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Regional Financial Markets With Common Currency

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With the development of globalization and regional market integration, regional markets with common currency emerge. We develop a heterogeneous agents model based on the frameworks of Day and Huang (1990) as well as Westerhoff and Dieci (2006). Two markets using same currency are populated by chartists and fundamentalists. Market linkage is established by allowing investors to trade in both markets. One of the consequences of market linkage is market pooling, in which investors from each market interact with each other and determine the price movements of the market system. The market that is more stable initially exerts stabilizing force on the market system while itself might suffer from destabilizing effect. Market system based on the model demonstrates the capability to generate important stylized facts of financial markets, in particular the significant cross-correlation between two markets.

Keywords: Financial multi-market interactions, Market integration, Market Pooling, Chaos, Heterogeneous beliefs

JEL Classifications: C61 D84, G15

1. INTRODUCTION

With the development of regional market integration, linkages among markets become closer and common currency circulating within the region emerges. Euro is a typical example of such a kind of common currency. Different financial markets within euro-zone use euro as transaction currency. Besides euro-zone markets, there are also other asset markets using same currency for transaction, such as the Shanghai and Shenzhen stock exchange markets in China, and NASDAQ and New York stock exchanges in United States. The common transaction currency eliminates the need of currencies exchange in case of multi-markets with different currencies and removes noises impacting financial markets from the foreign exchange market.

The heterogeneous agents model of this paper has a setup of two markets with a common transaction currency. When market linkage is established, there is a mixed steady state for the market system in which one market falls into a fundamental steady state while the other is in a nonfundamental state. The stability analysis shows that the mixed steady state is unstable. Another result of this paper is market pooling upon market linkage. With market pooling effect, a market that is more stable initially will stabilize the market system while itself is subject to destabilizing effect from the market system. This mutual effect also applies to a market that is more unstable initially. In addition, this paper contributes to the financial market stylized facts calibration in terms of cross-correlation between markets.

Day and Huang (1990) introduce a stylized market maker framework in which two agent types, chartist and fundamentalist, invest in an asset market and a market maker updates price in each period. The model is in discrete time and exhibits complicated, chaotic price fluctuations around a fundamental price with randomly switching bear and bull market episodes. Instead of the market maker framework, Brock and Hommes (1998) apply the Walrasian equilibrium concept in heterogeneous agents model. In their model, micro-foundation is built on a fitness measure, which is determined by the past realized profit. Every period, agent composition is determined by fitness measures. Agent aims to maximize investment profit and decides her supply and demand according to the chosen strategy. Market clears at the end of each period. This model is capable of explaining some stylized financial behaviors such as irregular switching among phases of price movements. However, Blake and Lebaron (2006) argue that the market clearing Walrasian equilibrium in every period has limitations. One of the limitations is that it may not well represent the continuous trading financial market. Nevertheless, combination of market maker and micro-foundation based on fitness measure develops in later literature such as Westerhoff (2004), He and Westerhoff (2005), Westerhoff and Dieci (2006), and He and Li (2008).

Majority of the heterogeneous agents models focus on single market or one risky asset with reference to one riskless asset. Recently, the ideas of heterogeneous agents start to be extended to price dynamics of multi-asset within a market, or even to the interactional dynamics of multi-markets. For example, Bohm and Wenzelburger (2005) investigate the performance of efficient portfolios in a financial market in which heterogeneous investors including rational traders, noise traders, and chartists are active. Brock et al. (2009) introduce additional Arrow securities into the stylized evolutionary equilibrium model of Brock and Hommes (1998) and demonstrate that more hedging instruments may destabilize markets with heterogeneous agents and performance-based reinforcement learning. Westerhoff and Dieci (2006) develop a model in which chartists and fundamentalists invest in two speculative markets. The composition of investors varies according to profit fitness measurement. After stability conditions for the fundamental steady state are derived, the model generates complex price dynamics resembling to actual speculative prices. Dieci and Westerhoff (2010) build up a three-market model in which two stock markets are linked via foreign exchange market. The foreign exchange market is populated with chartists and fundamentalists while the two stock markets have only fundamentalists. It is concluded that upon market interactions, stock markets may be destabilized while the stabilizing effect on the foreign exchange market and the whole market system can be observed.

Heterogeneous agents models have managed to calibrate some of the financial market stylized facts such as large trading volume, unpredictable asset returns with almost no autocorrelation, returns distribution with fat tails, and cluster volatility. We cite, in particular, Lux (1995, 1998), Lux and Marchesi (2000), Brock and Lebaron (1996), and Farmer and Joshi (2002). However, all these stylized facts are related to single market. The field of stylized facts calibration for multi-market such as cross-correlation needs further exploration. Our model can cover this research gap.

This paper follows the framework of Day and Huang (1990) and Westerhoff and Dieci (2006). Two types of investors, chartists and fundamentalists, invest in two speculative markets with the same transaction currency. Each investor can invest in either market and chooses a chartist or fundamentalist strategy in each market. The difference in this paper is that agents of the same type are inhomogeneous across markets. That is, chartists or fundamentalists from different markets have different demand strengths. For the purpose of multi-market interaction analysis, we postulate a new fitness measure based on the magnitude of price deviation. Stationary states including fundamental and nonfundamental steady states are derived. Through both analytical study and numerical experiments, it is shown that market-pooling emerges upon market linkage.

This paper is structured as follows. In section 2 we build a market maker model for the isolated market and derive the stability conditions. In section 3 we extend the model to two-market system. Market pooling effect emerges. In section 4, simulation based on the two-market model reveals its capability to generate time series data matching some of the stylized facts of financial market, especially the cross-correlation. Section 5 concludes the paper and proposes the directions for future research.

2. ISOLATED MARKET

In this section, a two-dimension discrete-time dynamic model in which chartists and fundamentalists invest in two stock markets with the same transaction currency is developed. Fundamentalists behave in a way that they sell over-priced asset and purchase under-priced one. In contrast, chartists simply assume the persistence of bullish and bearish market episodes in the short run. Following this expectation, they purchase the over-priced asset and sell the under-valued one. The two stock markets are denoted by A and B. An isolated market is modelled such that investors can only invest in their home market. The composition of chartists and fundamentalists among the investors depends on market circumstance. A larger asset price deviation triggers more agents to rely on fundamentalist strategy.

2.1. Model setup

For the isolated market i, $i = A$ or B, the log price of the asset at time t is denoted by $P_{i,t}$. The constant log fundamental value is denoted by F_i. For convenience, we define the log price deviation as

$$x_{i,t} \triangleq P_{i,t} - F_i. \tag{1}$$

Moreover, the state $x_{i,t} = \bar{x}_i = 0$ will be referred to as *the fundamental steady state* to indicate the fact of $P_{i,t} = F_i$.

For a chartist (c) or a fundamentalist (f) from market j, $j = A$ or B, the excess demand for the asset i is $D^c_{ij,t}$ and $D^f_{ij,t}$, respectively[1]. To capture the facts that chartists purchase asset when the price deviation is positive and sell when it is negative while fundamentalists behave in an exactly opposite way, without loss of generality, we assume that the excess demand for chartist (fundamentalist) is positively (negatively) proportional to the price deviation. In other words, we have

$$D^c_{ij,t} = c_j \cdot x_{i,t} \text{ and } D^f_{ij,t} = -f_j \cdot x_{i,t},$$

[1] In the sequel, we shall adopt the same notation convention with the first subscript standing for the market asset demanded, the second subscript for the investors originated from, and the superscript (c or f) for type of investors.

where c_j and f_j reflect the strength of demand of chartist and fundamentalist from market j, respectively.

Investors can choose to be either chartists or fundamentalists by comparing the relevant strategy fitness measures. According to the investment strategy of fundamentalists, if asset price deviation is large, chance of earning harvest (for *positive deviation*) or encountering investment opportunity (for *negative deviation*) increases. In other words, more investors tend to adopt fundamentalist strategy when the asset price deviation $|x_{i,t}|$ becomes larger. Therefore, it is reasonable to assume that the strategy fitness measure of fundamentalists from market j to invest in market i at period t, denoted as $m^f_{ij,t}$, is an increasing function of magnitude of price deviation. For simplicity, we express $m^f_{ij,t}$ as

$$m^f_{ij,t} = |x_{i,t}| \, .$$

In contrast, chartist faces increasing chance of investment loss (for *positive deviation*) or missing investment opportunity (for *negative deviation*) with price deviation. Fewer investors will adopt chartist strategy when the asset price deviation $|x_{i,t}|$ becomes larger. Therefore, the chartist strategy fitness measure to invest in market i, $m^c_{ij,t}$, should be a decreasing function of the magnitude of price deviation. Besides that, in the world of chartists, there are phenomena of support and resistance. Support (resistance) is a price level that may induce net increase of buying (selling) to prevent price from further declining (increasing). If a support/resistance is broken, a new support/resistance will be formed. Donaldson and Kim (1993) report empirical phenomena of "support" and "resistance" level in Dow Jones Industrial Average. As stock index approaches levels of support (resistance), stock sellers (buyers) become less aggressive with concern of a turn in the market. This implies that for a given price within support/resistance, investors will be more aggressive if asset price is far away from support/resistance. Then, for a given asset price, investors will be more aggressive with a bigger range of support/resistance compared to a smaller one. In another word, a bigger range of support/resistance will make chartists more confident, which implies a higher $m^c_{ij,t}$. For simplicity, we assume support and resistance are the same, so that

$$m^c_{ij,t} = s_j - |x_{i,t}| = \ln h_j - |x_{i,t}| \, ,$$

where $h_j \geq 1$ is *chartist adjustment parameter* while $s_j = \ln h_j$ takes account of the effects of support and resistance levels. It is worth to note that, for such formulation, at the fundamental steady state, $x_{i,t} = 0$, the strategy fitness measure for chartist strategy is larger ($h_j > 1$) or equal to ($h_j = 1$) the one for fundamentalist. Such formulation reflects the fact that, in the eyes of speculators and trend followers, the financial market always fulfils with the investment opportunities, regardless whether it is in the bull trend or bear trend. As h_j is related to support/resistance which is based on a common prevailing technique, chartists from different markets should derive similar h_j. It is reasonable to assume $h_A = h_B = h$. The difference among the chartists can be addressed by the chartist strengths.

We shall see later that the chartist adjustment parameter h essentially enhances the chartist strength in the sense that the *relative value of chartists' strengths* defined by

$$\beta_i \triangleq c_i h/f_i \tag{2}$$

plays an important role in determining the equilibrium of the financial markets.

Fitness measures affect investor composition in a way that investors are prone to adopt the strategy that has a comparatively higher fitness measure. Without loss of generality, we define chartist and fundamentalist *compositions* in isolated market i respectively as:

$$W^c_{ii,t} \triangleq \frac{\exp\left(m^c_{ii,t}\right)}{\exp(m^f_{ii,t}) + \exp\left(m^c_{ii,t}\right)} = \frac{h \cdot \exp\left(-\left|x_{i,t}\right|\right)}{\exp\left(\left|x_{i,t}\right|\right) + h \cdot \exp\left(-\left|x_{A,t}\right|\right)},$$

$$W^f_{ii,t} \triangleq \frac{\exp(m^f_{ii,t})}{\exp(m^f_{ii,t}) + \exp\left(m^c_{ii,t}\right)} = \frac{\exp\left(\left|x_{i,t}\right|\right)}{\exp\left(\left|x_{i,t}\right|\right) + h \cdot \exp\left(-\left|x_{i,t}\right|\right)}.$$

The aggregate excess demand in market i, denoted by $D_{i,t}$, is contributed by both chartists and fundamentalists from the respective market:

$$D_{i,t} = W^f_{ii,t} D^f_{ii,t} + W^c_{ii,t} D^c_{ii,t}.$$

Following Day and Huang (1990), we assume the existence of a *market maker* who updates the market price at each period adaptively with

$$P_{i,t+1} = P_{i,t} + a_i D_{i,t} \tag{3}$$

where a_i is the *price adjustment parameter* in market i.

2.2. Steady States and Stability

Substituting with relevant components, Eq. (3) leads to a nonlinear dynamics of $x_{i,t}$:

$$x_{i,t+1} = x_{i,t} + a_i \frac{h \cdot c_i \exp(-\left|x_{i,t}\right|) - f_i \exp(\left|x_{i,t}\right|)}{\exp(\left|x_{i,t}\right|) + h \cdot \exp(-\left|x_{i,t}\right|)} x_{i,t} \tag{4}$$

for $i = A$ or B.

We then arrive at the following conclusions.

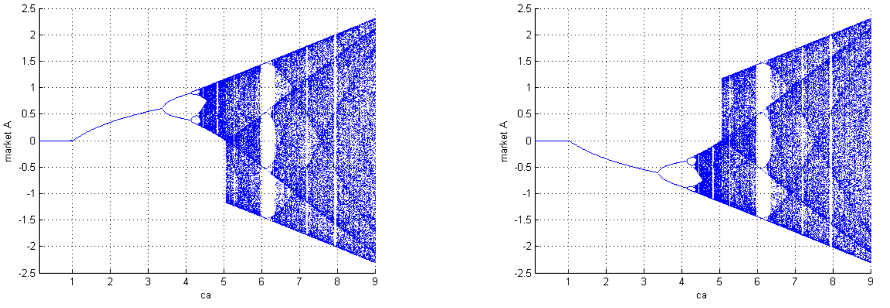
Proposition 1. *(a) There exists a unique fundamental steady state* $\bar{x}_i = 0$*, which is stable if and only if* $\beta^l < \beta_i < 1$ *with* $\beta^l = \max\{1 - 2(h_i + 1)/(a_i f_i), 0\}$;

(b) There exist two nonfundamental steady states: $\bar{x}_i = \pm \ln \sqrt{\beta_i}$*, which are stable if* $1 < \beta_i < \beta^u$ *with* $\beta^u = \exp(2(c_i + f_i)/(a_i c_i f_i))$*;*

(c) A pitchfork bifurcation occurs at $\beta_i = 1$ *while a flip bifurcation arises at* $\beta_i = \beta^u$*.*

Remark 1. To reach a stable fundamental steady state, we must have $\beta_i < 1$, that is, the fundamentalist strength f_i must exceed the enhanced chartist strength $h \cdot c_i$. If the enhanced chartist strength $h \cdot c_i$ increases and surpasses fundamentalist strength f_i, the fundamental steady state loses its stability while two symmetrical stable nonfundamental steady states appear. The magnitude of the nonfundamental steady states are determined by β_i, the relative value between enhanced chartist strength and fundamentalist strength. If chartist strength further increases such that $\beta_i = \beta^u$, nonfundamental steady states lose their stabilities. Flip bifurcation occurs and period-doubling phenomenon is observed.

A typical bifurcation diagram for Market A is provided in Figure 1, in which c_A is the bifurcation parameter with conditions $a_A = 2.131$, $f_A = 1$, and $h_A = 1$. The diagrams report the attractors corresponding to two different initial conditions, above and below the fundamental value (Figure 1.a and 1.b). According to the given conditions, pitchfork transition occurs at $c_A = 1$. Before and after pitchfork transition, stable steady states are fundamental and nonfundamental ones, respectively. At $c_A = 3.375$, nonfundamental steady states lose stability and deviation transits to two-period orbit; flip bifurcation arises. If c_A increases further, the bifurcation attractors experience a sequence of period-doubling bifurcations to chaos states. At some point, the deviations start to wander across positive and negative portions. These regions are characterized by intrinsic fluctuations and erratic switching between positive and negative regions. The attractors experience a transition from chaotic state to four-orbit state across positive and negative portions, followed by a transition to chaotic fluctuation again. In a short summary, the current analysis shows that chartists have destabilizing effect on market and market bifurcation state depends on the interaction between chartists and fundamentalists.



(a) $x_{A,0} > 0$ (b) $x_{A,0} < 0$

Fig. 1. Bifurcation diagram of isolated market A with respect to strength of chartist c_A.

3. MARKET LINKAGE

If the two stock markets open their markets to each other, investors are allowed to invest in both markets. Setup of the isolated market model is extended to a two-market system.

3.1. Model setup

Investors have more investment options in terms of market and agent strategy choices. Each investor from market j will compare four strategy fitness measures, $m_{ij,t}^k$, $i = A$ or B and $k = c$ or f, to make investment decision. The composition of investors originating from market j is thus defined by.

$$W_{ij,t}^k \triangleq \frac{\exp\left(m_{ij,t}^k\right)}{\sum_i \sum_k \exp\left(m_{ij,t}^k\right)}.$$

With more investment sources, the excess demands on market i change to

$$D_{i,t} = W_{iA,t}^c D_{iA,t}^c + W_{iA,t}^f D_{iA,t}^f + W_{iB,t}^c D_{iB,t}^c + W_{iB,t}^f D_{iB,t}^f$$

where

$$D_{ij,t}^c = c_j \cdot x_{i,t} \text{ and } D_{ij,t}^f = -f_j \cdot x_{i,t}$$

for $i, j = A$ or B.

Discerning the capital flow and linkage between markets, both market makers take into account of excess demands of the two markets and weigh them in their price updating mechanisms. Let g_i be a coupling factor, $0 \leq g_i < 0.5$ for $i = A$ or B. For the simplicity of presentation, we assume that the asset prices are updated according to the following mechanisms[2]:

$$P_{A,t+1} = P_{A,t} + a_A \left((1 - g_A) D_{A,t} + g_A D_{B,t} \right)$$
$$P_{B,t+1} = P_{B,t} + a_B \left((1 - g_B) D_{B,t} + g_B D_{A,t} \right)$$

Also, we define the *relative value of chartists' strengths* for this market system consisting of A and B by

$$\beta_{A+B} \triangleq h (c_A + c_B) / (f_A + f_B)$$

3.2. Steady States and Stability

After market linkage is established, two markets A and B form a market system that can be represented by a two-dimensional dynamical system.

$$x_{A,t+1} = x_{A,t} + a_A \left((1 - g_A) D_{A,t} + g_A D_{B,t} \right) \quad (5)$$
$$x_{B,t+1} = x_{B,t} + a_B \left((1 - g_B) D_{B,t} + g_B D_{A,t} \right)$$

The steady states for this system for arbitrary a_A and a_B can be derived and their stability conditions can be established. However, we shall confine ourselves to a more economically meaningful situation in which $a_A = a_B = a$. The very assumption is essential and indispensable to exclude the arbitrage opportunities due to unequal prices when the excess demands for two markets are identical ($D_{A,t} = D_{B,t}$).

Under this assumption, the steady states of two markets, denoted as $(\overline{x}_A, \overline{x}_B)$, and the corresponding stability conditions can be summarized in the following

PROPOSITION 2. *(a) There exists a unique fundamental steady state* $(\overline{x}_A, \overline{x}_B) = (0, 0)$, *which is stable if and only if* $\beta^l < \beta_{A+B} < 1$ *with* $\beta^l = \max\{1 - 4(h+1) / (a \cdot (f_A + f_B)), 0\}$.

(b) There exist four nonfundamental steady states: $(\overline{x}_A, \overline{x}_B) = \left(\pm \ln \sqrt{\beta_{A+B}}, \pm \ln \sqrt{\beta_{A+B}} \right)$, *which are stable if* $1 < \beta_{A+B} < \beta^u$ *with* $\beta^u = \exp\left[(4 (c_A + c_B + f_A + f_B)) / (a (c_A + c_B)(f_A + f_B)) \right]$;

(c) Mixed fundamental and nonfundamental steady states $(\overline{x}_A, \overline{x}_B) = \left(\pm \ln \sqrt{\beta_{A+B}}, 0 \right)$ *or* $\left(0, \pm \ln \sqrt{\beta_{A+B}} \right)$ *are all unstable.*

Proof. (Omitted) ∎

[2] Alternative coupling mechanisms such as (for Market A)
$P_{A,t+1} = g_A P_{A,t} + (1 - g_A) P_{B,t} + a_A D_{A,t}$ or
$P_{A,t+1} = P_{A,t} + g_A a_A D_{A,t} + (1 - g_A) D_{B,t}$ can also be considered. But they will not alter the main conclusions of this paper.

Remark 2. With market linkage, market system can have either stable fundamental or nonfundamental steady state depending on the range of β_{A+B}. The situation for one market being in the fundamental steady state while the other being in a nonfundamental one will never occur.

Once the two markets are linked and open to each other, both markets synchronize to be in an identical fundamental state or one of the four nonfundamental steady states. It is infeasible to expect that one market is stable in the fundamental steady state while the other remains in the nonfundamental steady state. It is interesting to examine further the investor composition for these cases.

In Table 1, for investors from market A, their compositions are categorized according to investment destination market and agent strategies. Compositions ratios $W^f_{AA,t} : W^c_{AA,t} : W^f_{BA,t} : W^c_{BA,t}$ are calculated. The composition ratios for investors from Market B are the same with those from market A. We use destination market A for analysis.

In the fundamental steady state $(\overline{x}_A, \overline{x}_B) = (0, 0)$, composition of chartists $(W^c_{AA,t} + W^c_{AB,t})$ is h times of that of fundamentalist $(W^f_{AA,t} + W^f_{AB,t})$ in Market A while the sum of chartist strength $(c_A + c_B)$ is less than that of fundamentalist $(f_A + f_B)$. Excess demand of all chartists $h\,(c_A + c_B)\,x_{A,t}$ is less than that of all fundamentalists $-\,(f_A + f_B)\,x_{A,t}$ in magnitude. Fundamentalists dominate in each market and hence market system is stable.

In the nonfundamental steady state, although relative composition of chartists to fundamentalists $(f_A + f_B)\,/\,(c_A + c_B)$ is less than h, excess demand of all chartists $(f_A + f_B)\,x_{A,t}$ is equal to that of all fundamentalists $-\,(f_A + f_B)\,x_{A,t}$ in magnitude. Chartists balance fundamentalists in terms of excess demand.

When market system is in the state where Market A is in fundamental steady state and Market B in nonfundamental steady state, composition of chartists is larger than that of fundamentalists by h folds; excess demand of all chartists $h\,(c_A + c_B)\,x_{A,t}$ is larger than that of fundamentalists $-\,(f_A + f_B)\,x_{A,t}$ in magnitude. Hence Market A is unstable.

The pivotal factor for stability of market system is the relative value of chartists' strength, β_{A+B}, which is similar to the isolated market. If $\beta_{A+B} < 1$, the two-market system is stable in the fundamental steady state. With β_{A+B} exceeding 1, the system deviates from the fundamental steady state and stabilizes in one of the nonfundamental steady states.

TABLE 1
Compositions of investors originating from market A

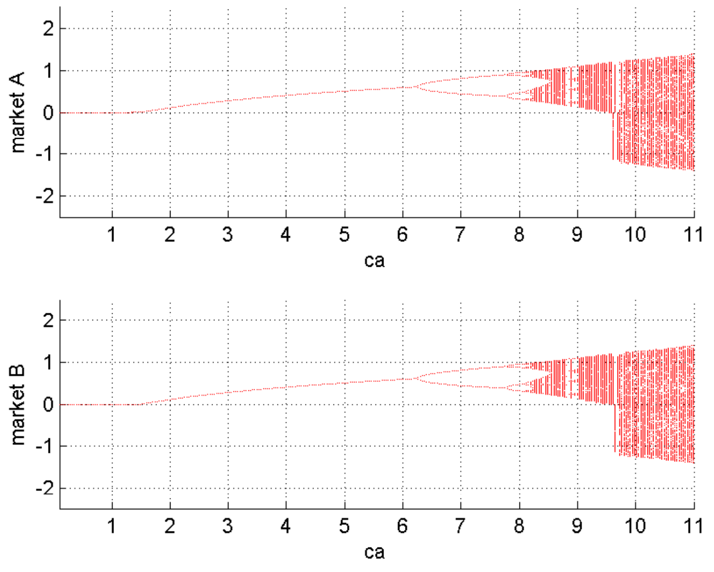
Steady state	$W^f_{AA,t} = W^f_{AB,t}$	$W^c_{AA,t} = W^c_{AB,t}$	$W^f_{BA,t} = W^f_{BB,t}$	$W^c_{BA,t} = W^c_{BB,t}$	Condition
Fundamental	1	h	1	h	$\frac{f_A+f_B}{c_A+c_B} > h$
Nonfundamental	1	$\frac{f_A+f_B}{c_A+c_B}$	1	$\frac{f_A+f_B}{c_A+c_B}$	$\frac{f_A+f_B}{c_A+c_B} < h$
Mixed	1	h	$\sqrt{\frac{h\,(c_A+c_B)}{f_A+f_B}}$	$\sqrt{\frac{h\,(f_A+f_B)}{c_A+c_B}}$	$\frac{f_A+f_B}{c_A+c_B} < h$

Further increasing the chartist strength will violate the stability condition; the system then experiences a series of state transitions such as periodic, quasi-periodic and even chaos. These facts can be numerically illustrated. To preserve continuity and unity, a *default parameter set* ($a_A = a_B = 2.131$, $c_A = 5$, $c_B = 0.5$, $f_A = f_B = 1$, $g_A = g_B = 0.2$ and $h = 1$) will be used through whole paper unless the parameters are specified.

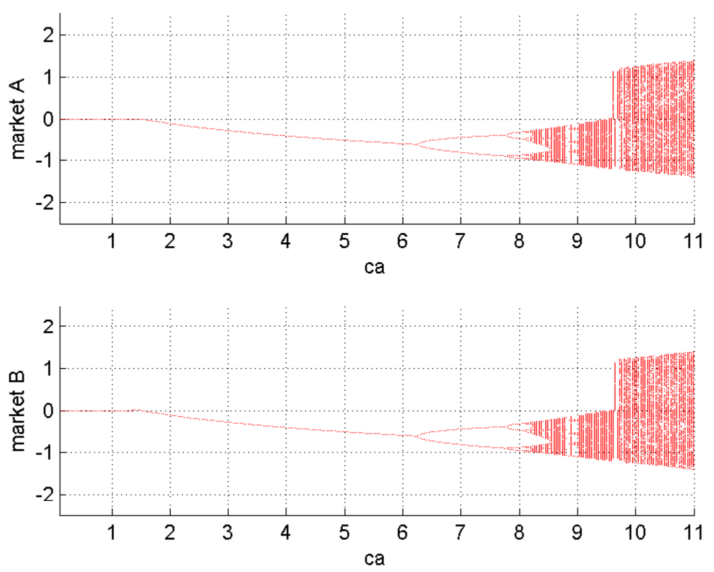
Figure 2 presents the bifurcation diagrams for the chartist strength from Market A, c_A, with positive and negative initial market values. Along with the increment of c_A, both markets experience a series of state transitions from fundamental steady state to chaos.

To understand the interactions of the two markets in chaos state, Figure 3 plots the strange attractor in $x_{B,t}$-$x_{A,t}$ space for $c_A = 10$. The strange attractor is symmetric at point $(x_{A,t}, x_{B,t}) = (0, 0)$.

Market pooling is the prominent effect emerging from market linkage. We define market pooling as a process in which investors from individual market interact with each other and determine the overall price movement of the market system. The stability of the market system is determined by the contrast between the overall strength of the two agent types.



(a) $x_{A,0} > 0$, $x_{B,0} > 0$



(b) $x_{A,0} < 0$, $x_{B,0} < 0$

Fig. 2. Bifurcation diagram of individual market after market connection with respect to bifurcation parameter c_A.

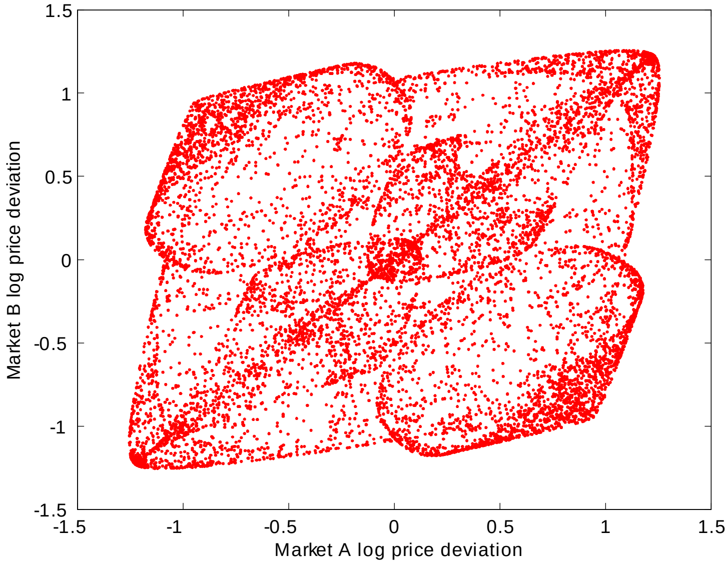
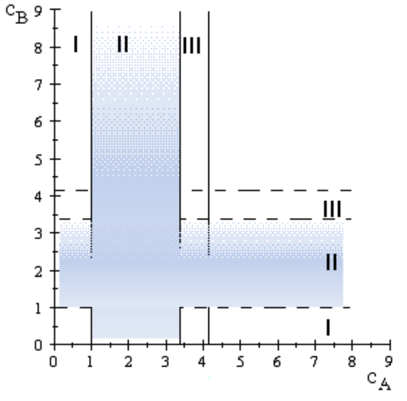
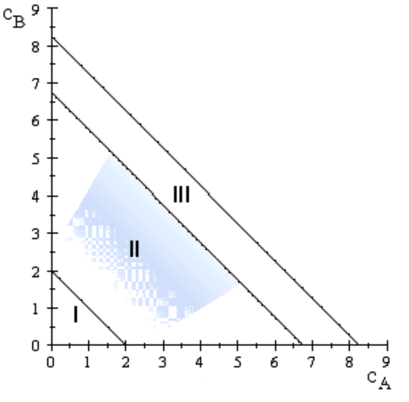


Fig. 3. Strange attractors symmetric at $(x_{A,t}, x_{B,t}) = (0, 0)$.

This implies that *the stable isolated market may be destabilized* and the initially unstable isolated market may be stabilized if the resultant market system is stable. In this pooling process, individual market's intrinsic characteristics may be overwritten. Chartist strength impact c_B vs c_A illustrates this pooling effect in Figure 4 using conditions of default parameters. In isolation, each market falls in fundamental (I), nonfundamental (II) and two-period (III) states, independent of the other market. c_B and c_A are independent of each other. For example, markets A and B are in fundamental steady state as long as the corresponding enhanced chartist strength is less than fundamentalist strength. Once market linkage is setup, c_B depends on c_A. Regions of market state changes. For illustration, at point $(c_A, c_B) = (0.5, 4)$, markets A and B are in fundamental steady state and two-period orbit state, respectively, when they are isolated. After market linkage is formed, the market system transforms into nonfundamental steady state. That is, both markets are in the nonfundamental steady state. In addition, mixed steady state of fundamental and nonfundamental market members does not exist in the figure.



(a) Isolated markets



(b) Linked markets

Fig. 4. Impact of chartist strength: c_B vs c_A.

To demonstrate this pooling effect, bifurcation diagrams of the individual market of the market system are compared with the isolation counterparts. The common bifurcation parameter is the chartist strength of Market A, c_A, given the conditions of default parameters. In Figure 5, blue color bifurcation diagrams represent the case of market isolation while red color ones are for the case of market linkage. Given the intrinsic condition $c_B = 0.5$ and $f_B = 1$, Market B is in the fundamental steady state with zero deviation and is not impacted by the changes of chartist strength in Market A when it is isolated. Translated into the bifurcation diagram, Market B is a zero value horizontal line while Market A exhibits the isolation bifurcation diagram as in Figure 1.

Once market linkage is established, bifurcation diagram of Market A is more stable compared to its isolation counterpart. Interval of fundamental steady state of Market A is extended; similar interval extensions can be observed in nonfundamental steady states and other states. Also, the deviation magnitudes of the bifurcation curve have been reduced. While for Market B, it is not longer non-reactive to the change of chartist strength of Market A. It experiences the similar bifurcation state transitions with Market A. Market B is destabilized and transits from the fundamental steady state to the nonfundamental steady state at $c_A = 1.5$.

In a short summary, the market that is more stable initially has a stabilizing effect on the market system after market linkage while itself is subject to destabilizing effect from the less stable one.

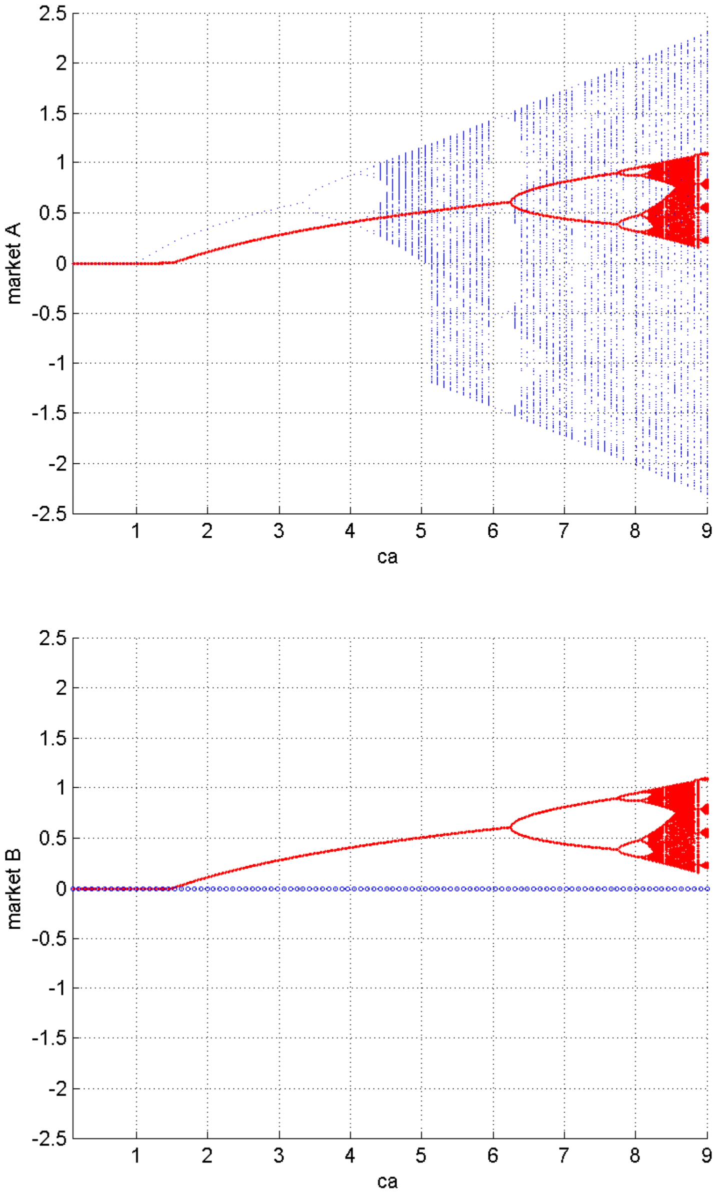


Fig. 5. Bifurcation diagram comparison between isolated market and market linkage with c_A as bifurcation parameter ($c_B = 0.5$).

To further illustrate the market pooling effect, we use another example with the same conditions except $c_B = 1.5$. Bifurcation diagrams of isolation and linkage cases are compared in Figure 6. Contrasting to the case $c_B = 0.5$, where each market seems to experience only stabilizing or destabilizing effect from market linkage, when $c_B = 1.5$, each market experiences both stabilizing and destabilizing effects from market linkage. For Market A, it is destabilized from the fundamental steady state into the nonfundamental steady state at

$c_A = 0.5$ and experiences the stabilizing effect from $c_A = 1.5$ onwards. While for Market B, it is in the nonfundamental steady state when it is isolated. With market linkage, it experiences stabilizing effect before c_A reaches value 1.5; when c_A is larger than 1.5, Market B suffers destabilizing effect. Again, an initially relative stable market will apply stabilizing effect on the market system and suffer destabilizing effect after market linkage is established. It is analogy to the zero-sum game where one market experiencing stabilizing effect from market linkage implies the other market suffering from destabilizing effect.

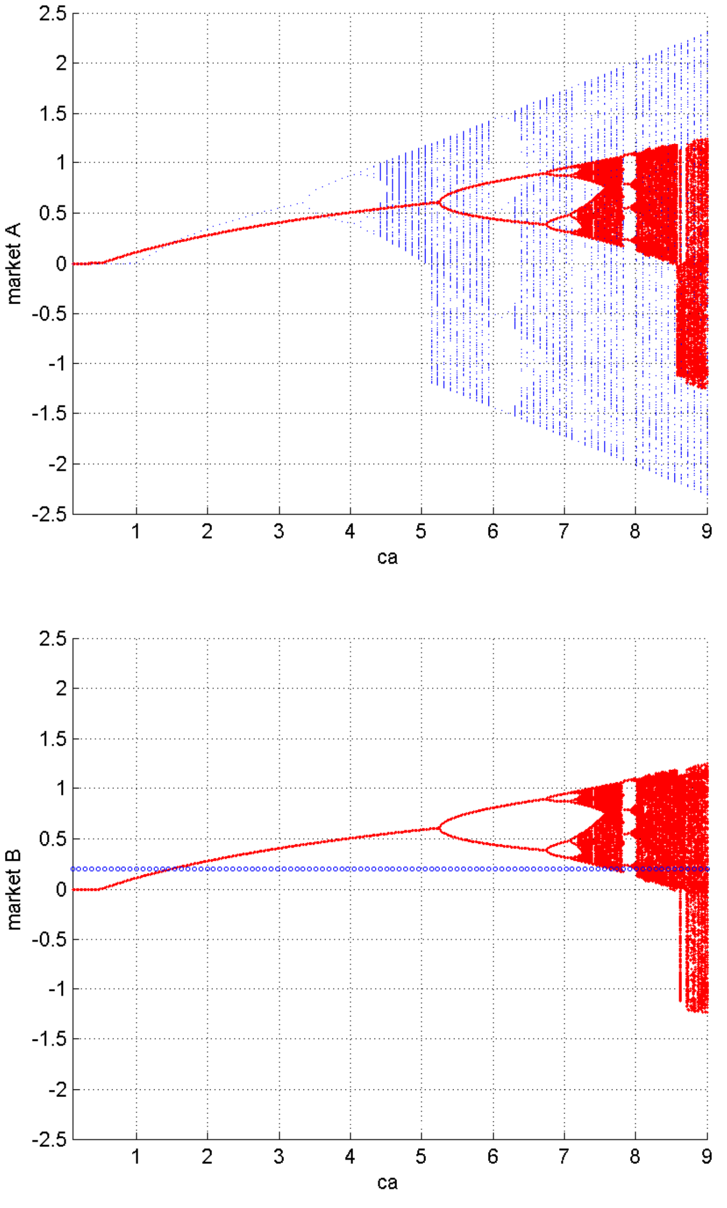


Fig. 6. Bifurcation diagram comparison between isolated market and market linkage with c_A as bifurcation parameter ($c_B = 1.5$).

4. STYLIZED FACTS

Next we conduct simulations to check the capability of our market system to match the statistical properties of real speculative financial markets.

4.1. Typical stylized facts

As per Cont (2001), T. Lux (2002) and Westerhoff and Dieci (2006), real world speculative markets have following characteristics: (1) prices have randomly switching bearish and bullish episodes; (2) volatility cluster phenomena are observed in which high-volatility events tend to cluster in time; (3) the distribution of returns has fat tails; (4) daily return autocorrelation tends to be insignificant; (5) absolute daily returns exhibit strong autocorrelation. Besides the above stylized fact for single market, empirical studies already show the existence of cross-correlation between markets. Egert and Kocenda (2011) find strong correlation among returns of Germany, France and UK. The correlation can be even up to 0.9.

4.2. Calibration results

According to T. Lux (2002), return r can be defined as log price changes. Here we have

$$r_{i,t} = P_{i,t} - P_{i,t-1},$$

where i stands for Market A or B.

We use parameters setting ($a_A = a_B = 1$, $c_A = 41.325$, $c_B = 18$, $f_A = f_B = 1$, $F_A = F_B = 0$, $g_A = g_B = 0.465$, $h = 25$, $x_{A,0} = 0.21$, and $x_{B,0} = 0.2$) to conduct the simulation. 15,000 observations of each market are generated. For clearer visualization, log price movements of last 300 time steps are plotted to check the randomly switching market episodes. For the same 300 time steps, return time series are plotted to demonstrate the volatility clustering phenomena.

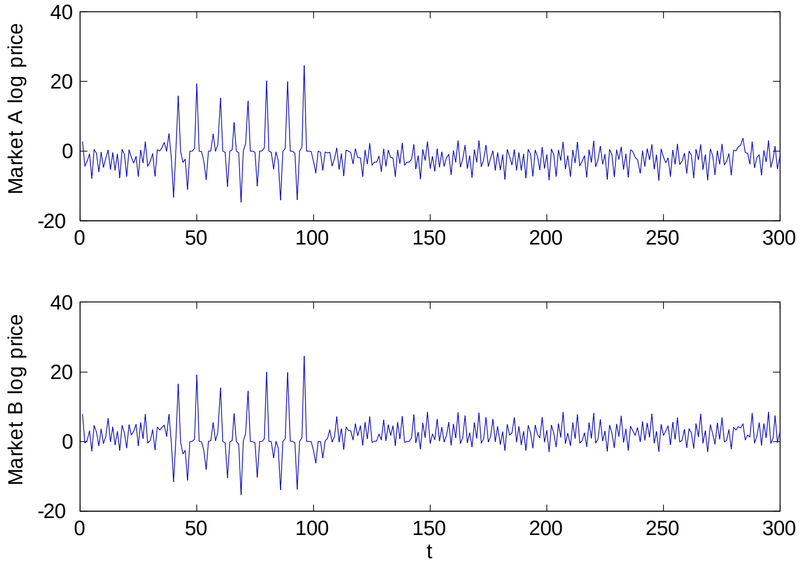
Figure 7 plots log price and return trajectories. Log price trajectories demonstrate features of randomly switching of bullish and bearish market episodes. Most of the time, the two markets have the same price movement, especially in a period with large volatility. This period of large volatility corresponds to the volatility clustering which is observed in the return trajectories.

Figure 8 compares the return distribution to the normal one for each market member. Fat tails are observed from the return distribution (Kurtosis of markets A and B are 4.6159 and 4.6155, respectively.), which indicates there are more extreme returns compared to the normal distribution.

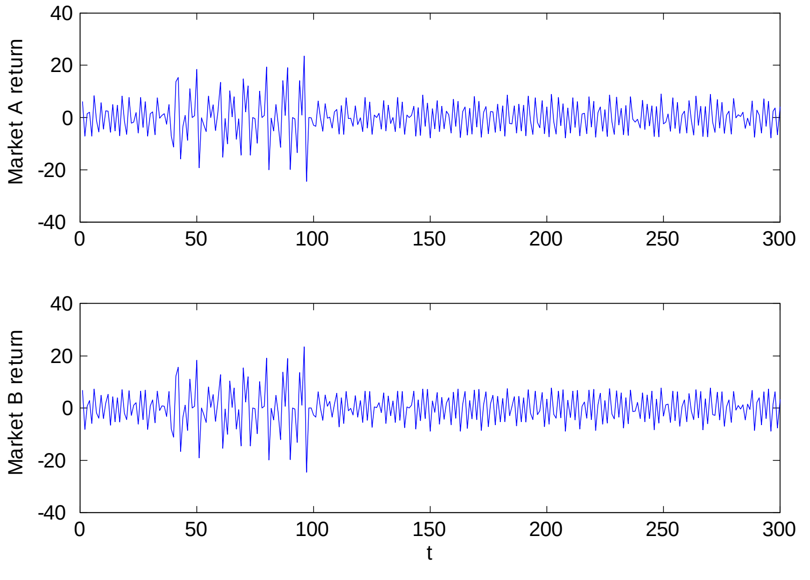
Figure 9 investigates the behavior of autocorrelation. For both market members, autocorrelations of returns tend to be insignificant across lags while the ones of absolute returns are significant and decrease slowly with lags.

Figure 10 examines cross-correlation of return between the two market members. At 95% confidence interval, cross-correlation exists across lags, especially for lag zero. The strong cross-correlation (value = 1) at lag zero explains the co-movement of the log price trajectories.

The above stylized facts demonstrate our model's capability to generate some most important stylized facts observed in financial markets and those related to individual market are similar to the results calibrated by Zhu et al. (2009).

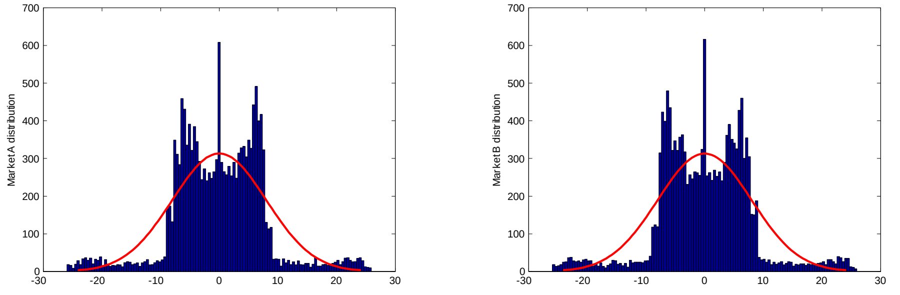


(a) Log price trajectory



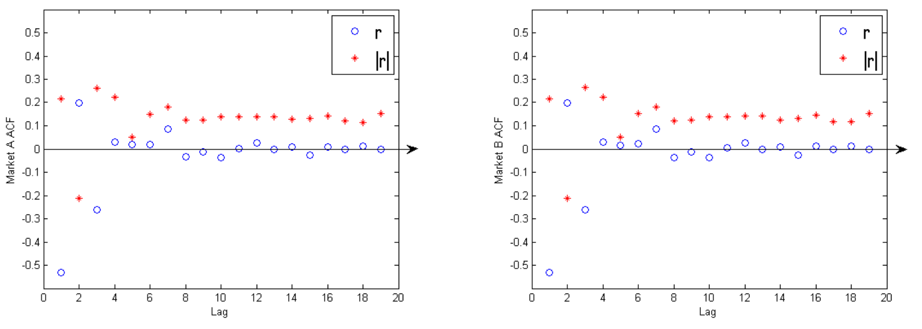
(b) Return trajectory

Fig. 7. Trajectories of last 300 steps.



(a) Market A (b) Market B

Fig. 8. "Fat tails" of returns distribution.



(a) Market A (b) Market B

Fig. 9. Autocorrelations vs lags.

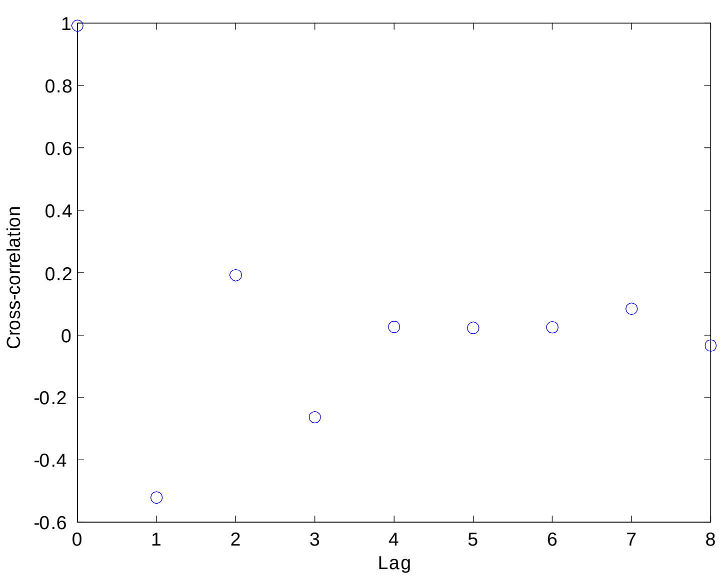


Fig. 10. Cross-correlation vs lags.

5. CONCLUSION

A two-market heterogeneous agents model is developed in this paper. Each market has a market maker and two types of agents: chartists and fundamentalists. Agents of the same type are inhomogeneous across markets. Market linkage is established by allowing investors to invest in each market. Aware of capital movement of the investors, individual market maker updates price for her market by weighing excess demands of both markets to take account of the impact from the other market. Stationary states of the market system and the corresponding stability conditions are derived. It is proven that mixed steady state, where one market member falls into a fundamental steady state while the other into a nonfundamental steady state, is unstable.

One of the important consequences of market linkage is market pooling. In the process of market pooling, investors from each market determine the asset prices of the whole market system. In addition, individual market's intrinsic properties may be overwritten. The market that is more stable initially in isolation will exert stabilizing effect on the market system while itself is subject to destabilizing effect from the resultant market system. The market pooling can provide policy implication for financial market opening. In a world consisting of a small market and a large market (or market agglomeration), if the small market is stable compared to the large market, market opening of the small market will destabilize itself. Small market will benefit from market opening only if it is unstable compared to the large market. This example indicates that market opening is a double-edged sword. Decision of market opening should base on the impact assessment on internal and external markets.

Lastly, numerical simulations demonstrate the model capability to generate some of the stylized facts of speculative financial market, especially the cross-correlation between markets. To our best knowledge, this is the first HAM model that is capable of generating the significant cross-correlation effect.

Future researches will be targeted to extend this model to cases more than two markets so that the impacts of financial market liberalization can be examined. Moreover, empirical verification and application of market pooling need further exploration.

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