other economic relationships that require the agent to make uncontractible

[26]See Malanowski (2007) for examples on how some companies have created a reputation for rewarding employees for inventions that ended up being patented.

experimentation. For example, in a long-term procurement relationship, from the result in TM, the incentives for the seller to make buyer-specific innovation attempts diminish when the innovation gains are expected to be observed and appropriated by the buyer. This might explain why buyers would choose suppliers who are located far away (analogous to FW) despite the transportation costs incurred.

The notion can also be related to the delegation-versus-centralization problem. Under delegation, the principal gives up decision rights to the agent; while under centralization, the principal keeps the decision rights. Shin and Strausz (2014) suggest that the party with the decision rights to the input should also have private information about its evolution. Hence delegation, analogous to FW, facilitates the agent to accrue private information; while centralization, analogous to TM, does not. The results here then suggest that innovations by the agent is more likely to take place under delegation. This hypothesis is supported by Acemoglu et al. (2007) and Kastl et al. (2013) who find a positive correlation between the degree of decentralization and the investment level in firms.

7.5 Team Production and Knowledge Spillover

One issue that is not addressed in this paper is team production and the related innovation knowledge spillover in firms. Many firms have multiple workers who serve (broadly) the same production functions; hence it is just as important to encourage process innovation as to spread the new knowledge to the other workers. The idea here that a firm can restore its workers' innovation incentives through allowing them to accrue private information about their innovations remain under team production, but each worker's compensation at the post-innovation stage should also depend on the joint output of the team to encourage the innovator worker to share the new knowledge. However, this raises the well-known "moral hazard in teams" issue (Holmström, 1982) whereby workers might free-ride on others to innovate. One potential solution to this is to have the innovator worker's compensation (which is tied to the team's joint output) being higher and riskier – this is analogous to making the innovator worker the leader of the team post-innovation. The higher compensation provides the incentives for workers to be the innovator, and the higher risk exposure deters workers from pretending to have an innovation in the absence of one. A formal study of this is left to future work.

8 Conclusion

This paper studies motivating process innovation when the innovation is not contractible ex-ante and the firm is unable to commit to future rewards. I illustrate how the monitoring environment affects a worker's incentives to innovate and improve the efficiency inside the firm, and I characterize the incentive scheme required to complement the monitoring environment. By allowing its workers to accrue private information about their innovation, the firm is compelled to share any innovation surplus with its workers via an information rent. This helps to explain why workers perform better when given more freedom at work, and how workplaces are sometimes organized to indirectly distribute rents to the workers.

A Appendix: Proofs for Section 5

Proof of Lemma 1.

Proof. $\varepsilon(\cdot)$ is differentiable using the implicit function theorem on (5.1). Differentiability of $u(\cdot)$ then follows. Omitting the arguments, the partial derivatives of $\varepsilon(\cdot)$ and $u(\cdot)$ are: $\varepsilon_\beta = \frac{\theta^2}{\psi''} > 0$, $\varepsilon_\theta = \frac{\psi'}{\psi''} + \frac{\varepsilon}{\theta} > 0$, $\varepsilon_{\beta\beta} = -\frac{\theta^3\psi'''}{(\psi'')^3} < 0$, $\varepsilon_{\theta\theta} = \frac{\psi'[2(\psi'')^2-\psi'.\psi''']}{\theta(\psi'')^3} > 0$, $\varepsilon_{\beta\theta} = \frac{\theta[2(\psi'')^2-\psi'.\psi''']}{(\psi'')^3} > 0$; and $u_\beta = \varepsilon > 0$, $u_\theta = \psi' \cdot \frac{\varepsilon}{\theta^2} > 0$, $u_{\beta\beta} = \varepsilon_\beta > 0$, $u_{\theta\theta} = \frac{(\psi')^2}{\theta^2(\psi'')^2} > 0$, $u_{\beta\theta} = \varepsilon_\theta > 0$. □

Proof of Proposition 2.

The proof proceeds in two steps. In Step 1, I consider the auxiliary problem $\tilde{\mathcal{P}}'$:

$$\Phi\left(\ddot{\beta}\right) := \max_{\beta(\cdot)\in\mathcal{B};\ \beta(\theta^o)\geq\ddot{\beta}} \int_{\theta^o}^{\bar{\theta}} \phi\left(\beta(\theta),\theta\right) f(\theta)d\theta. \qquad (\tilde{\mathcal{P}}')$$

$\Phi\left(\ddot{\beta}\right)$ is the principal's maximum expected payoff from the successful innovators when she has to give at least bonus $\ddot{\beta}$ to the failed innovator type θ^o. In Step 2, the optimal $\beta^*(\theta^o)$ is pinned down by solving:

$$\max_{\ddot{\beta}}\ \Phi\left(\ddot{\beta}\right) + s\left(\ddot{\beta},\theta^o\right) F\left(\theta^o\right). \qquad \text{(A.1)}$$

Because the solution to $\tilde{\mathcal{P}}'$ is unique, the solution to (A.1) together with its corresponding solution for $\tilde{\mathcal{P}}'$ is the solution to program $\mathcal{P}'$ in (5.6).

(Step 1) Solving the Problem $\tilde{\mathcal{P}}'$:

Lemma 5 below gives the solution to $\tilde{\mathcal{P}}'$; Lemma 4 provides the necessary properties to prove Lemma 5 and Step 2 later.

Lemma 4. *$\phi(\beta,\theta)$ is continuously differentiable in both its arguments. For any $\forall\theta\in\left[\theta^o,\bar{\theta}\right]$, $\arg\max_{\beta\geq 0}\phi(\beta,\theta)$ has a unique solution. Define $\hat{\beta}(\theta) := \arg\max_{\beta\geq 0}\phi(\beta,\theta)$. $\forall\theta\in\left[\theta^o,\bar{\theta}\right]$: (i) $\phi\left(\beta,\theta\right)$ is strictly concave in β for $\beta\in[0,Y]$; (ii) $\hat{\beta}\left(\theta\right)$ is characterized by $\phi_\beta\Big(\hat{\beta}(\theta),\theta\Big)=0$; (iii) $\hat{\beta}(\bar{\theta})=Y$; (iv) $\hat{\beta}\left(\theta\right)$ is differentiable and $\frac{d}{d\theta}\hat{\beta}\left(\theta\right)$ is continuous; (v) $\hat{\beta}\left(\theta\right)$ is strictly increasing; (vi) $\phi\Big(\hat{\beta}(\theta),\theta\Big)$ is continuous.*

Proof. Differentiability of $\phi\left(\cdot\right)$ follows from differentiability of $\varepsilon\left(\cdot\right)$ and $\psi\left(\cdot\right)$.

$$\begin{aligned}\phi_\beta\left(\beta,\theta\right) &= \varepsilon_\beta(\beta,\theta)Y - \frac{1}{\theta}\psi'\left(\frac{\varepsilon(\beta,\theta)}{\theta}\right) - u_{\beta\theta}(\beta,\theta)H(\theta)\\ &= \varepsilon_\beta(\beta,\theta)\left[Y-\beta\right] - \varepsilon_\theta(\beta,\theta)H(\theta), \qquad \text{(A.2)}\end{aligned}$$

$$\phi_{\beta\beta}(\beta,\theta) \;=\; \varepsilon_{\beta\beta}(\beta,\theta)\left[Y-\beta\right]-\varepsilon_{\beta}(\beta,\theta)-\varepsilon_{\beta\theta}\Big(\beta,\theta\Big)H(\theta). \tag{A.3}$$

Hence $\phi(\cdot)$ is strictly concave in β for $\beta \in [0,Y]$ (point i). From (A.2), $\phi_{\beta}(\beta,\theta) < 0$ for $\beta > Y$. Hence the optimal β must be less than Y.[27] Together with point i, $\hat{\beta}(\theta)$ is unique and is characterized by the FOC (point ii). Point iii then follows from $H(\bar{\theta}) = 0$. Next:

$$\phi_{\theta\beta}(\beta,\theta) \;=\; \varepsilon_{\theta\beta}(\beta,\theta)\left[Y-\beta\right]-\varepsilon_{\theta\theta}(\beta,\theta)H(\theta)-\varepsilon_{\theta}(\beta,\theta)H_{\theta}(\theta). \tag{A.4}$$

Using point ii and FOC (A.2), $Y-\hat{\beta}(\theta) = \frac{\varepsilon_{\theta}\left(\hat{\beta}(\theta),\theta\right)H(\theta)}{\varepsilon_{\beta}\left(\hat{\beta}(\theta),\theta\right)}$. Substitute this into (A.4) and with some algebra (and dropping the arguments without confusion):

$$\begin{aligned}
\phi_{\theta\beta}(\beta,\theta)\Big|_{\beta=\hat{\beta}(\theta)} &= \varepsilon_{\beta\theta}\frac{\varepsilon_{\theta}H(\theta)}{\varepsilon_{\beta}}-\varepsilon_{\theta\theta}H(\theta)-\varepsilon_{\theta}H_{\theta}(\theta)\\
&= \frac{\left[2(\psi'')^2-\psi'\psi'''\right]}{\theta^2(\psi'')^2}\varepsilon H(\theta)-\varepsilon_{\theta}H_{\theta}(\theta) > 0
\end{aligned}$$

where the inequality follows Assumptions 1 and 2. By the implicit function theorem, $\frac{d}{d\theta}\hat{\beta}(\theta) = -\frac{\phi_{\theta\beta}\left(\hat{\beta}(\theta),\theta\right)}{\phi_{\beta\beta}\left(\hat{\beta}(\theta),\theta\right)} > 0$ which is also continuous. This establishes points iv and v. Since $\phi(\cdot)$ is continuous in $\theta \in \left[\theta^o,\bar{\theta}\right]$, by the Maximum theorem, $\phi\Big(\hat{\beta}(\theta),\theta\Big)$ is continuous (point vi). $\square$

Lemma 5. *Fix a $\ddot{\beta} > 0$ and let $\tilde{\beta}^*(\cdot)$ be the solution to program $\tilde{\mathcal{P}}'$. $\tilde{\beta}^*(\cdot)$ is uniquely characterized as the following:*

1. *If $\ddot{\beta} \geq Y$, then $\tilde{\beta}^*(\theta) = \ddot{\beta}\ \forall\theta \in \left[\theta^o,\bar{\theta}\right]$.*

2. *If $\ddot{\beta} \leq \hat{\beta}(\theta^o)$, then $\tilde{\beta}^*(\theta) = \hat{\beta}(\theta)\ \forall\theta \in \left[\theta^o,\bar{\theta}\right]$.*

[27] From here, it is readily noted that the principal's objective is strictly quasi-concave in β. Hence offering stochastic contracts will never be optimal for the principal.

3. *If* $\ddot{\beta} \in \left(\hat{\beta}(\theta^o), Y\right)$, *then*

$$\tilde{\beta}^*(\theta) = \begin{cases} \hat{\beta}(\theta) & , \forall \theta \geq \ddot{\theta} \\ \ddot{\beta} & , \forall \theta < \ddot{\theta} \end{cases} \tag{A.5}$$

where $\ddot{\theta} := \hat{\beta}^{-1}\left(\ddot{\beta}\right)$.

Proof. Case 1 is trivially true since $\phi_\beta\left(\beta, \theta\right) < 0 \; \forall \theta$ for $\beta > Y$. Case 2 follows from $\hat{\beta}(\cdot)$ being point-wise optimal for all θ. For Case 3, first note that $\ddot{\theta}$ is well-defined since $\hat{\beta}(\cdot)$ is strictly increasing. Ignore the monotonicity constraint for the moment and just consider the constraint that $\beta\left(\theta\right) \geq \ddot{\beta} \; \forall \theta$. For any $\theta \geq \ddot{\theta}$, $\tilde{\beta}^*$ maximizes ϕ since $\hat{\beta}$ is the point-wise optimal. Next, consider any $\theta < \ddot{\theta}$. Since ϕ is strictly concave in β for $\beta \leq Y$ and $\phi_\beta\left(\hat{\beta}\left(\theta\right), \theta\right) = 0$, this implies that for any $\beta', \beta'' \leq Y$ such that $\hat{\beta}\left(\theta\right) \leq \beta' < \beta''$, $\phi\left(\beta', \theta\right) > \phi\left(\beta'', \theta\right)$. Since the bonus must be at least $\ddot{\beta}$, $\ddot{\beta}$ is the constrained optimal for any $\theta < \ddot{\theta}$. Hence $\tilde{\beta}^*$ maximizes ϕ for any θ under the constraint that $\beta\left(\theta\right) \geq \ddot{\beta} \; \forall \theta$. Since it is also monotonic, it is the solution to program $\tilde{\mathcal{P}}'$. □

(Step 2) Solve for $\ddot{\beta}$:

Lemma 6. $\Phi\left(\ddot{\beta}\right)$ *is differentiable, strictly decreasing and strictly concave for* $\ddot{\beta} \leq Y$.

Proof. $\Phi\left(\ddot{\beta}\right)$ can be written as:

$$\Phi\left(\ddot{\beta}\right) = \int_{\theta^o}^{\hat{\beta}^{-1}\left(\ddot{\beta}\right)} \phi\left(\ddot{\beta}, \theta\right) f(\theta) d\theta + \int_{\hat{\beta}^{-1}\left(\ddot{\beta}\right)}^{\bar{\theta}} \phi\left(\hat{\beta}(\theta), \theta\right) f(\theta) d\theta.$$

Since $\hat{\beta}(\cdot)$ is strictly increasing and differentiable, $\hat{\beta}^{-1}(\beta)$ is also strictly increasing and differentiable with derivative $\frac{1}{\hat{\beta}'\left(\hat{\beta}^{-1}(\beta)\right)}$ which is continuous since $\hat{\beta}'(\cdot)$ is continuous. Hence

Φ is differentiable by Leibniz's rule:

$$\frac{d}{d\ddot{\beta}}\Phi\left(\ddot{\beta}\right) = \underbrace{\left(\phi\left(\ddot{\beta},\ddot{\theta}\right)f(\ddot{\theta}) - \phi\left(\hat{\beta}(\ddot{\theta}),\ddot{\theta}\right)\right)f(\ddot{\theta})\Big)}_{=0}\left(\frac{1}{\hat{\beta}'\left(\hat{\beta}^{-1}(\ddot{\beta})\right)}\right) + \int_{\theta^o}^{\hat{\beta}^{-1}(\ddot{\beta})}\underbrace{\phi_\beta\left(\ddot{\beta},\theta\right)}_{<0}f(\theta)d\theta < 0.$$

$\phi_\beta\left(\ddot{\beta},\theta\right) < 0\ \forall\theta \in \left[\theta^o,\ddot{\theta}\right)$ since $\phi_{\beta\beta} < 0$ and $\ddot{\beta} > \hat{\beta}(\theta)$. Differentiating again:

$$\frac{d^2}{d\ddot{\beta}^2}\Phi\left(\ddot{\beta}\right) = \underbrace{\phi_\beta\left(\hat{\beta}(\ddot{\theta}),\ddot{\theta}\right)}_{=0}f(\ddot{\theta})\left(\frac{1}{\hat{\beta}'\left(\hat{\beta}^{-1}(\ddot{\beta})\right)}\right) + \int_{\theta^o}^{\hat{\beta}^{-1}(\ddot{\beta})}\underbrace{\phi_{\beta\beta}\left(\ddot{\beta},\theta\right)}_{<0}f(\theta)d\theta < 0.$$

$\square$

Lemma 7. *$s\left(\ddot{\beta},\theta^o\right)F\left(\theta^o\right)$ is differentiable, strictly increasing and strictly concave for $\ddot{\beta} < Y$.*

Proof. Differentiability of $s(\cdot)$ follows from differentiability of $\varepsilon(\cdot)$ and $\psi(\cdot)$.

$$\begin{aligned}\frac{d}{d\ddot{\beta}}s\left(\ddot{\beta},\theta^o\right) &= \varepsilon_\beta\left(\ddot{\beta},\theta^o\right)Y - \frac{\varepsilon_\beta\left(\ddot{\beta},\theta^o\right)}{\theta^o}\psi'\left(\frac{\varepsilon\left(\ddot{\beta},\theta^o\right)}{\theta^o}\right)\\ &= \varepsilon_\beta\left(\ddot{\beta},\theta^o\right)\left[Y-\ddot{\beta}\right]\end{aligned}$$

which is positive whenever $\ddot{\beta} < Y$, and $\frac{d^2}{d\ddot{\beta}^2}s\left(\ddot{\beta},\theta^o\right) = \varepsilon_{\beta\beta}\left(\ddot{\beta},\theta^o\right)\left[Y-\ddot{\beta}\right] - \varepsilon_\beta\left(\ddot{\beta},\theta^o\right) < 0.$ $\square$

Lemmas 6 and 7 jointly imply that $\Phi\left(\ddot{\beta}\right) + s\left(\ddot{\beta},\theta\right)F\left(\theta^o\right)$ is strictly concave in $\ddot{\beta} \in \left[\hat{\beta}(\theta^o),Y\right]$. Moreover, the optimal must be in the interior of $\left[\hat{\beta}(\theta^o),Y\right]$. To see why, notice that $\frac{d}{d\ddot{\beta}}\Phi\left(\ddot{\beta}\right)\Big|_{\ddot{\beta}=\hat{\beta}(\theta^o)} = 0$ while $\frac{d}{d\ddot{\beta}}s\left(\ddot{\beta},\theta^o\right)\Big|_{\ddot{\beta}=\hat{\beta}(\theta^o)} > 0$. Hence $\ddot{\beta} > \hat{\beta}(\theta^o)$. Similarly, $\frac{d}{d\ddot{\beta}}\Phi\left(\ddot{\beta}\right)\Big|_{\ddot{\beta}=Y} < 0$ while $\frac{d}{d\ddot{\beta}}s\left(\ddot{\beta},\theta^o\right)\Big|_{\ddot{\beta}=Y} = 0$. Hence $\ddot{\beta} < Y$. Thus, the solution to (A.1) is characterized by its first-order condition which is condition (5.8) in the proposition. The strict concavity also implies that the solution is unique and thus, the solution to $\mathcal{P}'$ is unique. Notice that $\tilde{\beta}^*(\cdot)$ is continuous everywhere, differentiable everywhere except (possibly) at $\ddot{\theta}$, and strictly increasing for $\theta \geq \ddot{\theta}$. Since these properties hold for any $\ddot{\beta}$, they carry over to $\beta^*(\cdot)$. Finally, $\alpha^*(\cdot)$ is obtained from Lemma 2, with $\frac{d}{d\theta}\alpha^*(\theta) = -u_\beta(\beta^*(\theta), \theta)\,\beta^*_\theta(\theta) \leq 0$ whenever the derivative exists.

Proof of Lemma 3.

Proof. Since $W^\tau(e,b)$ is super-modular, $e^\tau(b)$ is strictly increasing. Concavity of $W^\tau(e,b)$ in e implies that $e^\tau(b)$ is characterized by the respective first-order condition:

$$b = \frac{1}{\theta^o}\psi'\left(\frac{e^o(b)}{\theta^o}\right) \quad , \quad b = E\left[\frac{1}{\theta^n}\psi'\left(\frac{e^n(b)}{\theta^n}\right)\right] \tag{A.6}$$

This implies that for any $b > 0$:

$$\frac{1}{\theta^o}\psi'\left(\frac{e^o(b)}{\theta^o}\right) \;=\; E\left[\frac{1}{\theta^n}\psi'\left(\frac{e^n(b)}{\theta^n}\right)\right] \;>\; \frac{1}{E[\theta^n]}\psi'\left(\frac{e^n(b)}{E[\theta^n]}\right) \;\geq\; \frac{1}{\theta^o}\psi'\left(\frac{e^n(b)}{\theta^o}\right).$$

Since $\psi'(\cdot)$ is strictly increasing, $e^o(b) > e^n(b)$.

Next let $\Delta(b) = \bar{W}^o(b) - \bar{W}^n(b)$. Since $\lim_{b\to 0} e^\tau(b) = 0\ \forall\tau$, $\lim_{b\to 0}\Delta(b) = -R < 0$. Hence there exists small enough b such that $\Delta(b) < 0$. By the envelope theorem, $\Delta'(b) = e^o(b) - e^n(b) > 0\ \forall b > 0$; this implies that $\Delta(b)$ is strictly increasing and thus can cross 0 at most once. If $\Delta(b)$ crosses 0, then $\bar{b}$ is defined by $\bar{b} = \Delta^{-1}(0)$. If $\Delta(\cdot)$ is bounded above by 0, then $\bar{b} = \infty$. That $\bar{b}$ is non-decreasing in R is immediate from the observation that $\Delta(b)$ is point-wise decreasing in R. □

Proof of Proposition 3.

Proof. It suffices to show that the expected period-1 surplus generated by b is strictly concave. $\frac{d}{db}\left[e^n(b)Y - E\left[\psi\left(\frac{e^n(b)}{\theta^n}\right)\right]\right] = \frac{de^n(b)}{db}(Y - b)$; hence, $b \leq Y$. $\frac{d^2}{db^2}\left[e^n(b)Y - E\left[\psi\left(\frac{e^n(b)}{\theta^n}\right)\right]\right] = -\frac{de^n(b)}{db} + \frac{d^2e^n(b)}{db^2}(Y - b)$; hence the surplus is strictly concave if $\frac{d^2e^n(b)}{db^2} \leq 0$. Differentiating the FOC for $e^n(b)$ in (A.6) with b twice gives:

$$-E\left[\left(\frac{1}{\theta^n}\right)^3 \psi'''\left(\frac{e^n(b)}{\theta^n}\right)\right]\left(\frac{d}{db}\left(e^n(b)\right)\right)^2 - E\left[\left(\frac{1}{\theta^n}\right)^2 \psi''\left(\frac{e^n(b)}{\theta^n}\right)\right]\frac{d^2}{db^2}\left(e^n(b)\right) = 0$$

Since $\psi'''(\cdot) \geq 0$, it must hold that $\frac{d^2}{db^2}\left(e^n(b)\right) \leq 0$. $\square$

Proof of Proposition 4.

Proof. Program $\mathcal{P}$ has a feasible solution that achieves $s(Y, \theta^o)$: set $\beta(\theta) = Y$ and $\alpha(\theta) = -s(Y, \theta^o)\ \forall\theta$. This implies that $\int_{\theta^o}^{\bar{\theta}} s\left(\beta^*(\theta), \theta\right) dF(\theta) - R \geq s(Y, \theta^o)$. Since $R > 0$, $E\left[Innov(\theta)\right] > 0$. The other properties are immediate. $\square$

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B Supplementary Appendix: Proofs for Section 6 (for online publication only)

The following property will be useful for the proofs.

Lemma B.1. *$s_\theta(\beta,\theta) = \varepsilon_\theta(\beta,\theta)[Y-\beta] + u_\theta(\beta,\theta)$ and $s_{\theta\beta}(\beta,\theta) = \varepsilon_{\theta\beta}(\beta,\theta)[Y-\beta]$. Hence $\forall\beta \leq Y$, $s_\theta(\beta,\theta) > 0$ and $s_{\theta\beta}(\beta,\theta) > 0$.*

Proof.

$$\begin{aligned}
s_\theta(\beta,\theta) &= \varepsilon_\theta(\beta,\theta)Y - \psi'\left(\frac{\varepsilon(\beta,\theta)}{\theta}\right)\left[\frac{\theta\varepsilon_\theta(\beta,\theta)-\varepsilon(\beta,\theta)}{\theta^2}\right] \\
&= \varepsilon_\theta(\beta,\theta)\left[Y - \frac{1}{\theta}\psi'\left(\frac{\varepsilon(\beta,\theta)}{\theta}\right)\right] + \psi'\left(\frac{\varepsilon(\beta,\theta)}{\theta}\right)\frac{\varepsilon(\beta,\theta)}{\theta^2} \\
&= \varepsilon_\theta(\beta,\theta)[Y-\beta] + u_\theta(\beta,\theta)
\end{aligned}$$

By noting that $u_{\theta\beta}(\beta,\theta) = \varepsilon_\theta(\beta,\theta)$, it is immediate that $s_{\theta\beta}(\beta,\theta) = \varepsilon_{\theta\beta}(\beta,\theta)[Y-\beta]$. $\square$

Proof of Remark 3.

Proof. Since $F(\theta;k') = 1$ for $\theta \in \left[\bar{\theta}(k'), \bar{\theta}(k'')\right]$ and $f(\cdot;k'')$ is strictly positive over its support $\left[\underline{\theta}, \bar{\theta}(k'')\right]$, the lemma is trivially true in $\left[\bar{\theta}(k'), \bar{\theta}(k'')\right]$. Consider $\theta \in \left[\underline{\theta}, \bar{\theta}(k')\right]$ next. Notice that $\int_{\underline{\theta}}^{\theta} \frac{1}{H(t;k)}dt = -\log[1-F(\theta;k)]$. For any $\theta \in \left[\underline{\theta}, \bar{\theta}(k')\right]$:

$$\begin{aligned}
& \textstyle\int_{\underline{\theta}}^{\theta} \frac{1}{H(t;k')}dt > \int_{\underline{\theta}}^{\theta} \frac{1}{H(t;k'')}dt \\
\Longrightarrow \quad & -\log[1-F(\theta;k')] > -\log[1-F(\theta;k'')] \\
\Longrightarrow \quad & F(\theta;k') > F(\theta;k'')
\end{aligned}$$

$\square$

Proof of Proposition 5.

Throughout this proof, let $\hat{\beta}(\cdot;k)$ be the schedule in (5.7) for $F(\cdot;k)$, let $\ddot{\theta}(k)$ be the corresponding highest bunching type in (5.8), and let $\ddot{\beta}(k)$ denote the bunching bonus level $\hat{\beta}\left(\ddot{\theta}(k);k\right)$.

Proof of Proposition 5.1.

Proof. Since $\varepsilon_{\beta\beta} < 0$ and $\varepsilon_{\beta\theta} > 0$, inspection of (5.7) reveals $H(\theta;k'') > H(\theta;k') \implies \hat{\beta}(\theta;k'') < \hat{\beta}(\theta;k')$. Hence we only need to show $\ddot{\beta}(k'') < \ddot{\beta}(k')$. Rewrite (5.8) as:

$$\int_{\theta^o}^{\hat{\beta}^{-1}\left(\ddot{\beta}(k);k\right)} \phi_\beta\left(\ddot{\beta}(k),\theta\right) f(\theta;k)d\theta + s_\beta\left(\ddot{\beta}(k),\theta^o\right) F(\theta^o;k) = 0 \tag{B.1}$$

where $\hat{\beta}^{-1}(\cdot;k)$ is the inverse function of $\hat{\beta}(\cdot;k)$. By the implicit function theorem, $\frac{d}{dk}\ddot{\beta}(k) = -\frac{\frac{\partial(eq\ (B.1))}{\partial k}}{\frac{\partial(eq\ (B.1))}{\partial\ddot{\beta}}}$. From Lemma 6 and 7, $\frac{\partial(eq\ B.1)}{\partial\ddot{\beta}} < 0$. I show that $\frac{\partial(eq\ B.1)}{\partial k} < 0$ next.

$$\begin{aligned}\frac{\partial(eq\ B.1)}{\partial k} = \;& \underbrace{\phi_\beta\left(\ddot{\beta}(k),\ddot{\theta}(k)\right)}_{=0} f\left(\ddot{\theta}(k);k\right)\left(\frac{\partial}{\partial k}\hat{\beta}^{-1}\left(\ddot{\beta}(k);k\right)\right)\\ &+\int_{\theta^o}^{\hat{\beta}^{-1}\left(\ddot{\beta}(k);k\right)}\left[\underbrace{\varepsilon_\beta\left(\ddot{\beta}(k),\theta\right)}_{>0}\underbrace{\left(Y-\ddot{\beta}(k)\right)}_{>0} f_k(\theta;k) + \underbrace{\varepsilon_\theta\left(\ddot{\beta}(k),\theta\right)}_{>0}\underbrace{F_k(\theta;k)}_{<0}\right]d\theta\\ &+\underbrace{s_\beta\left(\ddot{\beta}(k),\theta^o\right)}_{>0}\underbrace{F_k(\theta^o;k)}_{<0}.\end{aligned}$$

To sign the second line, notice that if $f_k(\theta;k) \le 0$, then $\frac{\partial(eq\ B.1)}{\partial k} < 0$. Suppose that $f_k(\theta;k) > 0$ instead. Since $\theta \le \hat{\beta}^{-1}\left(\ddot{\beta}(k);k\right)$ in this region, $\ddot{\beta}(k) \ge \hat{\beta}(\theta;k)$. From (A.2) and (A.3):

$$\varepsilon_\beta\left(\ddot{\beta}(k),\theta\right)\left[Y-\ddot{\beta}(k)\right] - \varepsilon_\theta\left(\ddot{\beta}(k),\theta\right)H(\theta;k) \le 0 \iff Y-\ddot{\beta}(k) \le \frac{\varepsilon_\theta\left(\ddot{\beta}(k),\theta\right)H(\theta;k)}{\varepsilon_\beta\left(\ddot{\beta}(k),\theta\right)}$$

Substitute this into the above:

$$
\begin{aligned}
& \varepsilon_{\beta}\Big(\ddot{\beta}(k),\theta\Big)\Big(Y-\ddot{\beta}(k)\Big)f_k(\theta;k)+\varepsilon_{\theta}\left(\ddot{\beta}(k),\theta\right)F_k(\theta;k) \\
\leq\; & \varepsilon_{\beta}\Big(\ddot{\beta}(k),\theta\Big)\left(\frac{\varepsilon_{\theta}\left(\ddot{\beta}(k),\theta\right)H(\theta;k)}{\varepsilon_{\beta}\left(\ddot{\beta}(k),\theta\right)}\right)f_k(\theta;k)+\varepsilon_{\theta}\left(\ddot{\beta}(k),\theta\right)F_k(\theta;k) \\
=\; & \varepsilon_{\theta}\left(\ddot{\beta}(k),\theta\right)\left[H(\theta;k)f_k(\theta;k)+F_k(\theta;k)\right]
\end{aligned}
$$

which is negative since $\frac{d}{dk}H(\theta;k)>0$ and $\frac{d}{dk}H(\theta;k)=-\frac{1}{f(\theta;k)}\left[H(\theta;k)f_k(\theta;k)+F_k(\theta;k)\right]$.

$\square$

Proof of Proposition 5.2.[28]

Proof. By an integration by part, the objective in Program $\mathcal{P}'$ in (5.4) can be written as $\int_{\theta^o}^{\bar{\theta}} v\left(\beta(\cdot),\theta\right)dF(\theta)=v\left(\beta(\cdot),\bar{\theta}\right)-\int_{\theta^o}^{\bar{\theta}}\frac{d}{d\theta}\left[v\left(\beta^*(\cdot),\theta\right)\right]F(\theta)d\theta$. Suppose $k''>k'$. Let (without loss) $\beta^*(\theta;k')=Y$ for $\theta\in\left[\bar{\theta}(k'),\bar{\theta}(k'')\right]$. This implies:

$$
\begin{aligned}
\mathcal{V}(k'') &\geq v\left(\beta^*(\cdot;k'),\bar{\theta}(k'')\right)-\int_{\theta^o}^{\bar{\theta}(k'')}\frac{d}{d\theta}\left[v\left(\beta^*(\cdot;k'),\theta\right)\right]F(\theta;k'')d\theta, \\
\mathcal{V}(k') &= v\left(\beta^*(\cdot;k'),\bar{\theta}(k')\right)-\int_{\theta^o}^{\bar{\theta}(k')}\frac{d}{d\theta}\left[v\left(\beta^*(\cdot;k'),\theta\right)\right]F(\theta;k')d\theta.
\end{aligned}
$$

Hence:

$$
\begin{aligned}
\mathcal{V}(k'')-\mathcal{V}(k') \;\geq\; & v\left(\beta^*(\cdot;k'),\bar{\theta}(k'')\right)-v\left(\beta^*(\cdot;k'),\bar{\theta}(k')\right) && \text{(B.2)} \\
& -\int_{\bar{\theta}(k')}^{\bar{\theta}(k'')}\frac{d}{d\theta}\left[v\left(\beta^*(\cdot;k'),\theta\right)\right]F(\theta;k'')d\theta && \text{(B.3)} \\
& -\int_{\theta^o}^{\bar{\theta}(k')}\frac{d}{d\theta}\left[v\left(\beta^*(\cdot;k'),\theta\right)\right]\left(F(\theta;k'')-F(\theta;k')\right)d\theta. && \text{(B.4)}
\end{aligned}
$$

[28] Standard envelope theorem (eg. Milgrom and Segal (2002)) cannot be directly applied here because the feasible set $\mathcal{B}(k)$ is dependent on k. The proposition also does not require differentiability of $\mathcal{V}(k)$.

Using Lemma B.1, $s_\theta(Y,\theta) = u_\theta(Y,\theta)$. Hence:

$$\begin{aligned}
\text{(B.2)} &= v\left(\beta^*(\cdot;k'),\bar\theta(k'')\right) - v\left(\beta^*(\cdot;k'),\bar\theta(k')\right) \\
&= s\left(Y,\bar\theta(k'')\right) - s\left(Y,\bar\theta(k')\right) - \int_{\bar\theta(k')}^{\bar\theta(k'')} u_\theta\left(\beta^*(\theta;k'),\theta\right) d\theta \\
&= \int_{\bar\theta(k')}^{\bar\theta(k'')} s_\theta(Y,\theta)\, d\theta - \int_{\bar\theta(k')}^{\bar\theta(k'')} u_\theta(Y,\theta)\, d\theta \\
&= 0
\end{aligned}$$

$$\begin{aligned}
\text{(B.3)} &= \int_{\bar\theta(k')}^{\bar\theta(k'')} \frac{d}{d\theta}\left[v\left(\beta^*(\cdot;k'),\theta\right)\right] F(\theta;k'')dt \\
&= \int_{\bar\theta(k')}^{\bar\theta(k'')} \frac{d}{d\theta}\left[s\Big(\underbrace{\beta^*(\theta;k')}_{=Y},\theta\Big) - \int_{\theta^o}^{\theta} u_\theta\left(\beta(t;k'),t\right) dt\right] F(\theta;k'')d\theta \\
&= \int_{\bar\theta(k')}^{\bar\theta(k'')} \left[s_\theta(Y,\theta) - u_\theta(Y,\theta)\right] F(\theta;k'')d\theta \\
&= 0.
\end{aligned}$$

On the other hand, (B.4) is strictly positive:

$$\frac{d}{d\theta}v\left(\beta^*(\cdot;k'),\theta\right) = s_\beta\left(\beta^*(\theta;k'),\theta\right)\beta^*_\theta(\theta;k') + \varepsilon_\theta\left(\beta^*(\theta;k'),\theta\right)\left[Y - \beta^*(\theta;k')\right] > 0.$$

□

Proof of Proposition 6.

Let $R(k)$ be the agent's information rent under $F(\cdot;k)$.

Lemma B.2. *If $\hat\beta_{\theta k}(\theta;k) \le 0\ \forall\theta,k$, then $R(k)$ is strictly increasing in k.*[29]

Proof. By an integration by parts, $R(k) = \int_{\theta^o}^{\bar\theta(k)} u_\theta\left(\beta^*(\theta;k),\theta\right)\left[1 - F(\theta;k)\right] d\theta$. Note that

[29] Since $f(\theta;k)$ is absolutely continuous in k, $\hat\beta_\theta(\theta;k)$ is differentiable in k almost everywhere.

$u_\theta\left(\beta^*(\theta;k),\theta\right)$ is differentiable in θ everywhere except at $\ddot{\theta}(k)$. Define $Q(\theta;k) := \int_{\underline{\theta}}^{\theta}\left[1-F(t;k)\right]dt > 0$. Doing integration by parts again:

$$\begin{aligned}
R(k) &= u_\theta\left(\beta^*(\theta;k),\theta\right)Q(\theta;k)\Big|_{\theta^o}^{\bar{\theta}(k)} - \int_{\theta^o}^{\bar{\theta}(k)}\left[\frac{d}{d\theta}u_\theta\left(\beta^*(\theta;k),\theta\right)\right]Q(\theta;k)d\theta \\
&= u_\theta\left(Y,\bar{\theta}(k)\right)Q(\bar{\theta}(k);k) - u_\theta\left(\ddot{\beta}(k),\theta^o\right)Q(\theta^o;k) \\
&\quad - \int_{\ddot{\theta}(k)}^{\bar{\theta}(k)}\left[\frac{d}{d\theta}u_\theta\left(\hat{\beta}(\theta;k),\theta\right)\right]Q(\theta;k)d\theta - \int_{\theta^o}^{\ddot{\theta}(k)}\left[\frac{d}{d\theta}u_\theta\left(\ddot{\beta}(k),\theta\right)\right]Q(\theta;k)d\theta.
\end{aligned}$$

Differentiating with respect to k now:[30]

$$\begin{aligned}
&\frac{d}{dk}R(k) \\
&= \underbrace{\left[u_{\theta\theta}\left(Y,\bar{\theta}(k)\right)Q(\bar{\theta}(k);k) + u_\theta\left(Y,\bar{\theta}(k)\right)\underbrace{Q_\theta(\bar{\theta}(k);k)}_{=1-F(\bar{\theta}(k);k)=0} - \underbrace{\left[\frac{d}{d\theta}u_\theta\left(\hat{\beta}(\bar{\theta}(k);k),\bar{\theta}(k)\right)\right]}_{=u_{\theta\theta}\left(Y,\bar{\theta}(k)\right)}Q(\bar{\theta}(k);k)\right]\bar{\theta}'(k)}_{=0} \\
&\quad + \underbrace{u_\theta\left(Y,\bar{\theta}(k)\right)Q_k(\bar{\theta}(k);k) - u_\theta\left(\ddot{\beta}(k),\theta^o\right)Q_k(\theta^o;k)}_{=\text{Term 1}} - \underbrace{\left[\frac{d}{dk}u_\theta\left(\ddot{\beta}(k),\theta^o\right)\right]Q(\theta^o;k)}_{<0} \\
&\quad \underbrace{+\left[\frac{d}{d\theta}u_\theta\left(\hat{\beta}\left(\ddot{\theta}(k);k\right),\ddot{\theta}(k)\right)\right]Q\left(\ddot{\theta}(k);k\right)\cdot\ddot{\theta}'(k) - \left[\frac{d}{d\theta}u_\theta\left(\ddot{\beta}(k),\ddot{\theta}(k)\right)\right]Q\left(\ddot{\theta}(k);k\right)\cdot\ddot{\theta}'(k)}_{=0} \\
&\quad - \underbrace{\int_{\ddot{\theta}(k)}^{\bar{\theta}(k)}\frac{d}{dk}\left(\left[\frac{d}{d\theta}u_\theta\left(\hat{\beta}(\theta;k),\theta\right)\right]Q(\theta;k)\right)d\theta}_{=\text{Term 2}} - \underbrace{\int_{\theta^o}^{\ddot{\theta}(k)}\frac{d}{dk}\left(\left[\frac{d}{d\theta}u_\theta\left(\ddot{\beta}(k),\theta\right)\right]Q(\theta;k)\right)d\theta}_{=\text{Term 3}}
\end{aligned}$$

[30] The derivative exists almost everywhere.

Hence $\frac{d}{dk}R(k) >$ (Term 1) − (Term 2) − (Term 3). Next:

$$\begin{aligned}
\text{Term 1} \;&=\; u_\theta\Big(\underbrace{\hat\beta(\bar\theta(k);k)}_{=Y},\bar\theta(k)\Big)Q_k(\bar\theta(k);k) - u_\theta\Big(\underbrace{\hat\beta\left(\ddot\theta(k);k\right)}_{=\ddot\beta(k)},\ddot\theta(k)\Big)Q_k\left(\ddot\theta(k);k\right)\\
&\quad + u_\theta\left(\ddot\beta(k),\ddot\theta(k)\right)Q_k\left(\ddot\theta(k);k\right) - u_\theta\left(\ddot\beta(k),\theta^o\right)Q_k(\theta^o;k)\\
&= \int_{\ddot\theta(k)}^{\bar\theta(k)}\frac{d}{d\theta}\left(u_\theta\left(\hat\beta(\theta;k),\theta\right)Q_k(\theta;k)\right)d\theta + \int_{\theta^o}^{\ddot\theta(k)}\frac{d}{d\theta}\left(u_\theta\left(\ddot\beta(k),\theta\right)Q_k(\theta;k)\right)d\theta\\
&= \int_{\ddot\theta(k)}^{\bar\theta(k)}\left(\left[\frac{d}{d\theta}u_\theta\left(\hat\beta(\theta;k),\theta\right)\right]Q_k(\theta;k) \;+\; u_\theta\left(\hat\beta(\theta;k),\theta\right)Q_{k\theta}(\theta;k)\right)d\theta\\
&\quad + \int_{\theta^o}^{\ddot\theta(k)}\left(\left[\frac{d}{d\theta}u_\theta\left(\ddot\beta(k),\theta\right)\right]Q_k(\theta;k) \;+\; u_\theta\left(\ddot\beta(k),\theta\right)Q_{k\theta}(\theta;k)\right)d\theta
\end{aligned}$$

and

$$\begin{aligned}
\text{Term 2} \;&=\; \int_{\ddot\theta(k)}^{\bar\theta(k)}\left[\frac{d}{d\theta}u_\theta\left(\hat\beta(\theta;k),\theta\right)\right]Q_k(\theta;k)d\theta + \int_{\ddot\theta(k)}^{\bar\theta(k)}\left[\frac{d^2}{d\theta dk}u_\theta\left(\hat\beta(\theta;k),\theta\right)\right]Q(\theta;k)d\theta\\
\text{Term 3} \;&=\; \int_{\theta^o}^{\ddot\theta(k)}\left[\frac{d}{d\theta}u_\theta\left(\ddot\beta(k),\theta\right)\right]Q_k(\theta;k)d\theta + \int_{\theta^o}^{\ddot\theta(k)}\left[\frac{d^2}{d\theta dk}u_\theta\left(\ddot\beta(k),\theta\right)\right]Q(\theta;k)d\theta
\end{aligned}$$

Hence

$$\begin{aligned}
&\text{(Term 1)} - \text{(Term 2)} - \text{(Term 3)}\\
=\;& \int_{\ddot\theta(k)}^{\bar\theta(k)} u_\theta\left(\hat\beta(\theta;k),\theta\right)Q_{k\theta}(\theta;k)d\theta \;+\; \int_{\theta^o}^{\ddot\theta(k)} u_\theta\left(\ddot\beta(k),\theta\right)Q_{k\theta}(\theta;k)d\theta\\
&-\int_{\ddot\theta(k)}^{\bar\theta(k)}\underbrace{\left[u_{\theta\beta\beta}\left(\hat\beta(\theta;k),\theta\right)\hat\beta_\theta(\theta;k)\hat\beta_k(\theta;k) + u_{\theta\beta}\left(\hat\beta(\theta;k),\theta\right)\hat\beta_{\theta k}(\theta;k) + u_{\theta\theta\beta}\left(\hat\beta(\theta;k),\theta\right)\hat\beta_k(\theta;k)\right]}_{=\frac{d^2}{d\theta dk}u_\theta\left(\hat\beta(\theta;k),\theta\right)}Q(\theta;k)d\theta\\
&-\int_{\theta^o}^{\ddot\theta(k)}\underbrace{\left[u_{\theta\theta\beta}\left(\ddot\beta(k),\theta\right)\ddot\beta_k(k)\right]}_{=\frac{d^2}{d\theta dk}u_\theta\left(\ddot\beta(k),\theta\right)}Q(\theta;k)d\theta
\end{aligned}$$

Since $Q_{k\theta}(\theta;k) = -F_k(\theta;k) > 0$, the first line is positive. From the proof of Lemma 1,

$u_{\theta\beta} > 0$ and it is readily verified that $u_{\theta\theta\beta}, u_{\theta\beta\beta} > 0$; from the proof of Proposition 5.1, $\frac{d\ddot{\beta}(k)}{dk} < 0$. Hence the last line is positive. If $\hat{\beta}_{\theta k}(\theta; k) \leq 0$, then the second line is also positive. This in turn implies that $\frac{dR(k)}{dk} > 0$. $\square$

Proposition 6 is then established by checking that the condition $\hat{\beta}_{\theta k}(\theta; k) \leq 0$ holds under the given parametrization.

Proof. When $\psi(x) = \xi x^2$, it is readily verified that $\varepsilon(\beta, \theta) = \frac{1}{2\xi}\beta\theta^2$. From (5.7), $\hat{\beta}(\theta) = \frac{\theta Y}{2H(\theta)+\theta}$. When $\mathcal{F}$ is the family of uniform distributions in (6.1), $H(\theta; k) = k - \theta$. Hence $\hat{\beta}(\theta; k) = \frac{\theta}{2k-\theta} Y$ and it is readily verified that $\hat{\beta}_{\theta k}(\theta; k) < 0$. $\square$

C Supplementary Appendix: Additional Details and Extensions (for online publication Only)

This appendix considers other equilibria under FW besides the exploration equilibrium considered in the main text, and extensions to the baseline model with regards to general monitoring environments and innovative efforts by the agent.

C.1 Other Equilibria under FW

C.1.1 Exploitation Equilibrium under FW

This subsection considers the possibility of an exploitation equilibrium under FW in which the agent chooses o in period 1. In this case, θ^n is unknown in period 2 on the equilibrium path, hence the agent chooses o in period 2. The principal then optimally sets $\beta^* = Y$ and $\alpha^* = -s\left(Y, \theta^o\right)$. Under this period-2 contract, if the agent had deviated to $\tau = n$ in period 1 and privately knows θ^n, his period-2 payoff under innovation outcome $\theta = \max\left\{\theta^o, \theta^n\right\}$ is $\alpha^* + \max_\varepsilon \left\{\varepsilon Y - \psi\left(\frac{\varepsilon}{\theta}\right)\right\} = s\left(Y, \theta\right) - s\left(Y, \theta^o\right)$. His ex-ante expected period-2 payoff from deviation is thus $\int_{\theta^o}^{\bar{\theta}} s\left(Y, \theta\right) dF(\theta) - s\left(Y, \theta^o\right) > 0$.

Consider period 1 next. Analogous to (5.10) and (5.11), let the agent's total expected payoffs for choosing technology τ in period 1 together with effort e and under a contract (a, b) be $a + \tilde{W}^\tau(e, b)$, where:

$$\begin{aligned} \tilde{W}^n(e,b) &:= eb - E\left[\psi\left(\frac{e}{\theta^n}\right)\right] + \int_{\theta^o}^{\bar{\theta}} s\left(Y,\theta\right) dF(\theta) - s\left(Y, \theta^o\right), \\ \tilde{W}^o(e,b) &:= eb - \psi\left(\frac{e}{\theta^o}\right). \end{aligned}$$

Let $\tilde{e}^\tau(b) := \arg\max_e \tilde{W}^\tau(e,b)$,[31] and $\bar{\tilde{W}}^\tau(b) := \max_e \tilde{W}^\tau(e,b)$. Following the proof of Lemma 3, it is readily verified that $\bar{\tilde{W}}^o(b) - \bar{\tilde{W}}^n(b)$ is strictly increasing in b, and there exists $\underline{b} > 0$

[31] $\tilde{e}^\tau(\cdot)$ is equivalent to $e^\tau(\cdot)$ in Section 5.2.

such that $\tilde{\bar{\bar{W}}}^o(b) \leq \tilde{\bar{\bar{W}}}^n(b)$ if and only if $b \leq \underline{b}$. Analogous to the argument for Proposition 3, the following result is immediate:

Proposition C.1. *(Exploitation equilibrium under FW.) Under FW, an exploitation equilibrium (i.e. the agent chooses o in period 1) exists if and only if $\underline{b}$ is finite. When $\underline{b}$ is finite, the optimal period-1 contract that induces the agent to choose o in period 1 is unique: $a^* = -\tilde{\bar{\bar{W}}}^n(b^*)$ and $b^* = \max\{Y, \underline{b}\}$.*

Notice that under this equilibrium, the principal's period-2 payoff is $s(Y, \theta^o)$ which is exactly what she gets in period 2 from the optimal contract under TM as characterized in Proposition 1. On the other hand, her expected period-1 payoff must be weakly lower given the additional constraint of $b^* \geq \underline{b}$. Hence:

Corollary C.1. *If an exploitation equilibrium exists under FW, the principal's total expected payoffs from this equilibrium is weakly lower than her total expected payoffs from the unique equilibrium under TM as characterized in Proposition 1.*

C.1.2 Mixed Exploration-Exploitation under FW

This subsection considers equilibria in which the agent chooses mixed strategy in his technology choice in period 1 under FW. Consider an equilibrium in which the agent plays technology n in period 1 with probability ρ. As in Section 5, the agent's payoff-relevant private information is the cost-efficiency of his more efficient technology; if he had used o in period 1 (which occurs with probability $1-\rho$ on the equilibrium path), his cost-efficiency is θ^o. Hence the type space is still $\left[\theta^o, \bar{\theta}\right]$, but the principal's belief about the type distribution is different. In particular, for any $\theta \in \left[\theta^o, \bar{\theta}\right]$:

$$Pr\left(\theta \leq x\right) = 1 - \rho + \rho\left[F(\theta^o) + \int_{\theta^o}^{x} f(t)dt\right] \tag{C.1}$$

where f and F are, as previously, the pdf and cdf of θ^n respectively.

Consider the following density function:

$$\tilde{f}\left(\theta^n;\rho\right)=\begin{cases}\left[\frac{1-\rho}{F(\theta^o)}+\rho\right]f\left(\theta^n\right) & ,\ \theta^n\leq\theta^o\\ \rho f\left(\theta^n\right) & ,\ \theta^n>\theta^o\end{cases},$$

and let $\tilde{F}(\cdot;\rho)$ denote its corresponding cdf; it is readily verified that $\tilde{f}(\cdot;\rho)$ is a valid density for any $\rho\in[0,1]$. Notice that under $\tilde{f}(\cdot;\rho)$, for any $\theta\in\left[\theta^o,\bar{\theta}\right]$,

$$\begin{aligned}Pr\left(\theta\leq x\right) &= \int_{\underline{\theta}}^{\theta^o}\left[\frac{1-\rho}{F(\theta^o)}+\rho\right]f\left(t\right)dt\ +\ \int_{\theta^o}^{x}\rho f(t)dt\\ &= 1-\rho+\rho\left[F(\theta^o)+\int_{\theta^o}^{x}f(t)dt\right]\end{aligned}$$

as in (C.1). In other words, if the principal conjectures that the agent plays technology n in period 1 with probability ρ, it is equivalent to her designing a period-2 menu of contracts based on the belief that θ^n follows a distribution of $\tilde{f}(\cdot;\rho)$. Moreover, letting $\tilde{H}(\cdot;\rho):=\frac{1-\tilde{F}(\cdot;\rho)}{\tilde{f}(\cdot;\rho)}$ be the inverse hazard rate associated with $\tilde{f}(\cdot;\rho)$:

$$\tilde{H}\left(\theta^n;\rho\right)=\begin{cases}H\left(\theta^n\right) & ,\ \text{if}\ \theta^n>\theta^o\\ H\left(\theta^n\right)-\left[\frac{1-\rho}{1-\rho+\rho F(\theta^o)}\right]\frac{1-F(\theta^o)}{f(\theta^n)} & ,\ \text{if}\ \theta^n\leq\theta^o\end{cases}$$

It is readily verified that Assumption 2 is satisfied for $f\left(\theta^n;\rho\right)$ for all $\theta^n\in\left(\theta^o,\bar{\theta}\right]$ and $\rho\in(0,1]$. Hence all results from Section 5 hold for any $\rho\in(0,1]$.[32]

Let $R(\rho)$ be the agent's expected period-2 rent given the principal's conjecture ρ. For

[32]Note that because the agent reverts to o whenever $\theta^n\leq\theta^o$, the distribution of θ^n below θ^o does not have any substantial effect on the analysis in Section 5. Hence all the results there follow through even if the regularity conditions and the decreasing inverse hazard rate ratio property on the distribution of θ^n, as stated in Assumption 2, hold only for $\theta^n\in\left(\theta^o,\bar{\theta}\right]$.

the agent to mix between o and n in period 1, the bonus b in period 1 must satisfy:

$$\max_e \left\{ eb - E\left[\psi\left(\frac{e}{\theta^n}\right)\right] + R(\rho) \right\} = \max_e \left\{ eb - \psi\left(\frac{e}{\theta^o}\right) \right\}. \tag{C.2}$$

Let $\bar{b}(\rho)$ denote the bonus threshold characterized in Lemma 3 for $R = R(\rho)$. If $\bar{b}(\rho)$ is finite, then it is the unique solution bonus to (C.2); if $\bar{b}(\rho)$ is infinite, then there is no bonus level satisfying (C.2).

Proposition C.2. *Under FW, a mixed exploration-exploitation equilibrium in which the agent chooses n with probability ρ exists if and only if $\bar{b}(\rho)$ is finite.*

The final result in this section is on the comparison between the mixed equilibrium and the exploration equilibrium. Let $\Pi(\rho)$, with $\rho \in (0,1)$, be the principal's total payoff under a mixed exploration-exploitation equilibrium in which the agent chooses n with probability ρ.[33]

Proposition C.3. *Let ρ', ρ'' be such that $0 < \rho' < \rho'' < 1$, and suppose that the mixed exploration-exploitation equilibrium exist for both ρ' and ρ''. Then $\Pi(\rho') < \Pi(\rho'')$.*

The sketch of the proof of Proposition C.3 is given below. Proposition C.3 seems to suggest that any mixed equilibrium is better (in terms of the principal's payoff) than the exploitation equilibrium derived in Proposition C.1; this is *not* true. Roughly, this is because a mixed equilibrium puts a more stringent constraint on the period-1 bonus as given in (C.2), whereas an exploitation equilibrium requires the equivalence of this condition to only hold with inequality which in turn implies (possibly) less distortion on the period-1 surplus. Hence, the limit of $\Pi(\rho)$ as ρ approaches 0 (assuming the limit exists) is weakly lower than the principal's payoff under the exploitation equilibrium (when the exploitation equilibrium exists). By a similar argument, the limit of $\Pi(\rho)$ as ρ approaches 1 (again, assuming it

[33] If the equilibrium does not exist, then define $\Pi(\rho) = 0$.

exists) is also weakly lower than the principal's payoff under the exploration equilibrium derived in Section 5. One implication of this is that the comparison between the principal's payoff under the exploitation equilibrium and under the exploration equilibrium under FW is ambiguous. But as established in Corollary C.1, the exploitation equilibrium under FW is always weakly worse for the principal than the unique (exploitation) equilibrium under TM. The following corollary, which follows immediately from the discussion above, then establishes that it suffices to focus attention on the exploration equilibrium under FW (as is done in the main text) when one is concern with the principal's choice between TM or FW.

Corollary C.2. *Under FW, the principal's total payoff under the exploration equilibrium is always higher than her payoff under any mixed exploration-exploitation equilibrium.*

I sketch the proof of Proposition C.3 now. To ease exposition, I simply assume that the mixed equilibrium with ρ always exits. Notice first that $F(\theta^o) < 1$ implies that $\tilde{H}(\theta^n; \rho)$ is increasing in ρ for all $\theta^n \in \left(\theta^o, \bar{\theta}\right]$.[34] Hence $\rho \in (0, 1]$ parametrizes the distribution as in Assumption 3 (with the upper bound of the support fixed for all ρ), and it is readily verified that all results in Section 6 apply with weak inverse hazard ratio dominance.[35] Applying Proposition 5.2, the principal's expected period-2 payoff is strictly increasing in ρ.

Next, by the same argument given in Section 6, the principal's total payoff is increasing in ρ if $R(\rho)$ is always increasing. Notice that $\tilde{H}(\theta^n; \rho)$ is independent of ρ for $\theta > \theta^o$. This in turn implies that the associated $\hat{\beta}(\cdot)$ schedule, as characterized in (5.7), is independent of ρ, which in turn implies that $\frac{d^2}{d\theta d\rho}\hat{\beta}(\theta; \rho) = 0$. By Lemma B.2, $R(\rho)$ is indeed strictly increasing.

[34] $\frac{d}{d\rho}\left[\frac{1-\rho}{1-\rho+\rho F(\theta^o)}\right] = \frac{-(1-\rho+\rho F(\theta^o)-(1-\rho)(-1+F(\theta^o)}{[1-\rho+\rho F(\theta^o)]^2} = -\frac{F(\theta^o)}{[1-\rho+\rho F(\theta^o)]^2} < 0$. Hence $\frac{d}{d\rho}\tilde{H}(\theta^n; \rho) > 0$.

[35] That is, if Assumption 3.2 had been "$k'' > k'$ implies that $H(\theta^n; k'') \geq H(\theta^n; k')$ $\forall \theta^n$ and $H(\theta^n; k'') > H(\theta^n; k')$ $\forall \theta^n$ in some non-zero measure subset of $\left[\underline{\theta}, \bar{\theta}\right]$."

C.2 General Monitoring Environments

The key difference between TM and FW lies in the information available to the principal in period 2. In contrast to most principal-agent models, a more informative signal generating process, in the sense of Blackwell (1951), does not necessarily improve the social surplus nor the ex-ante expected payoffs of the principal. This is readily observed by noting that the perfectly informative signal generating process corresponds to TM while the perfectly uninformative one corresponds to FW.

One can consider more general types of monitoring environments by allowing for a more general structure of information flow to the principal post-experimentation. To formalize this idea, suppose now that the principal never observes the agent's effort nor technology choices in both periods, and she also does not observe the θ^n realization directly if it is realized. Instead, at the start of period 2, the principal observes a signal $\sigma \in \Sigma$, where Σ is a finite set. If the agent uses n in period 1, the conditional probability distribution of σ given the θ^n realization is $\Pi\left(\sigma \middle| \theta^n\right)$; and if the agent uses o in period 1, the conditional probability distribution of σ is $\Pi\left(\sigma \middle| \theta^o\right)$. Signal σ is also observed by the agent but is non-verifiable by a third party and hence not contractible. A $\{\Sigma, \Pi\}$-pair is thus a general form of monitoring environment.

On the equilibrium path of an exploration equilibrium, upon observing signal σ in period 2, the principal forms a posterior belief about θ^n. Using Bayes theorem, the density of the posterior is:

$$f^{\sigma}\left(\theta^n\right) = \frac{\Pi\left(\sigma \middle| \theta^n\right) f\left(\theta^n\right)}{\int_{\underline{\theta}}^{\bar{\theta}} \Pi\left(\sigma \middle| t\right) f\left(t\right) dt}, \qquad \forall \theta^n.$$

The principal's screening problem after observing σ will be program $\mathcal{P}$ in (5.3) but with the distribution replaced by her posterior now. The parties' ex-ante expected payoffs in period 2 is then obtained by taking expectation over the probability distribution of σ.[36]

[36]Many of the assumptions made here are without loss of generality. First, the assumption that the agent

The choice of $\{\Sigma, \Pi\}$ is a choice of the distribution over the principal's posterior, where each posterior is associated to a set of period-2 payoffs for the two parties. This is an information control problem and is related to the literature on Bayesian persuasion (Kamenica and Gentzkow, 2011). The difference is that in the persuasion literature, the principal chooses an information structure to "persuade" the agent to take an ex-post action in her favor; in contrast, here the principal chooses an information structure that commits herself to take an ex-post action (the bonus offers in period 2) which then influences the agent's actions both ex-ante (period 1) and ex-post (period 2).

C.3 Innovative Effort

The degree of innovation success in the baseline model is completely dictated by luck since it is just a draw of θ^n. In reality, a worker should be able to affect the probability of getting an innovation and its degree of success. This section extends the baseline model to incorporate innovative effort in a stylized way and develop some insights on the nature of innovative effort.

Suppose now that in period 1, when the agent chooses technology n, on top of the choice of e, the agent also gets to privately choose another dimension of effort denoted by $i \in [0, \bar{i}]$ with $\bar{i} < 1$. One should interpret i as an *innovative effort* which is exerted only during the innovative stage (period 1), as opposed to e and ε which are *productive efforts* that are exerted in both periods. Let p be a commonly known marginal productivity parameter for technology n with the assumption that $p \in (\bar{i}, 1-\bar{i})$. When the agent exerts innovative effort i and productive effort e on technology n in period 1, the probability of getting outcome Y in that period is $(p - i)e$. On the other hand, come period 2, the probability of getting

also observes σ is not important. Even if the agent does not observe σ, the menu offered by the principal, which depends on her now privately known posterior distribution of θ^n, must still satisfy the same set of agent's truth-telling constraints for all realizations of θ^n. Next, when the focus is on exploration equilibrium, what the principal observes when the agent uses $\tau = o$ in period 1 does not matter, because an agent who uses o always earns zero rent regardless of the posterior held by the principal.

outcome Y under a period-2 productive effort level ε becomes $(p+i)\varepsilon$. Hence the choice of i is a deterministic transfer of marginal productivity of productive effort for technology n across periods. The rest of the model remains unchanged from Section 3. The reason to why i is an appropriate representation of innovative effort will be made clear below after Lemma C.1.

Since technology o is unaltered, it is immediate that Proposition 1 continues to hold here. The focus thus remains on inducing the agent to explore in period 1 under FW and studying the effect of having the additional dimension of innovative effort i. Denote the marginal productivity of productive effort for technology n in period 2 by $\pi(i) := p+i$. Previously an innovation is considered successful if the θ^n realization is higher that θ^o. This is no longer the case here because of the difference in the marginal productivity of productive effort between the two technologies in period 2, which is dependent on the choice of i in period 1.

Lemma C.1. *For any $\beta > 0$, $\max_{\varepsilon}\left\{\pi(i)\varepsilon\beta - \psi\left(\frac{\varepsilon}{\theta^n}\right)\right\} \geq \max_{\varepsilon}\left\{\varepsilon\beta - \psi\left(\frac{\varepsilon}{\theta^o}\right)\right\}$ if and only if $\pi(i)\theta^n \geq \theta^o$.*

Proof. Let $p^n = \pi(i)$ and $p^o = 1$. For $\tau \in \{o, n\}$, let $\varepsilon^\tau = \arg\max_\varepsilon \left\{p^\tau\varepsilon\beta - \psi\left(\frac{\varepsilon}{\theta^\tau}\right)\right\}$ which is uniquely characterized by $p^\tau\beta = \frac{1}{\theta^\tau}\psi'\left(\frac{\varepsilon^\tau}{\theta^\tau}\right)$. Hence $\max_\varepsilon\left\{p^\tau\varepsilon\beta - \psi\left(\frac{\varepsilon^\tau}{\theta^\tau}\right)\right\} = \frac{\varepsilon^\tau}{\theta^\tau}\psi'\left(\frac{\varepsilon^\tau}{\theta^\tau}\right) - \psi\left(\frac{\varepsilon^\tau}{\theta^\tau}\right)$. Convexity of ψ implies that $x\psi'(x) - \psi(x)$ is increasing in x for any positive x. Hence for any $\beta > 0$:

$$
\begin{aligned}
& \max_\varepsilon\left\{(\pi(i)\varepsilon)\,\beta - \psi\left(\frac{\varepsilon}{\theta^n}\right)\right\} \geq \max_\varepsilon\left\{\varepsilon\beta - \psi\left(\frac{\varepsilon}{\theta^o}\right)\right\} \\
\iff\quad & \frac{\varepsilon^n}{\theta^n}\psi'\left(\frac{\varepsilon^n}{\theta^n}\right) - \psi\left(\frac{\varepsilon^n}{\theta^n}\right) \geq \frac{\varepsilon^o}{\theta^o}\psi'\left(\frac{\varepsilon^o}{\theta^o}\right) - \psi\left(\frac{\varepsilon^o}{\theta^o}\right) \\
\iff\quad & \frac{\varepsilon^n}{\theta^n} \geq \frac{\varepsilon^o}{\theta^o} \\
\iff\quad & p^n\theta^n\beta \geq p^o\theta^o\beta \\
\iff\quad & \pi(i)\theta^n \geq \theta^o
\end{aligned}
$$

□

Lemma C.1 describes the agent's period-2 technology choice after choosing $\tau = n$ in period 1 with innovative effort level i. It is also clear that $\max_{\varepsilon} \left\{\pi(i)\varepsilon\beta - \psi\left(\frac{\varepsilon}{\theta^n}\right)\right\}$ is increasing in $\pi(i)$. From this, the efficiency of technology n in period 2 (and hence the degree of innovation success) can be represented by $\pi(i)\theta^n$.

From a modeling perspective, an innovative effort should satisfy two criteria. First, it should increase the probability and the degree of innovation success; i here clearly satisfies this. Second, given that it is related to innovation, the effect of innovative effort should be stochastic, which is also satisfied by i – although the inter-temporal transfer of marginal productivity due to i is deterministic, the marginal effect of i on the degree of innovation success is θ^n, which is stochastic.

The final consideration is what is the cost of innovative effort. One obvious way to incorporate this is to have an exogenous cost for i; this can be readily incorporated here without changes to the result. Instead of having this, the model here considers the cost of innovative effort to be some forgone output during the innovative process. The distinction between effort for production and effort for better innovation here is motivated by the view that the process of generating output and that of generating innovation are different, and better innovation is often created at the expense of lowering productivity during the innovating stage. The choice of innovative effort i thus studies to what extent is a worker willing to neglect his current main job, which his current pay is dependent on, to focus on coming out with better innovations, to which he might benefit from in the future?

C.3.1 Period 2 with Innovative Effort

To fix idea, let us first suppose that the agent's choice of innovative effort i in period 1 is also observed by the principal. $\pi(i)$ is thus common knowledge in period 2. The agent's private type is now (with an abuse of notation on θ and $\bar{\theta}(\cdot)$) $\theta := \max\{\theta^o, \pi(i)\theta^n\} \in \left[\theta^o, \bar{\theta}(i)\right]$, where $\bar{\theta}(i) := \pi(i)\bar{\theta}$. Let $f(\cdot|i)$ and $F(\cdot|i)$ denote the pdf and cdf of $\pi(i)\theta^n$ respectively.[37] The distribution of θ is then $Pr(\theta \leq x|i) = F(\theta^o|i) + \int_{\theta^o}^{x} f(t|i)\, dt$.

As in expression (5.2), it will be convenient to work with the agent's indirect utility function. The analog of $u(\beta, \theta)$ in (5.2) is defined here as:

$$u(\beta, \theta|i) := \max_{\varepsilon} \left\{ \pi(i)\varepsilon\beta - \psi\left(\frac{\varepsilon}{\theta}\right) \right\}.$$

It is readily verified that the properties in Lemma 1 continue to hold for any i. The optimal menu of period-2 contracts is thus characterized as in Proposition 2 with the appropriate modifications to the distribution and indirect utility functions as described.

Now, when the principal does not observe the agent's choice of innovative effort i in period 1, she designs the menu of contracts in period 2 based on her conjecture on i. Note that the outcome of y in period 1 does not affect her conjecture. Consistency in beliefs in equilibrium requires that the principal's conjecture is correct, and this issue will be handled later.

Complications arise regarding which contract an agent who has deviated in period 1 to an innovative effort $\tilde{i} \neq i$ will optimally choose in period 2, and what is his resulting (off-equilibrium path) payoffs. In particular, the agent's private type space is now two-dimensional ($\tilde{i}$ and θ^n), and multidimensional screening problems are in general very difficult to handle.

The advantage of the formulation here is that under Lemma C.1, the agent's payoff-relevant private information can still be summarized by the one-dimensional variable (abusing notation of θ again) $\theta := \max\left\{\pi(\tilde{i})\theta^n, \theta^o\right\}$ where $\tilde{i}$ is his privately known innovative effort exerted in period 1. Notice that if θ is in $\left[\theta^o, \bar{\theta}(i)\right]$, which is the type-space according to the principal's conjecture, the agent will still truthfully report his type since each contract in

[37] $f(z|i) = \frac{1}{\pi(i)} f\left(\frac{z}{\pi(i)}\right)$ and $F(z|i) = F\left(\frac{z}{\pi(i)}\right)$.

the menu satisfies the truth-telling constraint. The only concern is what an agent with type $\theta > \bar{\theta}(i)$ will choose; such an agent can exist if $\tilde{i} > i$. The following lemma establishes that such an agent will optimally claim to be $\bar{\theta}(i)$, the highest type that the principal expects to exist:

Lemma C.2. *Suppose that the principal conjectures that the agent has exerted innovative effort i in period 1 and thus offers him a menu of contracts according to Proposition 2. Facing this menu of contracts, an agent with type $\theta \in \left[\theta^o, \bar{\theta}(i)\right]$ will report his type truthfully, and an agent with type $\theta > \bar{\theta}(i)$ will report his type to be $\bar{\theta}(i)$.*

Proof. Incentive compatibility for type $\bar{\theta}(i)$ implies that for all $\tilde{\theta} < \bar{\theta}(i)$:

$$\left(\beta\left(\bar{\theta}(i)\right), \bar{\theta}(i)\right) - u\left(\beta\left(\tilde{\theta}\right), \bar{\theta}(i)\right) \geq \alpha\left(\tilde{\theta}\right) - \alpha\left(\bar{\theta}(i)\right).$$

$u_{\theta\beta}(\beta, \theta) > 0$ (see Lemma 1) implies that for any $\theta > \bar{\theta}(i)$,

$$\begin{aligned} u\left(\beta\left(\bar{\theta}(i)\right), \theta\right) - u\left(\beta\left(\tilde{\theta}\right), \theta\right) &> u\left(\beta\left(\bar{\theta}(i)\right), \bar{\theta}(i)\right) - u\left(\beta\left(\tilde{\theta}\right), \bar{\theta}(i)\right) \\ &\geq \alpha\left(\tilde{\theta}\right) - \alpha\left(\bar{\theta}(i)\right). \end{aligned}$$

The last inequality then establishes that type $\theta > \bar{\theta}(i)$ prefers the contract of $\bar{\theta}(i)$ to that of any $\tilde{\theta} < \bar{\theta}(i)$. □

With this, the agent's expected period-2 information rent both on and off equilibrium path can be computed. Let $R\left(\tilde{i}\middle|i\right)$ be the agent's expected period-2 information rent under innovative effort level $\tilde{i}$ and facing a menu of contracts designed for an agent with innovative effort i (i.e. the principal conjectures i).

Lemma C.3. *The agent's expected information rent, $R\left(\cdot\middle|i\right)$, is strictly increasing and strictly convex for any principal's conjecture i.*

Proof. Suppose that the principal conjectures the innovative effort to be i and offers the optimal menu of contracts $\{\alpha^*(\cdot|i), \beta^*(\cdot|i)\}$ as characterized in Proposition 2. Let $rent\left(\theta^n, \tilde{i}\middle|i\right)$ be the information rent of an agent who has exerted innovative effort $\tilde{i}$ in period 1, has a θ^n realization and faces a menu of contracts designed by a principal with conjecture i. From the result in Lemma C.2:

$$rent\left(\theta^n, \tilde{i}\middle|i\right) = \begin{cases} 0 & \text{if } \pi(\tilde{i})\theta^n < \theta^o, \\ \int_{\theta^o}^{\pi(\tilde{i})\theta^n} u_\theta\left(\beta^*(t|i), t\right) dt & \text{if } \theta^o \leq \pi(\tilde{i})\theta^n \leq \bar{\theta}(i), \\ \int_{\theta^o}^{\bar{\theta}(i)} u_\theta\left(\beta^*(t|i), t\right) dt + \int_{\bar{\theta}(i)}^{\pi(\tilde{i})\theta^n} u_\theta\left(Y, t\right) dt & \text{if } \bar{\theta}(i) < \pi(\tilde{i})\theta^n. \end{cases}$$

$rent\left(\theta^n, \cdot|i\right)$ is continuous and piece-wise differentiable for all θ^n and i:

$$\frac{d}{d\tilde{i}} rent\left(\theta^n, \tilde{i}\middle|i\right) = \begin{cases} 0 & \text{if } \pi(\tilde{i})\theta^n < \theta^o, \\ u_\theta\left(\beta^*\left(\pi(\tilde{i})\theta^n\middle|i\right), \pi(\tilde{i})\theta^n\right) \cdot \theta^n & \text{if } \theta^o \leq \pi(\tilde{i})\theta^n \leq \bar{\theta}(i), \\ u_\theta\left(Y, \pi(\tilde{i})\theta^n\right) \cdot \theta^n & \text{if } \bar{\theta}(i) < \pi(\tilde{i})\theta^n. \end{cases}$$

By noting that u_θ is positive and strictly increasing in both its argument, and that $\beta^*\left(\pi(\tilde{i})\theta^n\middle|i\right) \leq Y$ for $\pi(\tilde{i})\theta^n \leq \bar{\theta}(i)$, the derivatives are strictly positive and increasing everywhere for $\pi(\tilde{i})\theta^n \geq \theta^o$. Hence $rent\left(\theta^n, \cdot|i\right)$ is increasing and convex. This implies that $E\left[rent\left(\theta^n, \tilde{i}|i\right)\right]$ is strictly increasing in $\tilde{i}$. It is also convex since expectation is a linear operator which preserves convexity. $\square$

C.3.2 Period 1 with Innovative Effort

With the agent's period-2 expected rent both on and off equilibrium paths accounted for, I consider the innovative effort level that can be induced in equilibrium next. Under a contract (a, b) that prescribes the agent to choose technology n in period 1 together with an innovative effort i (where i then forms the principal's conjecture in period 2), the agent's total expected payoff for choosing n together with productive effort e and an innovative effort $\tilde{i}$ is $a + W^n(\tilde{i}, e, b|i)$ where:

$$W^n(\tilde{i}, e, b|i) := (p - \tilde{i})eb - E\left[\psi\left(\frac{e}{\theta^n}\right)\right] + R\left(\tilde{i}\middle|i\right).$$

His total expected payoff for deviating to choose technology o instead is $a + W^o(e, b)$ as in (5.11). With the additional choice of innovative effort now, the equilibrium definition is slightly different from that in Definition 1. The differences are incorporated in the following definition:

Definition C.1. Under the setting with innovative effort, a contract (a, b), together with the agent choosing technology n, productive effort e^* and innovative effort i^*, forms an equilibrium if (i) $a + W^n(i^*, e^*, b|i^*) \geq 0$; (ii) $W^n(i^*, e^*, b|i^*) \geq \max_e W^o(e, b)$; (iii) $(e^*, i^*) \in \arg\max_{e,\tilde{i}} W^n(\tilde{i}, e, b|i^*)$.

The next proposition, illustrated in Figure C.1, characterizes the set of possible equilibria associated with each contract (a, b).

Proposition C.4. *Under FW in the setting with innovative effort, for any principal's conjecture $i \in [0, \bar{i}]$, there exist thresholds $\bar{b}^l(i)$ and $\bar{b}^h(i)$, with $0 < \bar{b}^l(i) \leq \bar{b}^h(i)$, such that:*

1. *For all $b \leq \bar{b}^l(i)$:*

$$\max_e W^o(e, b) \leq \max_{e,\tilde{i}} W^n(\tilde{i}, e, b|i) = \max_e W^n\left(\bar{i}, e, b|i\right).$$

In words, the agent's optimal actions are to choose technology n in period 1 and maximum innovative effort $i = \bar{i}$ (together with the appropriate productive effort e on n).

2. *If* $\bar{b}^l(i) < \bar{b}^h(i)$*, then for all* $b \in \left[\bar{b}^l(i), \bar{b}^h(i)\right]$*:*

$$\max_e W^o(e,b) \leq \max_{e,\tilde{i}} W^n(\tilde{i}, e, b|i) = \max_e W^n\left(0, e, b|i\right).$$

In words, the agent's optimal actions are to choose technology n *in period 1 and minimum innovative effort* $i = 0$ *(together with the appropriate productive effort* e *on* n*).*

3. *For all* $b \geq \bar{b}^h(i)$*:*

$$\max_e W^o(e,b) \geq \max_{e,\tilde{i}} W^n(\tilde{i}, e, b|i).$$

In words, the agent's optimal action is to choose technology o *(together with the appropriate productive effort* e *on* o*).*

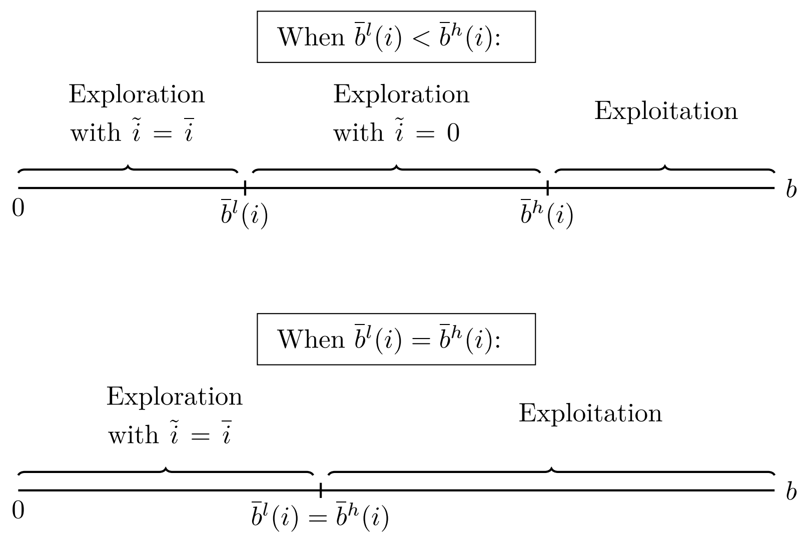

Figure C.1: The agent's best response given a bonus b under principal's conjecture i.

The following result is then immediate:

Corollary C.3. *Under FW in the setting with innovative effort, in an exploration equilibrium in which the agent uses technology* n *in period 1, the agent must be either choosing zero or maximum innovative effort.*

The intuition behind the result is similar to that for Lemma 3. When deciding whether to engage in the innovative activity or not, the agent trades off between future expected payoffs of innovation and current payoffs from production. In addition, here, there is the added consideration of "how intense should one be exploring the new technology". As in the baseline model, low-powered incentive in period 1 decreases the opportunity cost of forgoing early production. This encourages innovative initiative and moreover, it increases the intensity of the innovative initiative. Furthermore, because the gain from better innovation is convex (Lemma C.3), whenever the agent finds it profitable to exert additional intensity for exploration, he will exert maximum intensity which thus explains Corollary C.3. This result suggests an implicit need for specialization in organizations at each stage of their development in which the focus at any one time can only be on either innovation or production, but not both:

> "Both exploration and exploitation are essential for organizations, but they compete for scarce resources. As a result, organizations make explicit and implicit choice between the two" - March (1991), pg. 71.

The proof of Proposition C.4 is as follows:

Proof. First note that given any contract (a, b) with prescribed innovative effort i, the optimal $\tilde{i}$ and e choices for the agent exist since the choice set is compact and the objective is continuous. I first show that at the optimal, $\tilde{i}$ must be either 0 or $\bar{i}$ which proves Corollary C.3. Suppose for a contradiction that under some $b > 0$, the optimal choices are e and $\tilde{i} \in (0, \bar{i})$. Let $\tilde{i}' = \max\left\{0, 2\tilde{i} - \bar{i}\right\}$. $\tilde{i}$ is optimal implies that:

$$\begin{aligned} & W^n(\tilde{i}, e, b|i) \geq W^n(\tilde{i}', e, b|i) \\ \iff \quad & -\tilde{i}eb + R\left(\tilde{i}\middle|i\right) \geq -\tilde{i}'eb + R\left(\tilde{i}'\middle|i\right) \\ \iff \quad & \frac{R\left(\tilde{i}\middle|i\right) - R\left(\tilde{i}'\middle|i\right)}{\tilde{i} - \tilde{i}'} \geq eb \end{aligned}$$

Recall that $R\left(\cdot\middle|i\right)$ is strictly increasing and strictly convex (Lemma C.3). Since $\bar{i}-\tilde{i} \geq \tilde{i}-\tilde{i}'$, this implies that $\frac{R\left(\bar{i}\middle|i\right)-R\left(\tilde{i}\middle|i\right)}{\bar{i}-\tilde{i}} > \frac{R\left(\tilde{i}\middle|i\right)-R\left(\tilde{i}'\middle|i\right)}{\tilde{i}-\tilde{i}'}$. Hence:

$$
\begin{aligned}
& \frac{R\left(\bar{i}\middle|i\right)-R\left(\tilde{i}\middle|i\right)}{\bar{i}-\tilde{i}} > eb \\
\implies \quad & -\bar{i}eb + R\left(\bar{i}\middle|i\right) > -\tilde{i}eb + R\left(\tilde{i}\middle|i\right) \\
\implies \quad & W^n(\bar{i}, e, b|i) \geq W^n(\tilde{i}, e, b|i)
\end{aligned}
$$

which contradicts $\tilde{i}$ being optimal. This then implies that $\tilde{i} = 0$ or $\tilde{i} = \bar{i}$ in equilibrium.

Hence, given the contract, the agent is essentially only choosing from 3 options in period 1: (i) $\tau = n$ and $\tilde{i} = \bar{i}$, (ii) $\tau = n$ and $\tilde{i} = 0$, or (iii) $\tau = o$, each together with the corresponding optimal productive effort level. Using the same steps as in the proof of Lemma 3 and noting that $R(\cdot|i)$ is increasing, the following three claims can be established:

Claim 1. Suppose the agent is considering only between options (i) and (ii). Then there exists a threshold $\bar{b}^{(1)}(i) > 0$ such that $\max_e W^n(\bar{i}, e, b|i) \geq \max_e W^n(0, e, b|i)$ if and only if $b \leq \bar{b}^{(1)}(i)$.

Claim 2. Suppose the agent is considering only between options (i) and (iii). Then there exists a threshold $\bar{b}^{(2)}(i) > 0$ such that $\max_e W^n(\bar{i}, e, b|i) \geq \max_e W^o(e, b)$ if and only if $b \leq \bar{b}^{(2)}(i)$.

Claim 3. Suppose the agent is considering only between options (ii) and (iii). Then there exists a threshold $\bar{b}^{(3)}(i) > 0$ such that $\max_e W^n(0, e, b|i) \geq \max_e W^o(e, b)$ if and only if $b \leq \bar{b}^{(3)}(i)$.

Set $\bar{b}^l(i) = \min\{\bar{b}^{(1)}(i), \bar{b}^{(2)}(i)\}$ and $\bar{b}^h(i) = \max\{\bar{b}^{(2)}(i), \bar{b}^{(3)}(i)\}$, and the three statements in the proposition will follow. To see why, first notice that the statements 1 and 3 of the proposition are readily verified to be true. The non-obvious part is the second statement about $b \in \left[\bar{b}^l(i), \bar{b}^h(i)\right]$. Consider all 6 possible permutations on the order of $\bar{b}^{(1)}(i), \bar{b}^{(2)}(i)$

and $\bar{b}^{(3)}(i)$ now. Note that whenever $\bar{b}^l(i) = \bar{b}^h(i)$, the statement is trivially true. To ease notation, I drop the argument of i from now on. Also, recall that $\bar{W}^o(b) = \max_e W^o(e, b)$, and I denote $\bar{W}^n(\tilde{i}, b|i) := \max_e W^n(\tilde{i}, e, b|i)$.

Case 1: $\bar{b}^{(1)} \leq \bar{b}^{(2)} \leq \bar{b}^{(3)}$: Hence $\bar{b}^l = \bar{b}^{(1)}$ and $\bar{b}^h = \bar{b}^{(3)}$. For any $b \in \left[b^l, b^h\right]$, $\bar{W}^n(0, b|i) \geq \bar{W}^n(\bar{i}, b|i)$ since $b \geq \bar{b}^{(1)}$, and $\bar{W}^n(0, b|i) \geq \bar{W}^o(b)$ since $b \leq \bar{b}^{(3)}$.

Case 2: $\bar{b}^{(1)} \leq \bar{b}^{(3)} \leq \bar{b}^{(2)}$: Hence $\bar{b}^l = \bar{b}^{(1)}$ and $\bar{b}^h = \bar{b}^{(2)}$. If $\bar{b}^{(3)} = \bar{b}^{(2)}$, then the argument of Case 1 applies. Moreover, it cannot be the case that $\bar{b}^{(3)} < \bar{b}^{(2)}$. If not, consider a $b \in \left(\bar{b}^{(3)}, \bar{b}^{(2)}\right)$. $b > \bar{b}^{(3)}$ implies that $\bar{W}^n(0, b|i) < \bar{W}^o(b)$, $b < \bar{b}^{(2)}$ implies that $\bar{W}^o(b) < \bar{W}^n(\bar{i}, b|i)$, and $b > \bar{b}^{(1)}$ implies that $\bar{W}^n(\bar{i}, b|i) < \bar{W}^n(0, b|i)$, which then implies $\bar{W}^n(0, b|i) < \bar{W}^n(0, b|i)$ (contradiction).

Case 3: $\bar{b}^{(2)} \leq \bar{b}^{(1)} \leq \bar{b}^{(3)}$: Hence $\bar{b}^l = \bar{b}^{(2)}$ and $\bar{b}^h = \bar{b}^{(3)}$. If $\bar{b}^{(2)} = \bar{b}^{(1)}$, then the argument of Case 1 applies. Moreover, it cannot be the case where $\bar{b}^{(2)} < \bar{b}^{(1)}$. If not, consider a $b \in \left(\bar{b}^{(2)}, \bar{b}^{(1)}\right)$. $b < \bar{b}^{(1)}$ implies that $\bar{W}^n(0, b|i) < \bar{W}^n(\bar{i}, b|i)$, $b > \bar{b}^{(2)}$ implies that $\bar{W}^n(\bar{i}, b|i) < \bar{W}^o(b)$, and $b < \bar{b}^{(3)}$ implies that $\bar{W}^o(b) < \bar{W}^n(0, b|i)$, which then implies $\bar{W}^n(0, b|i) < \bar{W}^n(0, b|i)$ (contradiction).

Case 4: $\bar{b}^{(2)} \leq \bar{b}^{(3)} \leq \bar{b}^{(1)}$: Hence $\bar{b}^l = \bar{b}^{(2)}$ and $\bar{b}^h = \bar{b}^{(3)}$. If $\bar{b}^{(2)} = \bar{b}^{(3)}$, then $\bar{b}^l = \bar{b}^h$ and the statement is trivially true. Moreover, it cannot be the case where $\bar{b}^{(2)} < \bar{b}^{(3)}$. If not, consider a $b \in \left(\bar{b}^{(2)}, \bar{b}^{(3)}\right)$. $b_1 > \bar{b}^{(2)}$ implies that $\bar{W}^n(\bar{i}, b|i) < \bar{W}^o(b)$, $b < \bar{b}^{(3)}$ implies that $\bar{W}^o(b) < \bar{W}^n(0, b|i)$, and $b_1 < \bar{b}^{(1)}$ implies that $\bar{W}^n(0, b|i) < \bar{W}^n(\bar{i}, b|i)$, which then implies $\bar{W}^n(\bar{i}, b|i) < \bar{W}^n(\bar{i}, b|i)$ (contradiction).

Case 5: $\bar{b}^{(3)} \leq \bar{b}^{(1)} \leq \bar{b}^{(2)}$: Hence $\bar{b}^l = \bar{b}^{(1)}$ and $\bar{b}^h = \bar{b}^{(2)}$. If $\bar{b}^{(1)} = \bar{b}^{(2)}$, then $\bar{b}^l = \bar{b}^h$ and the statement is trivially true. Moreover, it cannot be the case where $\bar{b}^{(1)} < \bar{b}^{(2)}$. If not, consider a $b \in \left(\bar{b}^{(1)}, \bar{b}^{(2)}\right)$. $b > \bar{b}^{(3)}$ implies that $\bar{W}^n(0, b|i) < \bar{W}^o(b)$, $b < \bar{b}^{(2)}$ implies that $\bar{W}^o(b) < \bar{W}^n(\bar{i}, b|i)$, and $b > \bar{b}^{(1)}$ implies that $\bar{W}^n(\bar{i}, b|i) < \bar{W}^n(0, b|i)$, which then implies $\bar{W}^n(0, b|i) < \bar{W}^n(0, b|i)$ (contradiction).

Case 6: $\bar{b}^{(3)} \leq \bar{b}^{(2)} \leq \bar{b}^{(1)}$**:** Hence $\bar{b}^l = \bar{b}^h = \bar{b}^{(2)}$ and the statement is trivially true. $\square$