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Abstract

The famous Wall Street adage that states, “It takes volume to make prices move” has long suggested the existence of a positive correlation between absolute change in stock price and trading volume. For practitioners who make use of technical analysis see trading volume as an important signal for price change. While many existing studies have empirically examine the nonlinear relationship between price change and trading volume, very few studies are found to be able to replicate this relationship in a simple and yet useful theoretical models to provide persuasive explanations for such relationship. This paper aims to fill the gap and provide an explanation for such price-volume relationship using a heterogeneous agent model (HAM) with evolutionary switching mechanism. From the US stock market data, we first document some useful stylized facts on trading volume. We then aim to replicate these facts using our model and show that our model is able to (1) replicate the seemingly chaotic fluctuations of financial market and (2) explain how stock prices and trading volumes co-evolve with agent beliefs in our simulation.

Keywords: Heterogeneous beliefs, Price change, Trading volume, Stock market

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1 Introduction

In the analysis of financial market, much attention has been drawn to the study of assets price. Another important indicator for financial activities, trading volume, is ignored in many research. Volume represents the total amount of transactions in a risky asset or entire market during a specific period of time. In the technical analysis, volume is used to measure the relative worth of market move. A high volume during the price move always implies a strong market. Besides, the information roles of volume have been discovered by practitioners, and the relationship between price and volume has been widely studied by both researchers and technicians. As the old Wall Street adage asserts, “it takes the volume to move prices”.

The characteristics of trading volume have been commonly found in the financial markets. Some of these stylized facts can only be observed visually, and others can be examined statistically. One visualized stylized fact concerning volume is that a dramatic change in prices always accompanied by significant volumes. This feature is easily found in crisis periods. Figure 1 is the illustration of this phenomenon in 2008 financial crisis. We can clearly see that the Dow Jones Index experiences a sharp decline in the middle of 2008 and falls to the bottom subsequently in the early of 2009. At the same time the volume also surges and hits to an unprecedented level. Similar examples also occur like Black Monday in 1987 and Asian Financial crisis in 1997. Another visualized stylized fact is that volume can be regarded as a signal to confirm the price trend in technical analysis. For example, if investors observed a sudden increased trading volume, they can confirm the breakout of price to the trend-line.(see Murphy (1999) and Bulkowski (2011)). Figure 2 illustrates this fact in stock of Google. Its stock price fluctuates below the resistance line (around 575) for a long time until suddenly jumping up to around 660 at July,17 2015. Meanwhile, the volume almost five times the average value. After that the price moves up and down above the previous resistance level. The huge volume can be recognized as the signal to confirm that price breaks out the old regime and enters a new one.

In addition, the statistical relationship between price and volume has received considerable attention in the empirical researches. Karpoff (1987) makes the points that it is not conflict that volume could correlates positively with the elements of both the absolute change of price or the price change *per se*. Although an early empirical study by Granger and Morgenstern (1963) fails to find a correlation between price index and aggregated volume in New York Stock Exchange(NYSE). Succeeding studies have found evidence of positive correlation. Since 1990s, the dynamic(causal) correlation between price and volume has drawn a lot of attention. Bivariate vector autoregressive(VAR) models and Granger causality tests are used to investigate the price-volume relation. Saatcioglu and Starks (1998) find evidence that volume leads to price change. Statman et al. (2006) use

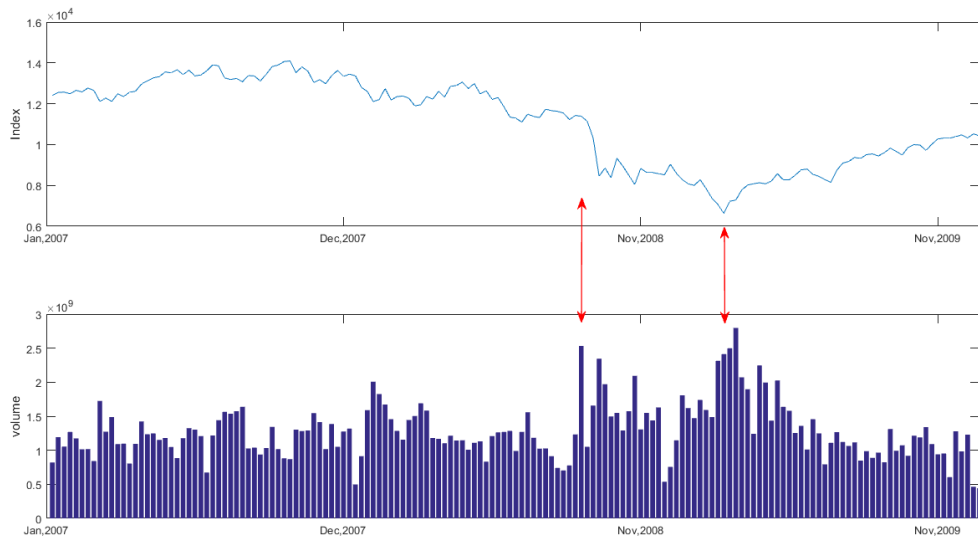


Figure 1: Dow Jones Index and trading volume during 2008 Global Financial Crisis

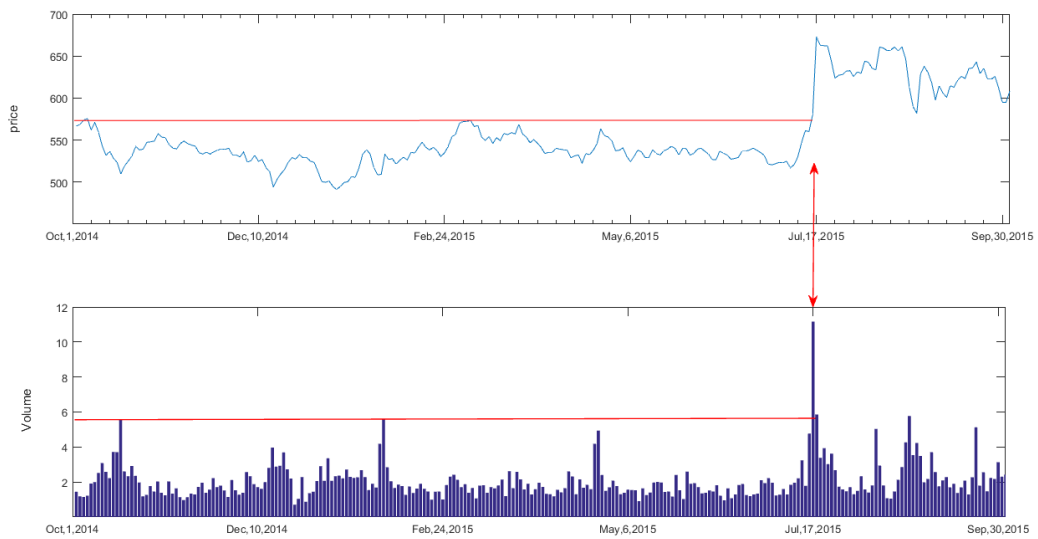


Figure 2: Google stock and trading volume(2014-2015)

monthly data from NYSE and find the positive relation between trading activities and lagged return. By using S&P 500 data from 1973 to 2008, Chen (2012) finds that trading volume does not Granger cause stock return, but return Granger causes volume. Departing from the linear model, Hiemstra and Jones (1994) apply nonlinear Granger causality tests to investigate the price-volume relationship in the US market. They find evidence of significant bidirectional nonlinear causality between returns and volumes. Moreover, Diks and Panchenko (2006) modify the method of nonlinear causality test to improve the performance, and find weak nonlinear causality between returns and volumes. Although many literatures have proven that Granger causalities exist between volumes and returns, the significance and directions are still controversial. To have a clearer view on price-volume relation, we reexamine the linear and nonlinear Granger causalities between return and volume change. We use both S&P 500 and Dow Jones index daily data, the range of the sample is from 1/1/2010 to 12/31/2016. The method to run linear and nonlinear Granger causality tests are provided in Appendix A and the results are reported in Table 1 and Table 2, respectively. We find evidence that returns Granger cause volumes in the linear test, which is consistent with the results in Chen (2012). For the nonlinear test, we find weak Granger causalities between specific lagged returns and volume changes in S&P 500 data, but fail to find the causality relation in Dow Jones index.

Beside the price-volume causal relation, it is also worth to investigate the correlation between volume and volatility. Clark (1973) shows that the squared price changes are positive related to volumes. Daigler and Wiley (1999) find positive volume and volatility relation in the future market, and argue that this relation is driven by the existence of different types of investors. In practice, the positive relation between volume and volatility can be easily found in current stock market. Recent example is the positive correlation of volatility and volume in US stock market during 2008 financial crisis. As shown in Figure 3, the fluctuation of Volatility Index(VIX) is followed by the similar movement of trading volume, and the volatility of the stock market is measured by Chicago Board Options Exchange (CBOE) Volatility Index. To further check the relation, we plot the correlation chart in Figure 4, and find that the correlation coefficient between volume and VIX is 0.725, which suggests a strong positive relation.

While various studies focus on the detection of price-volume relationship, few of them have explained why such patterns exist and how the relation evolves in the market. And most of the previous models are not able to address all the stylized facts mentioned above. To better simulate the financial market, nonlinear dynamic models have been drawn attention in the past decades. Granger (2014) argues that univariate and multivariate nonlinear models represent the proper way to model a real world that is almost certainly nonlinear. After the sudden crisis in 1987, great amount of literatures have been worked on the nonlinear model to detect and simulate the well known stylized facts in financial market. The heterogeneous agent model(HAM) are widely used in this area and proven

Table 1: Linear Granger causality test

Dependent variable	S&P 500		Dow Jones Index	
	(1) R_t	(2) V_t	(3) R_t	(4) V_t
Constant	-0.660 (2.664)	5.321*** (0.478)	-1.301 (1.143)	3.168*** (0.361)
R_{t-1}	-0.047* (0.024)	-0.011*** (0.004)	-0.048** (0.024)	-0.026*** (0.008)
R_{t-2}	0.020 (0.024)	-0.003 (0.004)	0.031 (0.024)	-0.016** (0.008)
R_{t-3}	-0.066*** (0.024)	-0.002 (0.004)	-0.042* (0.024)	-0.006 (0.008)
R_{t-4}	-0.005 (0.024)	-0.010 (0.004)	-0.015 (0.024)	-0.004 (0.008)
V_{t-1}	-0.104 (0.133)	0.471*** (0.024)	-0.011 (0.075)	0.378*** (0.024)
V_{t-2}	0.252* (0.147)	0.456*** (0.026)	0.134* (0.080)	0.188*** (0.025)
V_{t-3}	-0.247* (0.147)	0.051* (0.026)	-0.084 (0.081)	0.152*** (0.025)
V_{t-4}	0.131 (0.133)	0.079*** (0.024)	0.032 (0.076)	0.113*** (0.024)
Wald-stat	7.923	11.340***	4.218	15.678***
p-value	0.295	0.023	0.377	0.003

Notes: (1) The entries in brackets are the standard errors. The Wald-stat and p-value are tests of Granger causality. (2)* denotes rejection at the 10% level, ** rejection at the 5% level, ***rejection at the 1% level.

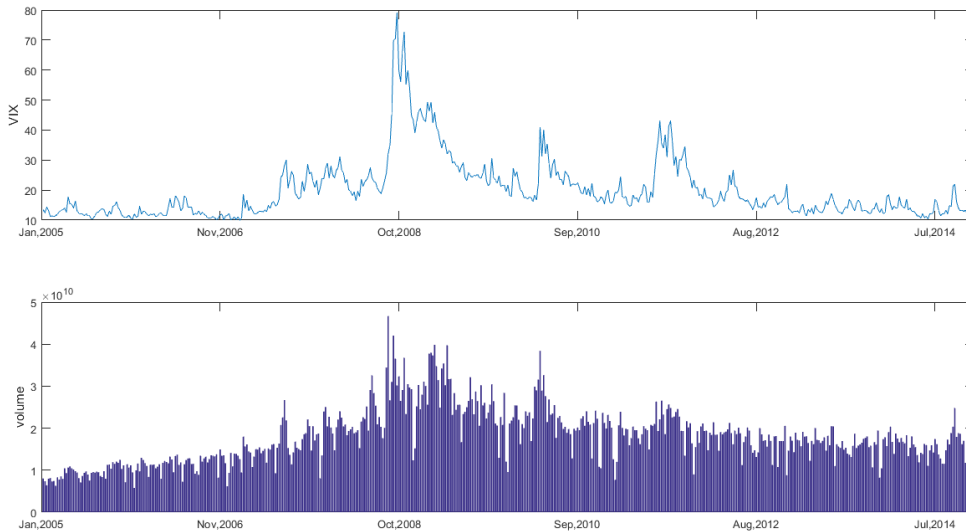


Figure 3: S&P volume and the VIX

Table 2: Nonlinear Granger causality test

Lags	H_0 : Stock Returns Do not Cause Volume Changes		H_0 : Volume Changes Do not Cause Stock Returns	
	T-statistic	p-value	T-statistics	p-value
S&P 500				
1	3.027	0.001	1.059	0.145
2	2.237	0.013	0.960	0.169
3	1.975	0.024	0.403	0.343
4	2.076	0.019	1.241	0.101
5	1.175	0.120	1.115	0.132
6	0.748	0.227	1.372	0.085
7	0.966	0.167	1.924	0.027
8	0.566	0.286	1.575	0.058
Dow Jones Index				
1	1.026	0.153	-0.613	0.730
2	0.874	0.191	-0.575	0.717
3	0.128	0.449	0.248	0.402
4	0.249	0.401	-0.041	0.516
5	0.609	0.271	-0.086	0.534
6	-0.436	0.668	0.118	0.453
7	-0.422	0.663	0.239	0.405
8	-0.222	0.588	0.872	0.192

Note: Test critical value for T statistics are 2.326 (1%), 1.645 (5%) and 1.282 (10%).

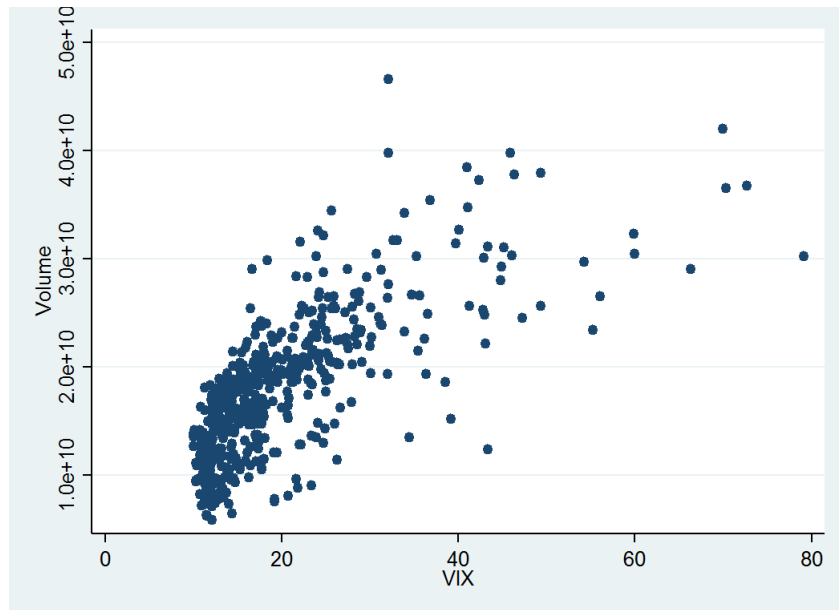


Figure 4: Correlation volume and the VIX

to be very powerful on asset pricing, replicating the stylized facts and explaining different features of financial market, such as crises, crashes and bubbles.(see Beja and Goldman (1980),Day and Huang (1990), Brock and Hommes (1998), Lux (1995,9), Farmer and Joshi (2002), Chiarella and He (2003), Huang et al. (2010)).

In this paper, we intend to develop a HAM with trading volume, and basing on the determinate chaotic model we investigate how the prices,volumes and beliefs co-evolves in the stock market. To prove our model fitting the real financial market well, we check its capability of replicating the well known stylized facts in the market. As shown in the current literatures, most of the HAMs have the ability to explain stylized facts concerning price or return. These stylized facts and literatures include: non-stationary price and stationary return (Hommes (2002)), fat tail (Lux (1998)),volatility clustering (Lux and Marchesi (2000),Hommes (2002)), leverage effect(Huang et al. (2013);Chen et al. (2013)), asymmetric returns(Huang et al. (2013)) and the power law of return(He and Li (2007);He and Zheng (2016)). Regrettably, little attentions have been paid so far to volume except Brock and LeBaron (1995) and Chen and Liao (2005). Brock and LeBaron (1995) build an adaptive beliefs model, and the model is able to reproduce the slowly decaying autocorrelation function of volatility and trading volume, but other stylized facts and co-evolvement of price and volume has not been fully investigated. Chen and Liao (2005) attempt to use a agent-based stock markets(ABMs) model to determine the price-volume series and reproduce the presence of the nonlinear Granger causality relation between the price and volume, but the simulation results are unpersuasive.

To fill the gap of HAM on volume, we develop a simple HAM with trading volume. Comparing to previous HAMs, our model have greater potential to simulate the features of financial market:

- (1) Our model are able to generate most of stylized facts both on price and volume.
- (2) The co-evolvement of prices, volumes and beliefs can be found in our simulation.
- (3) Our model can explain the formation of different chart patterns and crises, which provide theoretical support for technical analysis.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 presents the simulation results and verifies model's capability to generate stylized facts documented in the literature. Based on the simulation, the co-evolvement of price changes, volumes and beliefs in different scenarios is analyzed. Section 4 conclude.

2 Model

2.1 Heterogeneous beliefs

Instead of representative and perfect rational, we assume the agents in stock market are boundedly rational and hold heterogeneous beliefs on future price trend. Following the standard HAM literatures (for example Day and Huang (1990),Lux (1995),Brock and Hommes (1998)and Chiarella and He (2003)), we consider a market with one risky asset and two types of distinguished agents, which are fundamentalists and chartists. Different agents use heterogeneous rule-of-thumb strategies in the market. Fundamentalists form their expectations on future price and carry out trading strategies based on the fundamental factors of financial market or the whole economy, such as dividends, profits and economic growth rate. In contrast, chartists take more time on technical analysis. Observed historical price patterns or chart patterns are important elements to help form their expectations and trading strategies. Moreover, a market maker is included in this model, who adjusts the market price in response to the aggregate demand of fundamentalists and chartists.

2.1.1 Fundamentalists

Fundamentalists are assumed to have full information about fundamental value μ_t^f , and they believe asset price will not deviate from this intrinsic value for long. They expect prices fluctuate in a reasonable zone (m, M) due to some short term external disturbance and will eventually converge to its fundamental value. Therefore, fundamentalists will buy in when the price is below μ_t^f and sell out when above . Their demand function is thus given by

$$D_{f,t} = \begin{cases} (\mu_t^f - p_{t-1})A(\mu_t^f, p_{t-1}), & p_t \in \mathbb{Q} \\ 0, & p_t \notin \mathbb{Q} \end{cases} \quad (1)$$

Where, $\mathbb{Q} = [m_t, M_t]$, is the reasonable zone. m_t and M_t are the minimum and maximum boundaries, respectively. $A(\cdot)$ is a chance function with respect to μ_t^f and p_t . It is firstly introduced by Day and Huang (1990) and depicts the psychological behavior of investors. When price is close to the upper boundary M_t , the chance of losing the exiting gains increase. When price is close to the lower boundary m_t , the chance of missing a capital gain by failing to buy is high. Details about the chance function are presented in Appendix B.

To obtain accurate fundamental value of risky assets, fundamentalists update information Ω_t for each period. Instead of using constant fundamental value, they adjust the value according to following rule

$$\mu_{t+1}^f = g(t)\mu_t^f \quad (2)$$

We simply use $g(t)$ to simulate business cycle.

$$g(t) = \begin{cases} g & t \in [4(i-1) \cdot s, (4i-1)s] \\ -\frac{g}{2} & t \in [(4i-1)s, 4i \cdot s] \end{cases} \quad i = 1, 2, 3 \dots \quad (3)$$

where, g is economic growth rate. We assume a business cycle consists of $4s$ period. Economy expands at an average growth rate g during first $3s$ periods, then a recession occurs and lasts for $1s$ periods with a growth rate of $(-\frac{g}{2})$. Although $g(t)$ can not simulate the business cycle accurately, it introduces macroeconomic factors into the model, which improves the ability of HAM to explain relationship between business cycle and stock market cycle.

2.1.2 Chartists

While the fundamentalists make trading decisions based on fundamental approaches, the chartists mostly rely on the observed past price of risky assets. Similar to the fundamentalists, they estimate the short-term ‘fundamental value’ v_t from historical price, then extrapolate next period market price by the price deviation from the short-term ‘fundamental value’(past estimation bias). Specifically

$$E_{c,t-1}(p_t) = p_{t-1} + \beta(p_{t-1} - v_{t-1}) \quad (4)$$

Where $\beta \neq 0$ measures the sensitivity of price expectation to the latest estimation bias $p_{t-1} - v_{t-1}$. The demand function of chartists is thus given by

$$D_{c,t} = \eta[E_{c,t-1}(p_t) - p_{t-1}] = \eta\beta(p_{t-1} - v_{t-1}) \quad (5)$$

We assume that all the chartists share the same belief on the reference price v_t , and we shall introduce how they estimate and update this value later. Instead of making identical trading decision, the chartists have two different trading strategies. The difference among them is captured by the sign of β . Some of them believe that $\beta = \beta_1 > 0$, that is the market price p_t will deviate further away from the current price p_{t-1} , and the price trend will persist. They behave like trend followers in other HAM literatures. Since they hold the positive bias, we call this kind of chartists as optimistic chartists. The excess demand function of optimistic chartists is given by

$$D_{c_A,t} = \eta_1\beta_1(p_{t-1} - v_{t-1}) \quad (6)$$

In contrast, the other chartists believe $\beta = \beta_2 < 0$. They hold the opposite belief with optimistic chartists, and they believe the price trend will reverse in the next period. They behave like contrarians (see Charella and He (2002)). As they take the negative

bias when make trading decision, we name them as pessimistic chartists. Their excess demand function is given by

$$D_{c_B,t} = \eta_2 \beta_2 (p_{t-1} - v_{t-1}) \quad (7)$$

As all chartists hold the identical short-term assets value v_t , we adopt the adaptive belief mechanism of v_t following Huang et al. (2010) and Huang and Zheng (2012). We assume the chartists update their expectation on short-term ‘fundamental value’ according to change of price regimes. Chartists usually set the support levels and resistance levels derived from common prevailing rules of technical analysis, and Donaldson and Kim (1993) have provided empirical evidence for existence of support and resistance levels in Dow Jones Industrial Average. We assume chartists divide the price domain $\mathbb{P} = [p_{min}, p_{max}]$ into n regimes so that

$$\mathbb{P} = \bigcup_{j=1}^n \mathbb{P}_j = [\bar{p}_0, \bar{p}_1) \cup [\bar{p}_1, \bar{p}_2) \cup \dots \cup [\bar{p}_{n-1}, \bar{p}_n] \quad (8)$$

where $\bar{p}_j (j = 1, 2, \dots, n)$ represents the different support and resistance levels chartists set. The short-term ‘fundamental value’ can be simply extrapolated to be the average of the top and the bottom threshold prices

$$v_t = (\bar{p}_{j-1} + \bar{p}_j)/2 \quad \text{if } p_t \in [\bar{p}_{j-1}, \bar{p}_j), j = 1, 2 \dots n \quad (9)$$

when the price moves within current regime, there are enough reasons for chartists to believe that the short-term assets value remains unchanged. However, once the price breaks through the support line or the resistance line, chartists are more likely to adjust their expectation on short-term fundamental v_t according to Eq.(9). The ‘regime switching’ phenomena can be commonly found in stock market, and chartist’s beliefs evolve with the regime switching. Following Huang et al. (2010), for each period, the short-term ‘fundamental value’ can be calculated as

$$v_t = (\lfloor p_t/\lambda \rfloor + \lceil p_t/\lambda \rceil) \cdot \frac{\lambda}{2} \quad \text{if } p_t \in [\bar{p}_{j-1}, \bar{p}_j), j = 1, 2 \dots n \quad (10)$$

2.2 Evolutionary belief switch

The belief evolvement of fundamentalists and chartists is not only derived from the updated expectation on price, but also induced by the agents’ interaction with each other. In our model, agents are allowed to switch their strategy, and these switches are driven by the discounted expected profit π . Specifically, some investors are forward looking, and they are prone to adopt the strategies with high expected profit, and abandon previous less attractive strategies when making trading decision. However, some investors still

stick to their original strategy even it perform poorly. The market structure updates along with the switching of different types of agents in each period. The discounted expected profit functions for fundamentalists and chartists can be written as

$$\begin{aligned}\pi_{f,t} &= s(p_{t-1}) \left| (u_t^f - (1+r)p_{t-1}) \right| - C \\ \pi_{c,t}^\alpha &= |\beta_1(p_{t-1} - v_{t-1}) - rp_{t-1}| \\ \pi_{c,t}^\beta &= |\beta_2(p_{t-1} - v_{t-1}) - rp_{t-1}|\end{aligned}\tag{11}$$

Where, $\pi_{f,t}$, $\pi_{c,t}^\alpha$, $\pi_{c,t}^\beta$ are expected profit for fundamentalists, optimistic chartists and pessimistic chartists, respectively. C is the information cost, which implies that it is costly to become a fundamentalist. $s(p_t)$ is the discount factor. For the chartist, they capitalize capital gains or loss immediately, their discount factor is 1. For fundamentalist, $s(p_t) = |(u_t^f - p_t)/3u_t^f|$.

Let $\omega_{i,t}$ be the market fraction for three different types of investors. $\omega_{i,t}$ evolves according to choice model with multinomial logit probabilities (Brock and Hommes (1998)):

$$\omega_{i,t}(p_t) = \frac{\exp(\rho\pi_{i,t}(p_t))}{\sum_k \exp(\rho\pi_{k,t}(p_t))}\tag{12}$$

where the parameter ρ is the intensity of choice, which measures the speed of transition between different beliefs. A high value of ρ means more investors will switch to fitter strategy, and the mutual switch is based on the endogenous selection of expectation rule. $\omega_{i,t}$ is always positive, which reflects that not all agents chase the strategy with high discounted expected profit.

2.3 Price dynamic

Another participant is the market maker who mediate transactions on the market out of equilibrium by providing liquidity. Market maker collect orders from fundamentalists and two types of chartists, then sets a price and supplies from (absorbs into) his inventory when there is positive (negative) excess demand. The market prices are updated at each period adaptively with price adjustment rule

$$p_t = p_{t-1} + \gamma(\omega_{1,t}D_{f,t} + \omega_{2,t}D_{c_A,t} + \omega_{3,t}D_{c_B,t})\tag{13}$$

where γ is speed of price adjustment to the excess demand. The price adjustment rule is determined by the financial institution, and market maker can be treated as a stylized version of the specialist at the New York Stock Exchange.

2.4 Volume dynamic

Volume is another important indicator in stock market besides price. In real world, the total number of all shares that changed hands in a market is named as trading volume. We simulate the trading volume in our model by calculating all transactions under the framework of market maker. As the optimistic chartists and pessimistic chartists hold the opposite expectation on the future price of the risky assets, they trade with each other first, then the chartists will form a positive or negative excess demand. Because fundamentalists and chartists hold different strategies when making decision, they may have positive demand or negative demand on the risky assets. Hence, there are two cases we need to consider. If chartists and fundamentalists share the same opinion on the future trend of price, both of them will trade with market maker and the trading volume should be the absolute value of aggregate excess demand. On the other hand, if they hold the opposite opinion on the price trend, they will trade with each other first and the market maker will provide the liquidity to satisfy the rest part of excess demand. In this case, maximum of the absolute value of the excess demand for each group should be taken as volume. Specifically, the volume can be defined as

$$V(P) = \begin{cases} |\tilde{D}_{f,t} + \tilde{D}_{c,t}| & \text{if } \tilde{D}_{f,t} \cdot \tilde{D}_{c,t} > 0 \\ \max(|\tilde{D}_{f,t}|, |\tilde{D}_{c,t}|) & \text{if } \tilde{D}_{f,t} \cdot \tilde{D}_{c,t} < 0 \end{cases} \quad (14)$$

Where

$$\tilde{D}_{c,t} = \begin{cases} \tilde{D}_{c_A,t} & \text{if } \tilde{D}_{c_A,t} > \tilde{D}_{c_B,t} \\ \tilde{D}_{c_B,t} & \text{if } \tilde{D}_{c_A,t} < \tilde{D}_{c_B,t} \end{cases} \quad (15)$$

$\tilde{D}_{f,t}$, $\tilde{D}_{c_A,t}$, $\tilde{D}_{c_B,t}$ are the weighted excess demand for fundamentalists, optimistic chartists and pessimistic chartists, respectively. $\tilde{D}_{f,t} = \omega_{1,t}D_{f,t}$, $\tilde{D}_{c_A,t} = \omega_{2,t}D_{c_A,t}$, $\tilde{D}_{c_B,t} = \omega_{3,t}D_{c_B,t}$.

3 Simulation results

In this section, we simulate the above HAM with volume dynamic. we innovatively combine prices, volumes and beliefs in our model, and the simulation results seem more interesting. Three-dimensional analysis in our model enhance the theoretical stringency of HAM and make the artificial model more close to realistic world. we also check the fitness of the model in capturing the complex dynamics of stock market by showing its power to reproducing the well-documented stylized facts mentioned in Section 1. In addition to the price related stylized facts, we also investigate whether our HAM has the capability of generating the stylized facts on trading volume, such as the correlation between volume and volatility, Granger causality between return and volume, information

role of volume and some chart patterns with volume. To verify the good fitness of the model in different situations and understand the market condition deeply, we demonstrate the co-movement of prices, volumes and beliefs during different chart patterns periods as well as crisis periods .

In order to demonstrate the robust of the model and remain consistency and unity, we simulate the price series and volume series using a set of parameters: $\mu_1^f = 50, d_1 = d_2 = -0.3, k = 2, s = 25, \lambda = 13.1787, C = 3, r = 10^{-4}, g = 0.008, a = 1, \beta_1 = 1.2, \beta_2 = -0.7, \eta_1 = 0.833, \eta_2 = 3.214, \rho = 0.9, \gamma = 1$. Unless otherwise specified, we maintain the same set of parameters throughout the paper. To see how well the simulated data matches with the real financial time series in terms of statical and qualitative properties, we take the daily price and volume of S&P 500 and Dow Jones Index as benchmark. The time range of these two dataset is from January 1,2010 to December 31,2016. Stock return are expressed in percentages: $r_t = \log(p_t/p_{t-1}) * 100$ and percentage changes in volume is $pv_t = \log(v_t/v_{t-1}) * 100$. Since some stylized facts are sensitive to the sample period chosen, we may refer to other literature to verify the explanatory ability of our model.

Before exploring the stylized facts generated by simulated data, we first illustrate the prices, returns and volumes dynamic of the model in Figure 5. It shows that price moves as random walk with occasional bubbles and crashes, and return fluctuates around zero and appears to cluster. The magnitude of largest positive return and negative return are not equal, which implies asymmetric returns. The volume changes period by period displayed by the different length value bars. The significant volumes are displayed as spikes in the figure. The overview of simulated price, return and volume indicates the potential of our model in generating the finical market data in terms of the general trend and pattern of price, return and volume.

3.1 Price evolvment

Our model is nonlinear due to the set up of chance function and short-term fundamental value for chartists. The chaotic system has the capability of generating rich price dynamic pattern. In order to gain some insights into the details of price evolvment in the model, we display the typical phase diagram of the simulated price in Figure 6. The chaotic multi-phase switches of prices can be readily observed. The price pattern crosses the 45 degree line ($p_t = P_{t-1}$) several times indicating that there are multiple equilibria along with the evolvment of the market structure.

As introduced by Huang and Zheng (2012), the price pattern can be divided into rising zones and declining zones. When the price falls above the 45 degree line, the price would rise in next period and if it falls blow the 45 degree line, the price would decline as expected. Combining the market condition, and taking a further look for the two step-

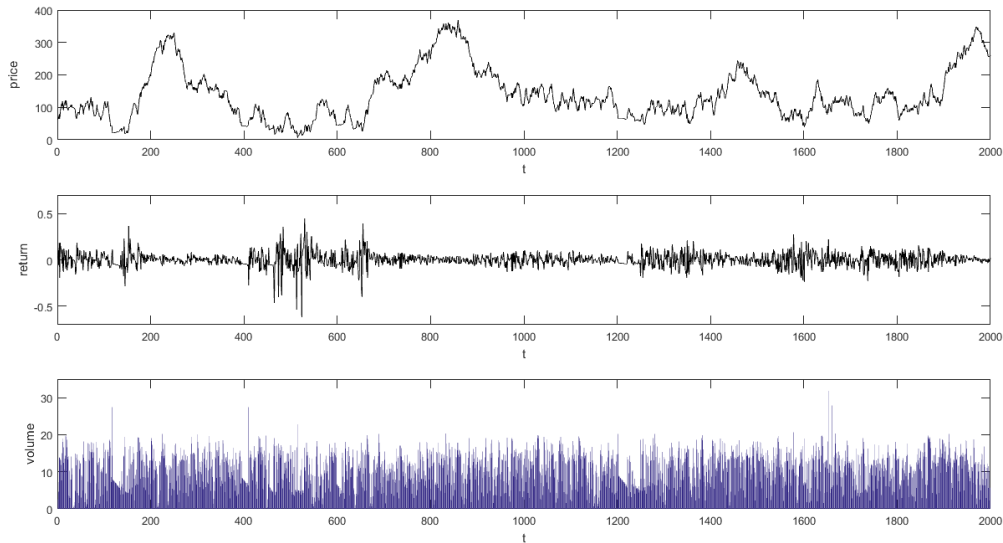


Figure 5: Simulated time series of price(top),return(middle) and volume(bottom)

wise dynamic, Huang and Zheng (2012) further classify the declining zones as sudden declining zone, smooth declining zone and disturbing declining zone. They also discuss the mechanism and conditions that price stay in the same regime or escape from one regime to another.

3.2 Stylized facts on price

3.2.1 Unit root

As been proven in many financial literature, the series of stock prices are not stationary but returns and volumes are usually stationary. To examine whether our model is compatible with the real financial data in this stylized fact, we check the unit root of SP&500 Dow Jones Index data as well as our simulated dataset, and the results are displayed in the Table 3. The augmented Dickey-Fuller(ADF) are used to test the existence of unit root. The ADF for three price time series are -0.661 , -0.675 and -2.158 , which are significantly greater than the critical value at 10% significant level. The corresponding p-values imply that all three price statistic are unit root process. In the return and volume series, the ADF tests significantly reject the null hypothesis of non-stationary at 1% level for all series. Therefore, we are confident that our simulation match the real financial data well in the terms of stationary for price, return and volume.

3.2.2 Fat tails

Fat-tailed distributions of financial asset returns are well documented in empirical studies (see Cont (2001) and Chakraborti et al. (2011)). The fat tail of return suggests that ex-

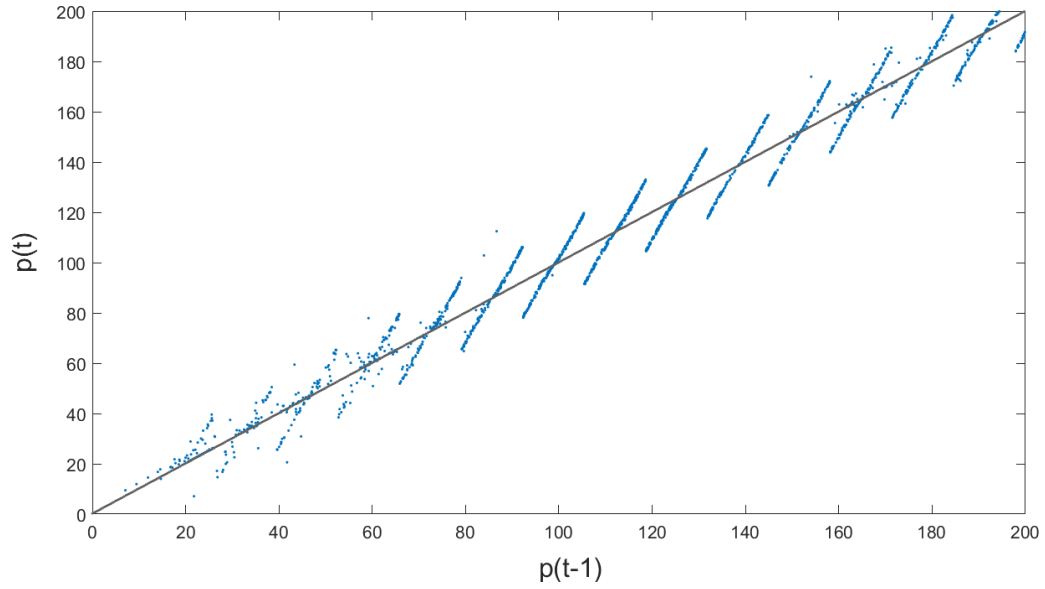


Figure 6: The phase diagram of price

Statistic	Price		Return		Volume	
	ADF	p-value	ADF	p-value	ADF	p-value
S&P 500	-0.661	0.857	-44.010	0.000	-18.742	0.000
Dow Jones Index	-0.675	0.853	-44.037	0.000	-21.444	0.000
Our simulation	-2.158	0.222	-37.604	0.000	-31.645	0.000

Notes: ADF critical value are:-3.43(1%), -2.86(5%), -2.57(10%)

Table 4: Fat tails

Statistic	Skewness	Kurtosis
S&P 500	-0.424	7.202
Dow Jones Index	-0.385	6.526
Our simulation	-0.440	10.514

treme returns appear more frequently than what are predicted by the normal distribution. To investigate the fatness of the simulated return, we calculate the skewness and kurtosis values and compare them with real financial data. Table 4 summarizes the statistic of fat tail tests on our simulation, S&P 500 and Dow Jones Index. The kurtosis (the fourth moments) for our simulation is 10.514, which is close to the value for real financial time series. The kurtosis for all three series are positive and greater than the benchmark value 3 when returns are normally distributed. It is consistent with finding in literature that distribution of returns displays a fat tail with positive excess kurtosis. The skewness (the third moments) of simulated returns is -0.440, which is very close to the skewness of the well-known indexes. The negative value of skewness implies that the price falls in great scales than it rises on average.

3.2.3 Volatility clustering

Another robust statistical property of financial market is the existence of volatility clustering. The volatility clustering phenomenon refers to significant changes of prices tend to cluster together, resulting in persistence of the amplitudes of price changes. Cont (2007) reveals that the different behavior of agents is the reason causes the volatility clustering. In order to check whether our model is able to account for this stylized fact, we plot the autocorrelation functions (ACF) for both simulated data and S&P 500 data.

The returns show almost insignificant autocorrelations for both real data and simulated data in Figure 7. No linear correlation does not mean independence of return. If we take non-linear functions of return into account, such as absolute or squared returns, the results will be different. As shown in Figure 7, the ACFs of simulated absolute returns and squared returns are slowly decaying as time lag increases. The persistent positive correlation is a quantitative feature of volatility clustering, meanwhile it also implies a long range dependence that one typically finds in financial time series. The slowly decaying patterns in the ACFs of simulated data are analogous to that for S&P 500 data, which demonstrates that our model has a great potential to generate the important stylized facts in financial market.

3.2.4 Asymmetric returns

Asymmetric returns is another stylized fact in financial market. In our simulated sample, the most positive return is 44% while the most negative return is -61%, which matches

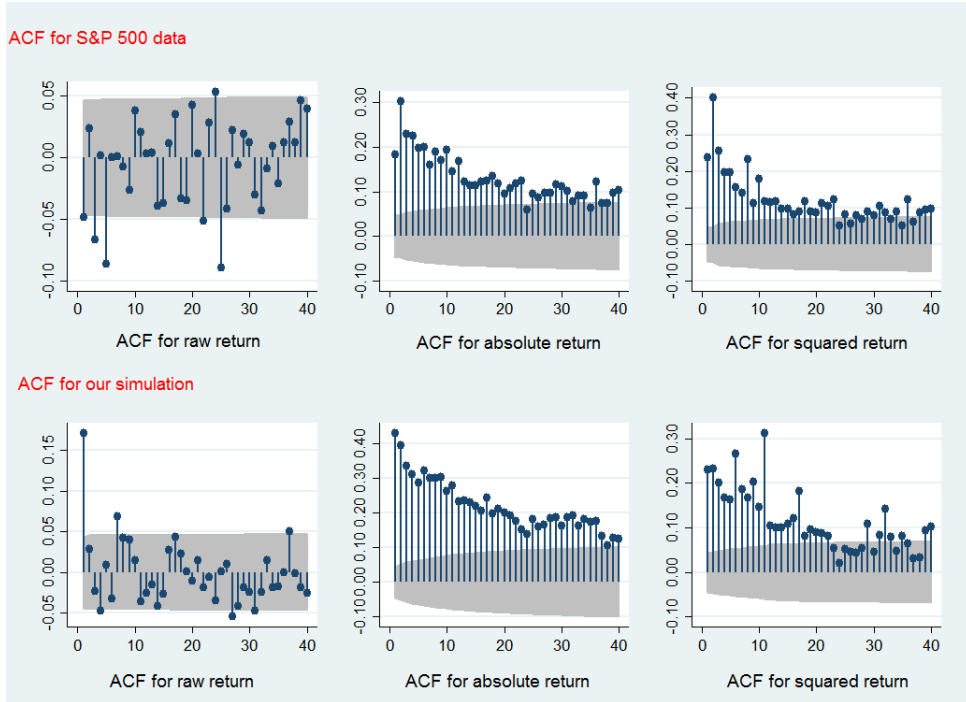


Figure 7: Autocorrelation functions for raw return, absolute return and squared return

Table 5: Shapiro-Wilk test

Variable	Observations	W	V	z	Prob>z
Return	2000	0.929	83.850	11.266	0.000

the documented asymmetry in returns. To test the asymmetry of the statistic, we run the Shapiro-Wilk test for normality. The null hypothesis for this test is that return is normally distributed. The result in the Table 5 significantly rejects the null hypothesis, and the simulated returns not follow the normal distribution. We also do the symmetric plot for the simulated return, and some points departing from the 45 degree line in Figure 8 strongly implies that the returns are asymmetric.

3.2.5 The power law of returns

The tail distribution of returns can be well approximated by the power law, which has been found and investigated in many literatures(Gabaix et al. (2003), He and Li (2007), Lux and Alfarano (2016)). In particular, the distribution of returns is found to decay according to

$$P(|r_t| > x) \sim X^{-\alpha} \quad (16)$$

where α is a constant parameter of the distribution known as the exponent or scaling parameter. To detect the power law distribution of return, we follow the method in Clauset et al. (2009), and the method involves maximum-likelihood fitting methods with goodness-of-fit tests based on the Kolmogorov-Smirnov statistic and likelihood ratios. As

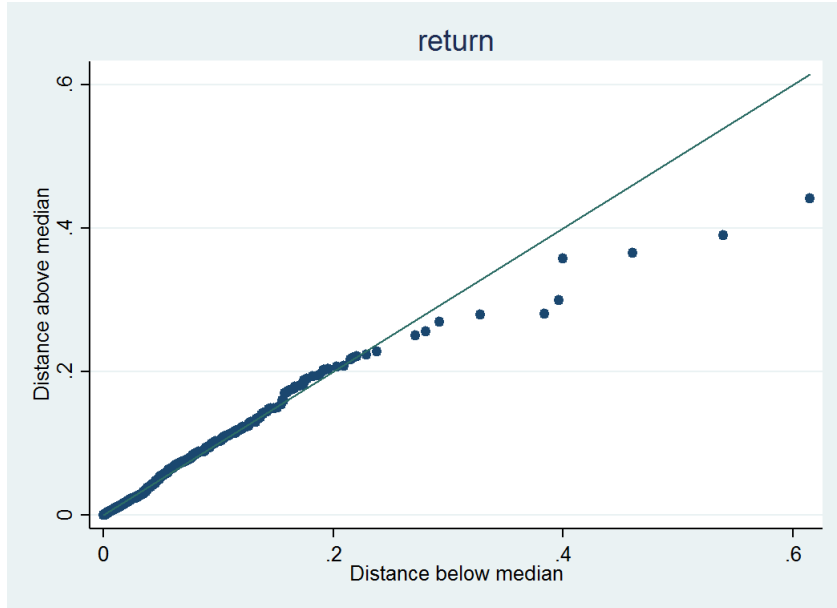


Figure 8: Symmetric plot of return

Table 6: The power law of return

Statistic	X_{min}	α	p-value
S&P 500(weekly)	3.45	3.83	0.77
Dow Jones Index(weekly)	4.07	4.44	0.82
Our simulation	0.10	3.90	0.13

Test hypothesis:
 H_0 : data is generated from a power law distribution.
 H_1 : data is not generated from a power law distribution.

the power law distribution of return is sensitive to the frequency of sample and time range, we test the the daily data(2010/1/1-2016/12/31) and weekly data(1983w1-2016w52) for both S&P 500 and Dow Jones Index, and find that the weekly data of these two indexes follow the power law distribution. To investigate whether our model could generate data with the power law distribution, we test the simulated data and find that p-value equals to 0.13, which suggests one can not reject the null hypothesis that data is generated from a power law distribution. The details of the results are shown in Table 6.

3.3 Stylized facts on volume

Most of the HAMs have the ability to generate the stylized facts on price, but few have investigated the performance of HAM on generating volume related stylized facts. As an important indicator for trading activities of financial market, volume should be carefully considered and studied by practitioners as well as researchers. In this section, we will test our model about the capability of replicating some stylized facts on trading volume. These stylized facts have been mentioned in Section 1, and some of them can be detected

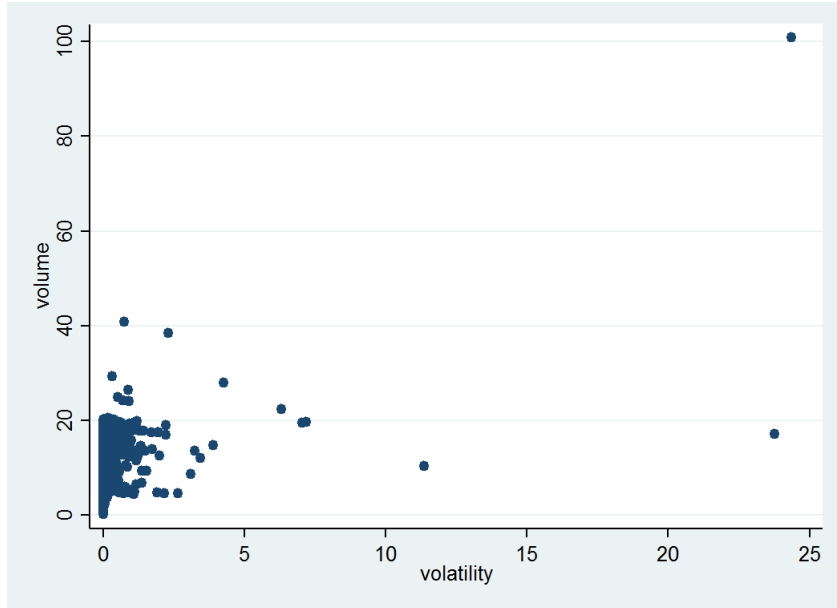


Figure 9: Correlation between volume and volatility

by statistic approaches, others will rely on the visualized analysis and previous literatures.

3.3.1 Correlation between volume and volatility

As shown in Section 1, there exists a strong positive linear relation between volume and VIX. Using the same method, we test the correlation between the simulated trading volume and volatility. Squared returns is used as the proxy of volatility. The correlation coefficient for the test is 0.337, which suggests a weak positive correlation. Although the correlation between simulated series is not as strong as that in real dataset, the significant positive correlation is consistent with the literature. In the past decades, the mechanism of positive correlation between volume and volatility has been always discussed by many researchers, the heterogeneous beliefs of agents in our model could provide a reasonable explanation for this question. As stated in Shalen (1993), the dispersion of beliefs is a factor contributing to the positive correlation between volume and volatility.

3.3.2 Granger causality between return and volume change

The linear and nonlinear relations between price and volume have been found in different markets and countries. To further examine the validity of our model, we conduct the linear and nonlinear Granger causality tests to investigate whether the causality relations exist in our simulation. The methods we used to test the linear and nonlinear Granger causality are same with those in Section 1. The results are reported in Table 7 and Table 8, respectively. In the linear Granger causality test, four lags for dependent and independent variables are considered in the VAR model. The Granger tests show strong evidence of unidirectional causality from return to volume changes. In particular, the Wald statistic

Table 7: Linear Granger causality test

Dependent varibale	(1) R_t	(2) V_t
Constant	0.005 (0.009)	1.438*** (0.074)
R_{t-1}	0.237*** (0.022)	0.183 (0.167)
R_{t-2}	0.065 (0.023)	0.565*** (0.172)
R_{t-3}	-0.002 (0.023)	-0.182 0.172
R_{t-4}	-0.145*** (0.022)	0.076 (0.168)
V_{t-1}	-0.002 (0.003)	0.337*** (0.022)
V_{t-2}	-0.001 (0.003)	0.045 (0.024)
V_{t-3}	0.002 (0.003)	-0.050*** (0.024)
V_{t-4}	-0.001 (0.003)	0.021 (0.022)
Wald-stat	0.937	17.829***
p-value	0.919	0.005

Notes: (1) The entries in brackets are the standard errors. The Wald-stat and p-value are tests of Granger causality. (2) * denotes rejection at the 10% level, ** rejection at the 5% level, ***rejection at the 1% level.

value equals 17.829, and suggests one can confidently reject the null hypothesis that no causality at 1% significant level. On the other hand, Granger noncausality from volume change to stock return can not be rejected, as the p-value of the test 0.919 is great enough. Hence, unidirectional causality from stock return to trading volume exists in our simulation.

To check whether the nonlinear relation exists in the simulated series, we further conduct the nonlinear Granger causality tests for return and volume change. We follow the set up for lags and other parameters in Section 1, and the results in Table 8 show us that all the T-statistics are larger than 1.645 (5% significant level), which implies there are strong evidence that one can reject the null hypothesis. Bidirectional nonlinear Granger causality between stock returns and volume changes is found in our simulation. The linear and nonlinear test results we get from the simulation are roughly consistent with the results using above S&P 500 and Dow Jones index dataset. Referring to the literature, the results are exactly consistent with the findings in Hiemstra and Jones (1994) both in linear and nonlinear Granger causality tests. The results also justify that

Table 8: Nonlinear Granger causality test

Lags	H_0 :Stock Returns Do not Cause Volume Changes		H_0 :Volume Changes Do not Cause Stock Returns	
	T-statistic	p-value	T-statistics	p-value
$L_x=L_y$				
1	1.654	0.049	3.393	0.000
2	1.733	0.042	2.990	0.001
3	1.976	0.024	3.303	0.000
4	2.180	0.015	2.211	0.014
5	2.190	0.014	2.348	0.009
6	1.806	0.035	2.177	0.015
7	1.895	0.029	1.678	0.047
8	1.818	0.034	1.811	0.035

Note:Test critical value for T statistics are 2.326 (1%),1.645 (5%) and 1.282 (10%).

our model is capable of simulating the relationships between returns and volume changes in stock market.

3.3.3 Significant volume with large price change

There are some visualized stylized facts on trading volume have drawn much attention from practitioners and researchers. The first one we intend to simulate is that significant volume is always along with price jump. Besides the statistical relationship we have tested using econometric approaches, the visualized relationship between prices fluctuation and volumes change during abnormal periods is also an important topic in the stock market. As shown in Section 1, the price jumps suddenly with significant volume during the crisis periods, which reflects a crucial characteristic of stock market. In our simulation, this kind of relation can be straightforwardly observed. As illustrated in the Figure 10, the dramatic changes of price (either positive or negative) are found with high trading volumes. When the price suddenly increases or decreases, the volumes almost double the daily average volume.

3.3.4 Informational role of trading volume

As one of informational tools, trading volume is widely adopted by technicians. Investors identify specific signal from the volume to confirm the future price trend and discover the selling and buying opportunities. In efficient market, price movement are attributed to the available of new piece of information. The previous example of Google stock demonstrates the the informational role of trading volume to predict the future price trend. In the Figure 11, we find similar price and volume pattern as in the real world. The trading volume of periods before $t = 436$ is less than 20, and prices fluctuate in a window from 30 to 100. When the price suddenly jumps up to the next level at $t = 436$,

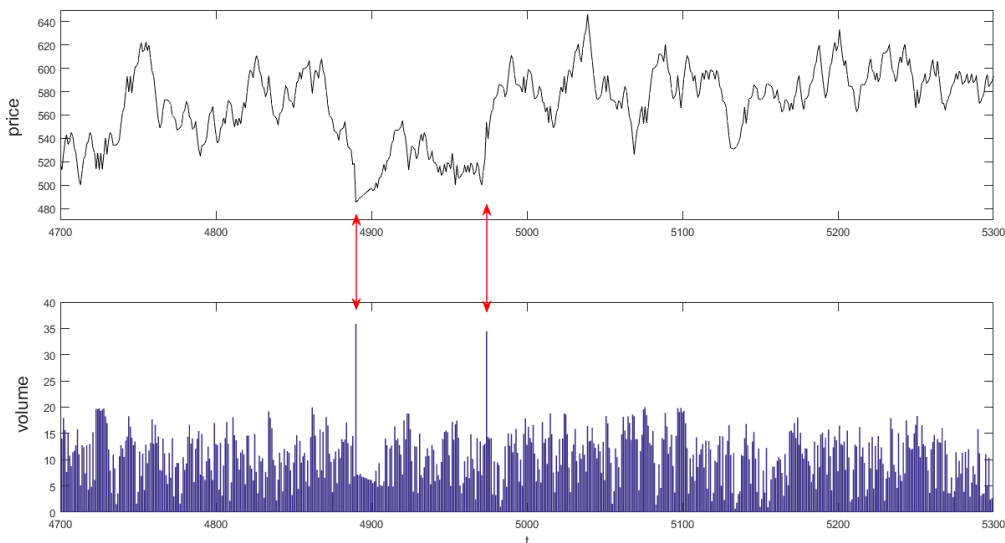


Figure 10: Significant volume with large price change

the trading volume is almost five times the daily average, which confirms the breaking-out of price to the resistance line. After jumping up, prices move above the $p = 100$, which has become to be a new support line.

3.4 Prices, volumes and beliefs co-evolve in different chart patterns and crises

Some literatures of HAM have investigated the co-movement of assets prices and agents beliefs, such as Boswijk et al. (2007) and Huang et al. (2010). Nevertheless, few of them have included the trading volumes into the analysis. In this section, we investigate the stock market from three dimensional viewpoint, and we observe the co-evolvement of prices, volumes and beliefs in our simulation. Furthermore, we introduce the chart patterns and crises into our analysis. As expected, our model has the ability to provide the reasonable explanation for the formation of different chart patterns and crises patterns.

Chart pattern is one of the most popular strategies for technicians, and they have summarized many patterns in the price series. Although not all of them appear during a specific period, some patterns such as double tops, double bottoms, head-and-shoulders and V tops are found in stock market frequently. We select some chart patterns from our simulation, and the co-evolvement of prices, volumes and beliefs within these pattern periods are displayed in Figure 12 and Figure 13. A double tops pattern is shown in the top of Figure 12. At the first stage of the pattern, the fundamentalists are in charge of the market as the price is lower than their fundamental value. The great excess demand of fundamentalists push up the price, and when the price hike gradually to the first top, the optimistic chartists dominate the market. The gradual bubble occurs with

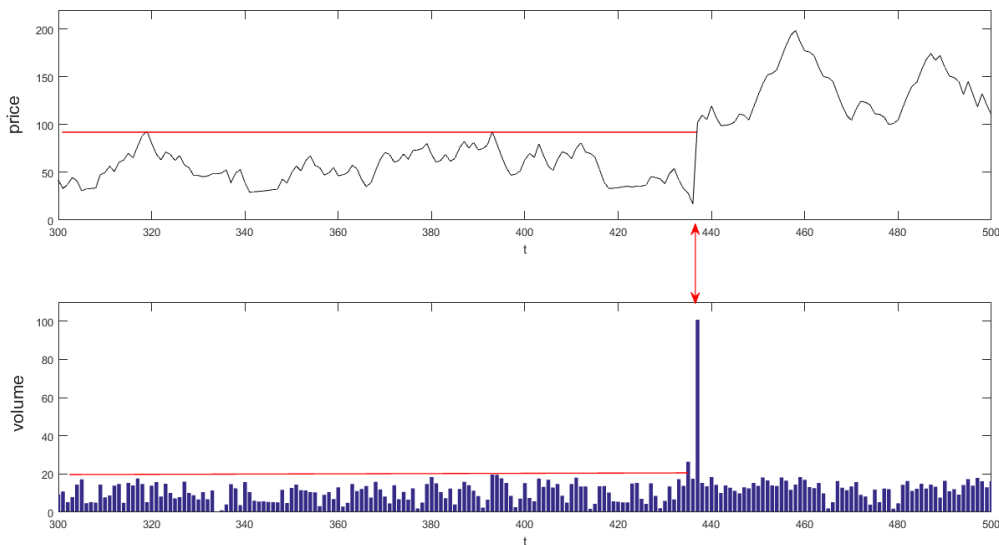


Figure 11: Informational role of trading volume

large trading volume. At the next stage, when the price is high enough, the fraction of pessimistic chartists increases. It follows that price falls with more chartists chasing the falling trend. At the bottom, the price is lower than the fundamental value, and the fundamentalists take over the market again. Huge demand of fundamentalists leads to large trading volume, and the price rise again with the increasing number of chartist. Finally, the second top appears. Double bottoms pattern is reversal pattern of double tops. As shown in Figure 13, the bottoms are formed with a sudden or smooth falling of price, and trading volume is also significant at the same time. The fundamentalists dominate the market at bottoms' period, and chartists take over during other periods.

HAMs have been widely used to explain the financial crises in many ways. Huang et al. (2010) has found that switches between trading strategies leads to price dynamic and cause different types financial crises, such as sudden crisis, smooth crisis and disturbing crisis. We examine the sudden crisis and smooth crisis in our simulation, and the co-evolution of prices, volumes and beliefs during crisis periods are demonstrated in Figure 14 and Figure 15. In the smooth crisis, the price falls smoothly from 300 to 100, and trading volume also decreases accordingly. When the price declines to a low level, fundamentalists replace the chartists to dominate the market. The lasting descending trading volume could be regarded as signal of smoothing crisis. During smoothing crisis, chartists dominate the market and induce the slowly decline of price. The existence of pessimistic chartists slow down the speed of price falling. Once price falls below the fundamental value, fundamentalists begin to increase and take over the market. In a sudden crisis, the price plunges from the peak precipitately down to the bottom, and trading volume surges at the same time. In the Figure 15, the sudden crisis occurs with the spike

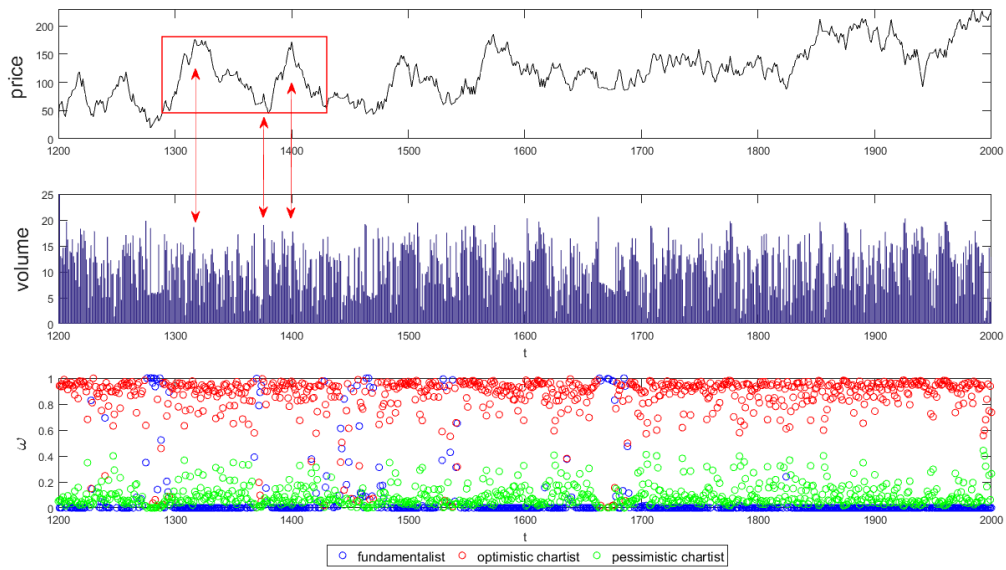


Figure 12: prices, volumes and beliefs co-evolve in double tops pattern

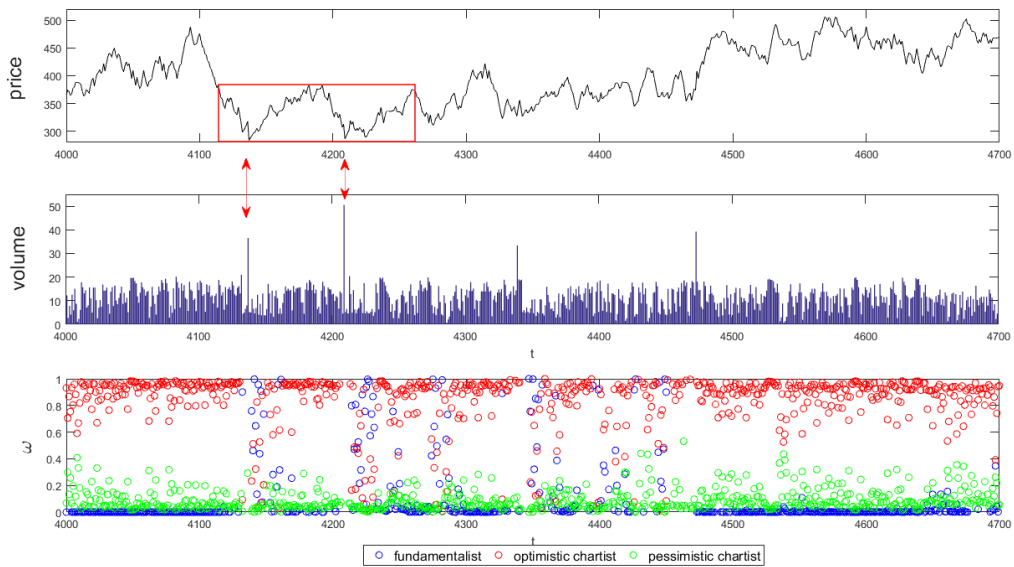


Figure 13: Prices, volumes and beliefs co-evolve in double bottoms pattern

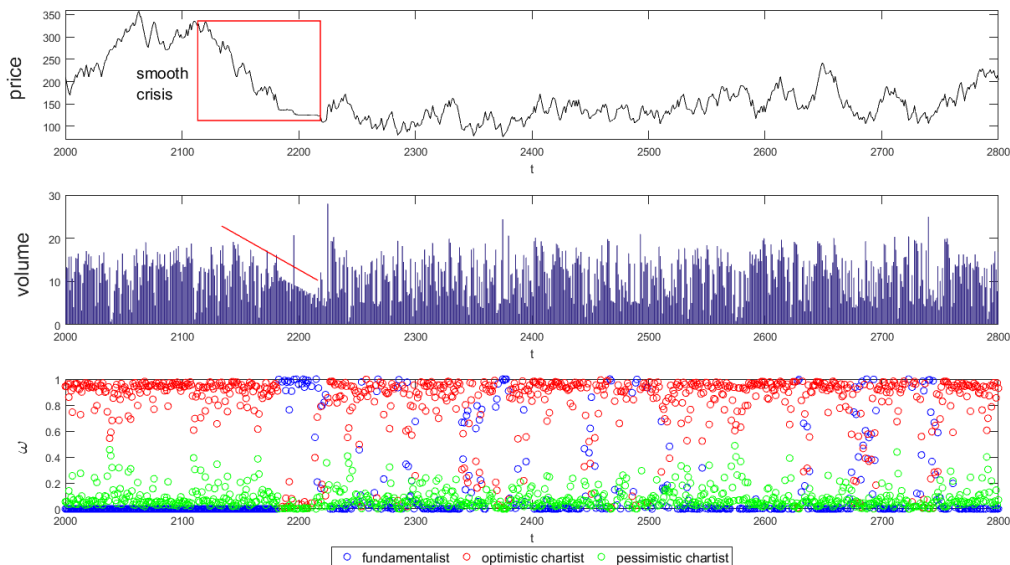


Figure 14: Prices, volumes and beliefs co-evolve in a smooth crisis

shape volumes, and the number of fundamentalists suddenly increase and become the majority in the market.

4 Conclusion

In this paper, we develop a HAM with trading volume to replicate qualitative and quantitative features commonly observed in stock market. Under the framework of market maker, fundamentalists and chartists hold heterogeneous beliefs on future price of risky assets. Agents are allowed to update their expected price basing on the different behaviors: the fundamentalists set their fundamental value refer to the costly internal information and economy growth rate, while chartists update their expected short-term fundamental value according to a series of psychological windows. To fit the real life case well, we introduced the adaptive evolutionary regime and agents freely switch to other group and choose the strategies that would optimize their discounted expected profit. The interaction between the fundamentalists and chartists could generate the price fluctuation and price-volume relationship. Meanwhile, the adaptive switching behavior of agents also increase market fluctuations both in price and volume.

Although we keep our model as simple as possible, it is capable of generating a wide range of stylized facts both on price and volume simultaneously. As documented in literature, many HAMs are capable of generating stylized facts on price or return, such as unit root process in prices, fat tails, asymmetric and volatility clustering returns. The HAM in Huang and Zheng (2012) even has the ability to simulate the strict power-law distribution of return. After successfully simulating all the “standard” stylized facts

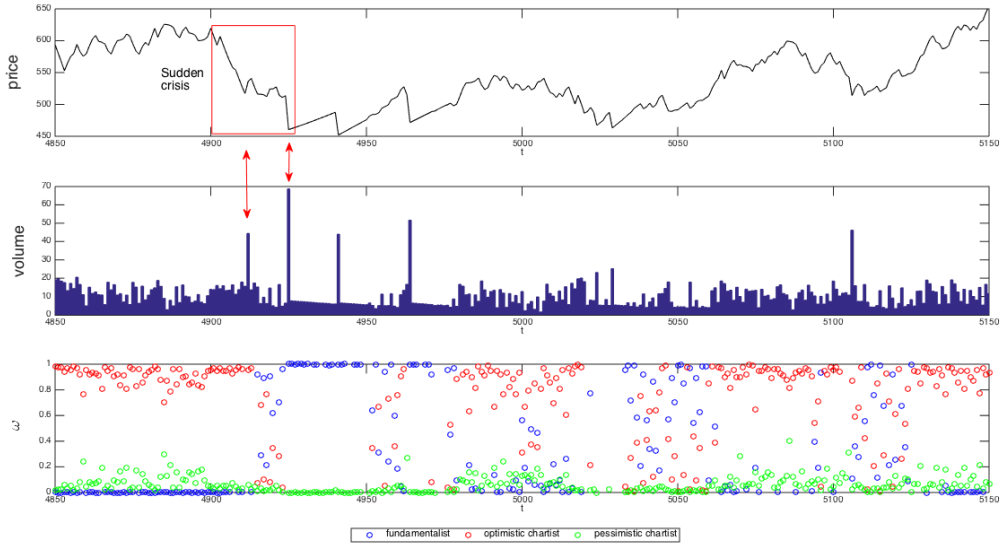


Figure 15: Prices, volumes and beliefs co-evolve in a sudden crisis

above, we further to explore the potential of our model in generating the stylized facts on volume. As expected, the chaotic model perform well in reproducing the stylized facts like stationary volume, positive correlation between volume and volatility, Granger causality between return and volume change. In addition, our model also successfully replicate some visualized stylized facts which are commonly found in financial market, such as significant volume with dramatic price change and information role of trading volume. To demonstrate the power of our model in explaining different patterns in stock market, we identify different chart patterns and crises pattern that frequently documented in technical analysis and literatures, then analyze the formation of these patterns. We are the first who use tridimensional analysis approach to investigate the co-evolvement of prices, volumes and beliefs in these patterns. The co-evolvement of these three elements comprehensively reflects the trading activities and the investors' behavior in financial market, which could give us a thorough view on financial market. By analyzing the chart patterns, our model could also provide theoretical underpinning for technical analysis.

Further exploration can be made as well. In the strategy of each group, none of them take the trading volume into their consideration, the self-fulfilling power of volume signals could be test in the future study. In addition, the profitability of different trading strategies has been documented in many empirical literature, and we are interested in explore this feature by HAM in the future research.

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Appendices

Appendix A

Linear Granger causality definition: Two stationary time series X_t and Y_t , let $F(X_t|\Omega_{t-1})$ be the conditional probability distribution of x_t given the bivariate information set Ω_{t-1} consisting of an L_x -length lagged vector of X_t and L_y -length lagged vector of Y_t . If:

$$F(X_t|\Omega_{t-1}) = F(X_t|(\Omega_{t-1} - Y^{L_y})), t = 1, 2, \dots \quad (\text{A.1})$$

We say, given lags L_x and L_y , the time series Y_t does not strictly Granger causality cause X_t . If the equality does not hold, Y is said to strictly Granger cause X . In plain words, X_t is said to Granger-cause Y_t if X cannot help predict future Y .

To test for Granger causality between stock return and volume change, we conduct the the following vector autoregressive(VAR) model:

$$R_t = A + B(L)R_t + C(L)V_t + U_t \quad (\text{A.2})$$

$$V_t = D + E(L)R_t + F(L)V_t + V_t \quad (\text{A.3})$$

Where, R_t is stock return and V_t is percentage change of volume. $B(L)$, $C(L)$, $E(L)$ and $F(L)$ are lag polynomials of R_t and V_t .

Nonlinear Granger causality definition: Consider two strictly stationary and weakly dependent time series $X_t, Y_t t = 1, 2, 3 \dots$, We then denote the m -length lead vector of X_t by \mathbf{X}_t^m and the L_x -length and L_y -length lag vectors of X_t and Y_t , respectively, by $\mathbf{X}_{t-L_x}^{L_x}$ and $\mathbf{Y}_{t-L_y}^{L_y}$. For given values of m, L_x , and $L_y \geq 1$ and for $e \geq 0$, Y does not strictly Granger-cause X if

$$\begin{aligned} Pr(\|\mathbf{X}_t^m - \mathbf{X}_s^m\| \leq e \mid \|\mathbf{X}_{t-L_x}^{L_x} - \mathbf{X}_{s-L_x}^{L_x}\| \leq e, \|\mathbf{Y}_{t-L_y}^{L_y} - \mathbf{Y}_{s-L_y}^{L_y}\| \leq e) \\ = Pr(\|\mathbf{X}_t^m - \mathbf{X}_s^m\| \leq e \mid \|\mathbf{X}_{t-L_x}^{L_x} - \mathbf{X}_{s-L_x}^{L_x}\| \leq e) \quad (\text{A.4}) \end{aligned}$$

Where $Pr(\cdot)$ denotes probability and $\|\cdot\|$ denotes the maximum norm. In order to transform equation (A.4) into a testable form, we denote the joint and marginal probabilities by:

$$\begin{aligned}
C1(m + Lx, Ly, e) &\equiv Pr(\mathbf{X}_{t-Lx}^{m+Lx} - \mathbf{X}_{s-Lx}^{m+Lx} \|\leq e, \|\mathbf{Y}_{t-Ly}^{Ly} - \mathbf{Y}_{s-Ly}^{Ly} \|\leq e) \\
C2(Lx, Ly, e) &\equiv Pr(\mathbf{X}_{t-Lx}^{Lx} - \mathbf{X}_{s-Lx}^{Lx} \|\leq e, \|\mathbf{Y}_{t-Ly}^{Ly} - \mathbf{Y}_{s-Ly}^{Ly} \|\leq e) \\
C3(m + Lx, e) &\equiv Pr(\mathbf{X}_{t-Lx}^{m+Lx} - \mathbf{X}_{s-Lx}^{m+Lx} \|\leq e) \\
C4(Lx, e) &\equiv Pr(\mathbf{X}_{t-Lx}^{Lx} - \mathbf{X}_{s-Lx}^{Lx} \|\leq e)
\end{aligned}$$

The strict Granger noncausality condition in equation (4) can be expressed as

$$\frac{C1(m + Lx, Ly, e)}{C2(Lx, Ly, e)} = \frac{C3(m + Lx, e)}{C4(Lx, e)} \quad (\text{A.5})$$

The null hypothesis for Y_t strictly Granger-causing X_t in equation (5) is

$$\sqrt{n} \left(\frac{C1(m + Lx, Ly, e, n)}{C2(Lx, Ly, e, n)} - \frac{C3(m + Lx, e, n)}{C4(Lx, e, n)} \right) \sim N(0, \sigma^2(m, Lx, Ly, e)) \quad (\text{A.6})$$

After getting two estimated residual series U_t and V_t from the linear VAR estimation, we use the modified HJ test(see Diks and Panchenko (2006)) to investigate the nonlinear Granger causality between stock return and trading volume.

Appendix B

The chance function shows the chance of lost opportunity either to buy when the assets price is low or fail to sell when the price is high. It can be expressed as

$$A(\mu_t^f, p_t) = a(p_t - m(\mu_t^f))^d (M(\mu_t^f) - p_t)^d \quad (\text{B.1})$$

where a and d are the parameters that describe the sensitiveness of fundamentalists when the price move close to the boundaries and $a > 0$, $d < 0$. Assuming μ_t^f as constant, the chance function can be simply illustrated as Figure B.1.

We define the boundaries as below

$$M_t = k\mu_t^f \text{ and } m_t = \frac{1}{k}\mu_t^f \quad (\text{B.2})$$

As defined in Black (1986), price fluctuates within a reasonable bond in efficient market. $k > 1$ is a pre-selected factor and μ_t^f is the fundamental value of risky assets.

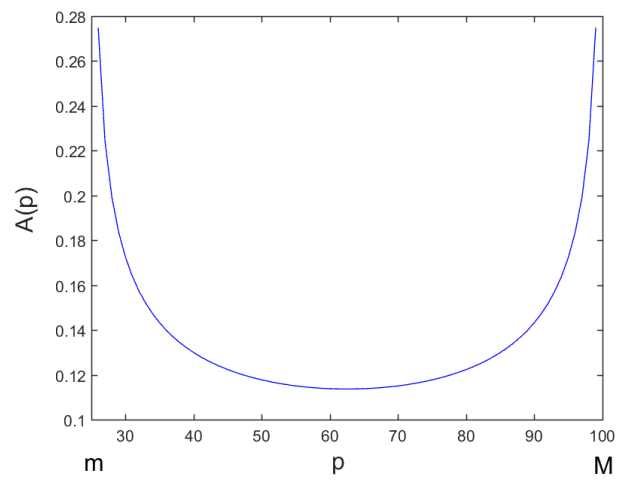


Figure B.1: The chance function