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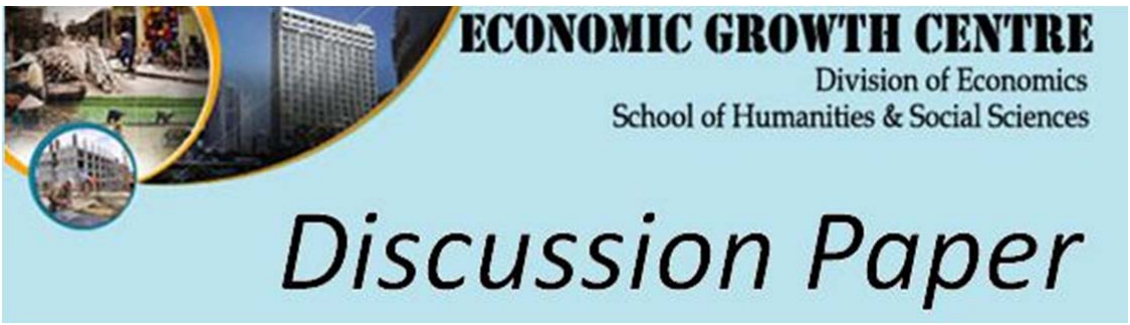

A concrete example of the transfer problem with multiple equilibria*

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Abstract

This paper presents a constructive example of the Balasko-type (1978, 2014) two-agent transfer problem, based on Shapley and Shubik (1977)'s example of a smooth trading economy with three competitive equilibria.

Keywords: Transfer problem; multiple equilibria; unstable equilibrium

JEL classification numbers: D5; F1; F4; O1

1 Introduction

The transfer problem refers to the possibility that a transfer of resources from one agent to another can make the donor better off and/or the recipient worse off. The main purpose of this paper is to present an example of the two-agent transfer problem in a smooth trading economy with multiple equilibria. Such a constructive example in this paper has not been presented before, even though the existence of the transfer problem has been proposed theoretically.

The phrase "transfer problem" was first introduced by Keynes (1929) and Ohlin (1929) in a debate over whether German reparation payments after World War I would improve or deteriorate Germany's terms of trade.

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Leontief (1936) pointed out that the change in terms of trade through international transfers can result in a paradoxical outcome in which the donor's utility increases and the recipient's utility decreases. Samuelson (1947, 1952) argued that the transfer problem is related to the existence of unstable equilibria. Balasko (1978, 2014) clarified Samuelson's argument in a regular and static economy with two agents; he showed that the existence of an unstable equilibrium is necessary and sufficient for the two-agent transfer paradox to occur in a static economy with any number of goods.

After Samuelson, the transfer problem was further developed in an economy with a unique and stable equilibrium with *three* agents. This was referred to as the "three-agent" transfer problem, distinct from the Balasko-type transfer problem with *two* agents. Gale (1974) first developed a concrete example of the transfer problem with three consumers. Subsequently, concrete examples of this topic have been shown by Chichilnisky (1980, 1983), Geanakoplos and Heal (1983), Polemarchakis (1983), Léonard and Manning (1983), Postlewaite and Webb (1984), and Kang and Ye (2014, 2015). Compared to a large volume of literature on the three-agent transfer problem, there has been no example of the Balasko-type two-agent transfer paradox thus far. This paper presents a constructive example of the Balasko-type transfer problem. In particular, it directly adopts the example of Shapley and Shubik (1977), originally developed in the context of a two-country currency-exchange model, and then incorporates a currency transfer to that example.

2 The Economy

The economy described in this paper is the same as that of Shapley and Shubik (1977), except for the addition of a currency transfer from one agent to the other. There are two consumers, Ivan and John. Ivan has 40 rubles and wants some dollars. John has 50 dollars and wants some rubles. Their utility functions ($x = rubles$, $y = dollars$) are

$$u^{John}(x^J, y^J) = y^J + 110(1 - e^{-x^J/10}) \tag{1}$$

and

$$u^{Ivan}(x^I, y^I) = x^I + 100(1 - e^{-y^I/10}) \tag{2}$$

Incorporating a transfer of t dollars from John to Ivan, their currency endowments can be expressed as

$$\text{John's endowment (rubles, dollars)} = (0, 50 - t), \tag{3}$$

and

$$\text{Ivan's endowment (rubles, dollars)} = (40, t). \tag{4}$$

The total amount of currencies are fixed as R 40 and \$ 50 for any amount of transfer t. The interior contract curve can be derived from utility functions (1) and (2):

$$y^J = x^J + 50 - 10 \log 10. \tag{5}$$

From (1) and (3), John's budget constraint can be expressed as

$$(11e^{-x^J/10})x^J + y^J = 50 - t, \tag{6}$$

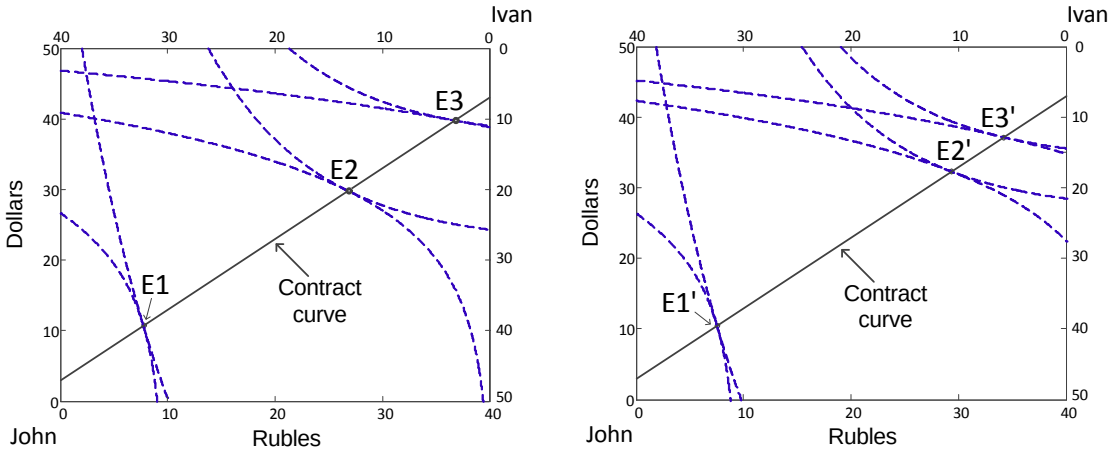
where $(11e^{-x^J/10}) = \left(\partial u^{John}/\partial x\right) / \left(\partial u^{John}/\partial y\right)$.

From equations (5) and (6), we have the following transcendental equation:

$$x^J \left(1 + 11e^{-x^J/10}\right) = 10 \log 110 - t. \tag{7}$$

3 The transfer problem

Numerically solving equation (7) where $t = 0$ results in three multiple equilibria as shown in Shapley and Shubik (1977). Therefore, for a sufficiently small value of t, the existence of three multiple equilibria is guaranteed. We assume that 1% of John's endowment is transferred to Ivan, i.e., $t = 0.5$. Figures 1 and 2 show the three equilibria in a Edgeworth box for the case of $t = 0$ and $t = 0.5$, respectively. In Figure 1, the indifference curves are indicated by the dashed curves. The three equilibria are indicated by $E1$, $E2$ and $E3$ where $t = 0$, and $E1'$, $E2'$ and $E3'$ where $t = 0.5$. $E1, E3, E1'$ and



(a) The case for t=0

(b) The case for t=0.5

Figure 1: The Edgeworth Box

$E3'$ are stable equilibria while $E2$ and $E2'$ are unstable equilibria.

Table 1 summarizes the equilibrium allocations with $t = 0$ and $t = 0.5$, respectively. The utility payoffs before and after the transfer are summarized in Table 2. Two types of the transfer problem can be observed in Table 2. Balasko (1978) distinguished the local transfer paradox, in which the equilibrium selection map is continuous, with the global transfer paradox, in which the selection map is permitted to be discontinuous: (a) the local transfer paradox can occur if and only if there exists an unstable equilibrium and (b) the global transfer can occur if and only if there exists multiple equilibria. As Balasko states, the (local) transfer paradox occurs in an unstable equilibria; John's (donor) utility in $E2'$ is higher than that in $E2$ while Ivan's (recipient) utility in $E2'$ is lower than that in $E2$. The global paradox across the two stable equilibria from $E1$ to $E3'$ is observed in Table 2; John's utility in $E3'$ is higher than that in $E1$ while Ivan's utility in $E3'$ is lower than that in $E3$.

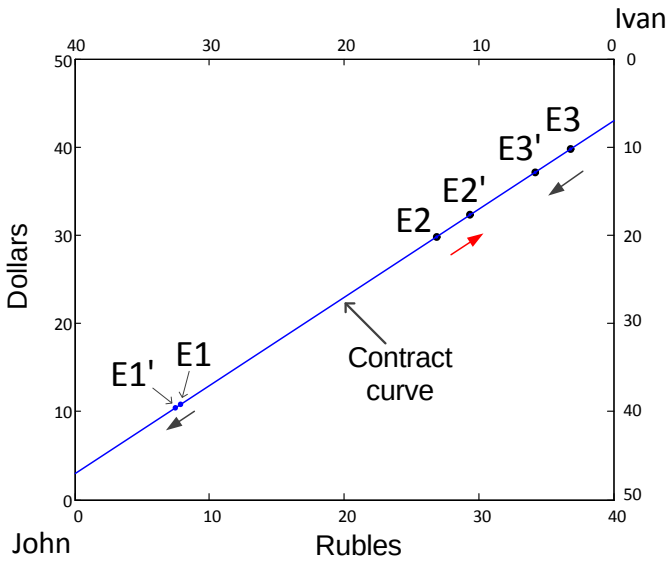


Figure 2: Equilibria before and after a transfer

	John	Ivan		John	Ivan
E1	(7.74,10.74)	(32.26,39.26)	$E1'$	(7.52,10.51)	(32.48,39.49)
E2	(26.83,29.82)	(13.17,20.18)	$E2'$	(29.34,32.33)	(10.66,17.67)
E3	(36.78,39.77)	(3.221,10.23)	$E3'$	(34.19,37.18)	(5.81,12.82)

Table 1. Equilibrium allocations (Rubles,Dollars)

	John's utility	Ivan's utility		John's utility	Ivan's utility
E1	70.011	130.290	$E1'$	68.628	130.56
E2	132.303	99.874	$E2'$	136.482	93.572
E3	146.993	67.256	$E3'$	143.580	78.057

Table 2. Equilibrium utility payoffs

4 Conclusion

The transfer problem in a static economy with two agents was shown to be theoretically possible. However, it has been difficult to show its applicability to a broader set of economic contexts due to the lack of concrete examples of the concept. In this regard, this paper has a clear contribution to the area of the transfer problem.

It is interesting that a similar type of transfer problem can also occur in a monetary economy with nominal taxes and transfers. Kang (2014) showed the existence of paradoxical outcomes in an economy with three consumers, in which inflation makes the tax payer worse off and the recipient of the subsidy better off. However, this "nominal" transfer problem has not been addressed in a two-agent economy, which should be an interesting topic for future research.

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