



Economic Growth Centre Working Paper Series

Global Yield Curves and Sovereign Bond Market Integration

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Working Paper No: 2009/02

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Second Version: January 2009

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Abstract

We extract global yield curve factors based on the affine arbitrage-free dynamic Nelson-Siegel model. The measure of integration proposed in the paper allows time-varying partial segmentation of national and global government bond markets. It takes into account the maturity structure of yields, therefore it is consistent in time series and cross-section as well. Though global factors and country-specific factors are highly correlated, the international bond market is less integrated than one might expected based on correlation analysis or prior knowledge of investment restrictions. The difference stems from 1) the integration asymmetry of factors: level factor is more integrated than slope and curvature factors ; 2) heterogeneous factors dynamics: one factor's integration may accompany the segmentation of other factors. Yet the expected integration is stable over the last two decades.

1 Introduction

Is the global government bond market integration time-varying? What are the impediments to government bond market integration after financial deregulation? Understanding the dynamic evolution of bond market integration is important for investors and policy makers for a variety of reasons, such as diversifying the risk in the world market, reducing the cost of capital, forecasting the future macroeconomic dynamics and interest rates, pricing the interest rate derivatives, making monetary policy and fiscal debt policy. The endeavor of understanding the yield curve dynamics has produced a large literature and a lot of models. However little attention has been directed to the integration dynamics of world government bond market. The difficulty associated with defining and measuring the integration is one reason. Another possible reason is bond markets are expected to be closely integrated. A few studies have focused on looking for the driving forces of the comovements (Engsted and Tanggaard (2007), Ilmanen (1995), Sutton(2000)).

There are many reasons in favor of a closely integrated global bond market, for instance, big institution age, financial deregulation, financial networks , free capital flow, among many others. Nevertheless, given the impediments in the global bond markets, we can't argue a priori that the global bond market is completely integrated. Home bias might exist in the bond market because of the information asymmetry about the real activity (Barr and Priestley 2004, hereafter BP). The hedging strategy of institutions with liabilities denominated in domestic currency is usually to manage domestic bond portfolio instead of global portfolio. Tax treatment difference provides one more reason for global bond market segmentation. Local currency denominated government bonds also constitutes a reason. In addition, liquidity and exchange rate

risk play a significant role in accounting for the failure of complete integration.

In the literature, the investigation of international linkages of yield curves can be broadly classified in three categories. The first line is the testing of uncovered interest rate parity (UIRP). The market efficiency hypothesis of UIRP is too restrictive and the testing just tells the failure or success of the null hypotheses, but the dichotomy doesn't show how well the model fits as an approximation and the probable time-varying dynamics of bond market integration. Furthermore the focus of the test is on the long-term interest rates parity differentials, so these studies neglect the information contained in the maturity structure of yield curves. As is well-known, the maturity spread helps forecast the real economic activity and interest rates (Ang, Piazzesi, and Wei (2006), Campbell and Shiller (1991), Diebold, Rudebusch, and Aruoba(2006), Estrella and Hardouvelis (1991), Hamilton and Kim (2002)).

The second line uses the one-factor asset pricing framework. The model is theoretically consistent and has firm micro-foundations. However, the one-factor model is difficult to interpret and the omitted factors might change the degree and trend of the market integration. Empirical econometric analysis constitutes the third line, but these models are not theoretically rigorous. Just like the first line literature, the second and third line studies don't pay much attention to the cross section of yield curves that are important for bond portfolio management and economic forecasting. Although the maturity structure is not considered, Sutton (2000) tries to relate the comovement of long-term and short-term yields by the expectation hypothesis of the term structure.

In this paper we apply the affine arbitrage-free dynamic Nelson-Siegel (1987) model (AFDNS) (Christensen, Diebold and Rudebusch(2007)), hereafter CDR) to modeling yield curves. The Nelson-Siegel model has good performance in fitting

maturity structure of yield curves, and it is extensively employed by financial institutions and central banks. Diebold and Li (2006) generalized the model to a dynamic specification that is consistent with the main stylized facts of yield curves. Forecasts based on the dynamic model are satisfactory, and three factors of the model, respectively level, slope and curvature, have close interactions with macroeconomic fundamentals (Diebold, Rudebusch and Aruoba (2006), Tam and Yu (2008)). CDR developed the no-arbitrage Nelson-Siegel model, the affine specification make it also a general equilibrium model (Duffie (2001), Chapter 10, Piazzesi (2003)). The price of risk is determined by the marginal utility process that links the risk-neutral measure and data-generating measure. The AFDNS model shows theoretical consistency and inherits the empirical fit of the Nelson-Siegel model.

There is strong evidence of cross-country bond market interactions (Barr and Priestley (2004), Diebold, Li and Yue (2007), DLY hereafter). In addition to the idiosyncratic factors, there are global yield curve factors driving the bond market in each individual country. We use latent factors for Germany, Japan, the U.K. and the U.S. to extract the global latent factors with the Kalman filter. As expected, the world factors are highly correlated with the country-specific factors by DCC-GARCH (Engle 2001) analysis. The idiosyncratic factors are given by the difference of the country-specific factors and the global factors.

Defining market integration is clearly challenging. There is no consensus. The bond market integration is defined here as movements in world factors determining movements in interest rates. This measure of integration is consistent with the uncovered interest rate parity if the expected exchange rate change is a martingale process, therefore uncovered interest rate parity is a polar case of our model. Following the idea of Bekaert and Harvey (1995), our specification, the Markov-switching

Nelson-Siegel model, allows time-varying segmentation of world bond markets, it hence circumvents the polar cases of complete segmented or integrated market and fixed integration. The interactions of global bond markets in this framework are more complicated because of the asymmetry and heterogeneity of three factors. The macroeconomic interpretation of factors hints at the potential impediments in the economic fundamentals. Our conjecture is the segmentation comes from the dynamics of the real economy rather than the nominal dynamics.

The integration is very volatile with our measure. It switches frequently between perfect integration and complete segmentation. This is no surprise since our measure requires markets to fluctuate together, and the first moment is employed for measurement purpose. In contrast, in the international CAPM model the second moment (volatility) is used to interpret the difference in expected return. From the perspective of making investment decisions and policy ex ante, the predicted integration is more of our interest. Therefore we suggest applying the expected integration as the measure instead of the filtered or smoothed integration of the Markov-switching Nelson-Siegel model. It also avoids the volatile situation. Our finding is interesting based on the expected integration measure. The relatively stable integration is consistent with results in BP. But our measure shows lower integration than in BP due to the asymmetry and heterogeneity of factors that are not captured by models not taking into account the maturity structure.

The article proceeds as follows. In section 2, the AFDNS model is presented and the empirical results of country local factors are reported. The global yield curve model is specified in section 3 and the global factors are extracted, then the nature of global factors and country factors are analyzed. In section 4, we present the Markov-switching Nelson-Siegel model for measuring the integration degree of the

bond markets, results are interpreted, and the potential impediments are considered. The final section offers some concluding remarks and some conjectures deserving further exploration.

2 Country-specific yield curve factors

2.1 Affine arbitrage-free dynamic Nelson-Siegel model

Most of term structure models use three factors to capture stylized facts of yields in cross-section and time series. By properly restricting the factor loadings in the statistical factor model, Diebold and Li (2006) proposed the dynamic Nelson-Siegel model:

$$y_{t(\tau)} = l_t + s_t \left(\frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} \right) + c_t \left(\frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} - e^{-\lambda_t \tau} \right) + \varepsilon_t \quad (1)$$

where the l_t is level factor, the s_t denotes slope factor and the c_t represents curvature factor. Empirically, the level factor is corresponding to the long-term interest rates, the slope factor is associated with the difference between the short-term yield and long-term yield, the curvature factor corresponds to two times of medium-term yields minus the sum of long- and short-term yields. Therefore, the level factor is a long-term factor, the slope factor is a short-term factor and the curvature is a medium-term factor. Three factors contain information of the macroeconomic dynamics and vice versa (Diebold, Rudebusch and Aruoba(2006), Tam and Yu(2008)). The λ_t is the rate of changes of factors loadings along the maturity horizons, it also determines the maturity at which the curvature loading achieves its maximum.

For the entire yield curve with different maturities (τ) at time t , the model can

be specified as:

$$\begin{pmatrix} y_{t(\tau_1)} \\ y_{t(\tau_2)} \\ \vdots \\ y_{t(\tau_N)} \end{pmatrix} = \begin{pmatrix} 1 & \frac{1-e^{-\lambda_t \tau_1}}{\lambda_t \tau_1} & \frac{1-e^{-\lambda_t \tau_1}}{\lambda_t \tau_1} - e^{-\lambda_t \tau_1} \\ 1 & \frac{1-e^{-\lambda_t \tau_2}}{\lambda_t \tau_2} & \frac{1-e^{-\lambda_t \tau_2}}{\lambda_t \tau_2} - e^{-\lambda_t \tau_2} \\ \vdots & \vdots & \vdots \\ 1 & \frac{1-e^{-\lambda_t \tau_N}}{\lambda_t \tau_N} & \frac{1-e^{-\lambda_t \tau_N}}{\lambda_t \tau_N} - e^{-\lambda_t \tau_N} \end{pmatrix} \begin{pmatrix} l_t \\ s_t \\ c_t \end{pmatrix} + \begin{pmatrix} \varepsilon_{t(\tau_1)} \\ \varepsilon_{t(\tau_2)} \\ \vdots \\ \varepsilon_{t(\tau_N)} \end{pmatrix}$$

The dynamic Nelson-siegel has superior out-of-sample forecasting performance, especiall at long horizon. In constrast, the affine term structure models, which is important for pricing interest rate derivatives, forecast poorly (Duffee, 2002). Although the dynamic Nelson-Siegel has the advantage in forecasting, it is neither general equilibrium model nor no-arbitrage model, hence theoretically inconsistent.

To achieve the theoretical rigor and keep the fit of the model, CDR develop the affine abritage-free class of Nelson-Siegel models. The three-dimension state variables X_t is assumed to be given by a stochastic differential equation (SDE)

$$dX_t = K^Q(t)[\theta^Q(t) - X_t]dt + \Sigma_t \begin{pmatrix} \sqrt{\gamma^1(t) + \delta^1(t)X_t} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \sqrt{\gamma^3(t) + \delta^3(t)X_t} \end{pmatrix} dW_t^Q$$

where Q is risk neutral-measure, W_t^Q is the standard Brownian motions in R^3 under measure Q . The relationship between data-generating measure P and risk-neutral measure Q is given by the follwoing equation:

$$dW_t^Q = dW_t^P + \Gamma_t dt$$

with

$$\Gamma_t = \begin{pmatrix} \gamma_1^0 \\ \gamma_2^0 \\ \gamma_3^0 \end{pmatrix} + \begin{pmatrix} \gamma_{11}^1 & \gamma_{12}^1 & \gamma_{13}^1 \\ \gamma_{21}^1 & \gamma_{22}^1 & \gamma_{23}^1 \\ \gamma_{31}^1 & \gamma_{32}^1 & \gamma_{33}^1 \end{pmatrix} \begin{pmatrix} X_t^1 \\ X_t^2 \\ X_t^3 \end{pmatrix}$$

then the SDE for the state variables X_t under data-generating measure P is given as follows:

$$dX_t = K^P(t)[\theta^P(t) - X_t]dt + \Sigma_t dW_t^P$$

This specification preserves the affine dynamics under data-generating measure. The connection between two measures determines the price of the risk. The above equation is corresponding to the $A_0(3)$ model in Dai and Singleton (2000), this type of model make the volatility of state variables independent of the state variables. With two ingredients of the affine term structure models at hand, the third ingredient, the risk-free short interest rate is given by

$$r_t = \delta_0^X + (\delta_1^X)' X_t \tag{2}$$

CDR follows Duffie and Kan (1996) framework where the zero-coupon bond prices are exponential-affine functions of the state variables, with appropriate assumptions in SDE of X_t specified in CDR, the zero-coupon bond yields are given by

$$y_{t(\tau)} = X_t^1 + \frac{1 - e^{-\lambda\tau}}{\lambda\tau} X_t^2 + \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right) X_t^3 - \frac{C(\tau)}{\tau}$$

Where (X_t^1, X_t^2, X_t^3) is the state variables vector corresponding the (l_t, s_t, c_t) in dynamic Nelson-Siegel model. This is the closest affine no-arbitrage approximate to the dynamic Nelson-Siegel model as discussed in CDR. The difference between two types models is the yield-adjustment term $-\frac{C(\tau)}{\tau}$, for the reason of identification, the volatility matrix is assumed to be triangular

$$\begin{pmatrix} \sigma_{11} & 0 & 0 \\ \sigma_{21} & \sigma_{22} & 0 \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix}$$

then the yield-adjustment term is given by

$$\begin{aligned} -\frac{C(\tau)}{\tau} = & -\sigma_{11} \frac{\tau^2}{6} - (\sigma_{21}^2 + \sigma_{22}^2) \left[\frac{1}{2\lambda^2} - \frac{1}{\lambda^3} \frac{1-e^{-\lambda\tau}}{\tau} + \frac{1}{4\lambda^3} \frac{1-e^{-2\lambda\tau}}{\tau} \right] \\ & -(\sigma_{31}^2 + \sigma_{32}^2 + \sigma_{33}^2) \times \left[\frac{1}{2\lambda^2} + \frac{1}{\lambda^2} e^{-\lambda\tau} - \frac{1}{4\lambda} \tau e^{-2\lambda\tau} \right. \\ & \quad \left. - \frac{3}{4\lambda^2} e^{-2\lambda\tau} - \frac{2}{\lambda^3} \frac{1-e^{-\lambda\tau}}{\tau} + \frac{5}{8\lambda^3} \frac{1-e^{-2\lambda\tau}}{\tau} \right] \\ & -\sigma_{11}\sigma_{21} \left[\frac{1}{2\lambda} \tau + \frac{1}{\lambda^2} e^{-\lambda\tau} - \frac{1}{\lambda^3} \frac{1-e^{-\lambda\tau}}{\tau} \right] \\ & -\sigma_{11}\sigma_{31} \left[\frac{1}{2\lambda} \tau + \frac{3}{\lambda^2} e^{-\lambda\tau} + \frac{1}{\lambda} \tau e^{-\lambda\tau} - \frac{3}{\lambda^3} \frac{1-e^{-\lambda\tau}}{\tau} \right] \\ & -(\sigma_{21}\sigma_{31} + \sigma_{22}\sigma_{32}) \left[\frac{1}{\lambda^2} + \frac{1}{\lambda^2} e^{-\lambda\tau} - \frac{1}{2\lambda^2} e^{-2\lambda\tau} \right. \\ & \quad \left. - \frac{3}{\lambda^3} \frac{1-e^{-\lambda\tau}}{\tau} + \frac{3}{4\lambda^3} \frac{1-e^{-2\lambda\tau}}{\tau} \right] \end{aligned} \tag{3}$$

The AFDNS model has natural state-space form, with the observation equation given by

$$\begin{pmatrix} y_{t(\tau_1)} \\ y_{t(\tau_2)} \\ \vdots \\ y_{t(\tau_N)} \end{pmatrix} = \begin{pmatrix} 1 & \frac{1-e^{-\lambda_t \tau_1}}{\lambda_t \tau_1} & \frac{1-e^{-\lambda_t \tau_1}}{\lambda_t \tau_1} e^{-\lambda_t \tau_1} \\ 1 & \frac{1-e^{-\lambda_t \tau_2}}{\lambda_t \tau_2} & \frac{1-e^{-\lambda_t \tau_2}}{\lambda_t \tau_2} e^{-\lambda_t \tau_2} \\ \vdots & \vdots & \vdots \\ 1 & \frac{1-e^{-\lambda_t \tau_N}}{\lambda_t \tau_N} & \frac{1-e^{-\lambda_t \tau_N}}{\lambda_t \tau_N} e^{-\lambda_t \tau_N} \end{pmatrix} \begin{pmatrix} X_t^1 \\ X_t^2 \\ X_t^3 \end{pmatrix} \\
- \begin{pmatrix} \frac{C(\tau_1)}{\tau_1} \\ \frac{C(\tau_2)}{\tau_2} \\ \vdots \\ \frac{C(\tau_N)}{\tau_N} \end{pmatrix} + \begin{pmatrix} \varepsilon_{t(\tau_1)} \\ \varepsilon_{t(\tau_2)} \\ \vdots \\ \varepsilon_{t(\tau_N)} \end{pmatrix} \quad (4)$$

and the state equation is

$$\begin{pmatrix} dX_t^1 \\ dX_t^2 \\ dX_t^3 \end{pmatrix} = \begin{pmatrix} \kappa_{11}^P & 0 & 0 \\ 0 & \kappa_{22}^P & 0 \\ 0 & 0 & \kappa_{33}^P \end{pmatrix} \left[\begin{pmatrix} \theta_1^P \\ \theta_2^P \\ \theta_3^P \end{pmatrix} - \begin{pmatrix} X_t^1 \\ X_t^2 \\ X_t^3 \end{pmatrix} \right] dt \\
+ \begin{pmatrix} \sigma_{11} & 0 & 0 \\ 0 & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} dW_t^{1,P} \\ dW_t^{2,P} \\ dW_t^{3,P} \end{pmatrix} \quad (5)$$

The state equation is the independent-factor AFDNS model in CDR. The independent specification facilitates the extraction of the global yield curve factors, as will be clear in section 3. The forecasting performance of independent model is not worse than the correlated-factor AFDNS model as indicated in CDR. The continuous model estimation is complicated, we can discretize the continuous state equation specifcaiton to have discrete-time state equation under P -measure:

$$X_t = (I - \exp(-K^P \Delta t_i))\mu^P + \exp(-K^P \Delta t_i)X_{t-1} + \eta_t$$

where Δt_i is the time span between two observations. To estimate the system, the Kalman filter is an efficient and consistent means.

The AFDNS model rules out the arbitrage opportunities in the financial market, in the big institutions age arbitrage rarely exist in the modern well-organized markets. At the same time, the model has the good fitness of dynamic Nelson-Siegel model empirically. Therefore, the AFDNS is theoretically rigorous and empirically appealing. Given the short interest rate given in equation (2), and the short interest rate is lower bounded by zero, then the model can be a general equilibrium (Duffie(2001)). Hence, the model is not only a no-arbitrage model but also a general equilibrium model. It is a fascinating advantage.

2.2 Data and summary statistics

The U.S. data consist of end-of-month observations of 1, 3, 6, 12, 24, 36, 60, 84, 120 months zero-coupon yields on treasury securities covering the period from January 1985 to March 2008. The data source is econstatsTM. The U.K. zero-coupon yields with maturities of 6, 9, 10, 11, 12, 24, 36, 48, 60, 72, 84, 96, 108, 120 months are retrieved from econstatsTM. It covers the same period as the U.S sample and all data are end-of-month observations. For the Germany zero-coupon government bond yields, the month-end observations with maturities 12, 24, 36, 48, 60, 72, 84, 96, 108, 120 months are retrieved from Deutsche Bundesbank, the central bank of Germany.

The Japanese dataset has two sources. The first sample covering the period

from January 1985 to December 1991 is from the Key Economic Statistics Files of the PACAP Database-JapanTM compiled by the Sandra Ann Morsilli Pacific-Basin Capital Markets Research Center at the University of Rhode Island. The end-of-month yields consist of government bond interest rates with maturities 12, 24, 36, 60, 84, 120 months. The second sample covers the period from January 1992 to March 2008. The dataset is downloaded from Bloomberg. The maturities are 6, 12, 24, 36, 48, 60, 72, 84, 96, 108, 120 months.

The summary statistics including skewness and kurtosis of yields for each maturity and for each country is presented in Table 1¹. The 3D plot of term structure of interest rates for each country is graphed in Figure 1. One stylized fact of interest rates is they tend to exhibit considerable persistence and are believed to be nonstationary or better approximated by the integrated process. This feature has profound implications for estimation and statistical inference.

The autocorrelation coefficients and augmented Dickey-Fuller tests in Table 1 provide evidence of persistence and non-stationarity². However, the yields are usually cointegrated, as implied by the rational expectation hypothesis. The Johansen cointegration analysis presents evidence of common trends in yields³. The cointegration may explain another important stylized fact of the yield curve: spreads are less persistent than yields. The skewness and kurtosis show yields don't deviate considerably from the normal distribution. The standard deviations in Table 1 tells us that short-term yields usually are more volatile than long-term yields with the exception of Japan. In Figure 9, we plot average yield curves, for Germany, Japan and U.S.,

¹The statistics for Japan are based on the sample retrieved from Bloomberg. This applies to results for the DNS estimates in Table 2 and the AFDNS estimates in Table 4.

²The ADF test rejects the unit root in the 6-month yield of Japan. Because of the low power of the ADF test, we also conduct the PP test, KPSS test, Ng-Perron tests, the results are mixing.

³The analysis results are available upon request.

the average yield curves are upward-sloping for the time period under analysis, in contrast, the U.K. average yield curve has S-shape.

Table 1 About Here

Figure 1 About Here

2.3 Country local factors

Non-linear least squares can be employed to estimate the dynamic Nelson-Siegel model in the first equation of section 1.4.2 in chapter 1. In Diebold and Li (2006), they fix the λ_t and set it equal to the value that maximizes the loading on the curvature factor at 30 months. In so doing, one can estimate the DNS model by ordinary least squares and make the numerical optimization more reliable. We follow this approach to estimate the DNS model and results are presented in Table 2 with λ_t fixed at value of 0.0600. The DNS model is capable of replicating a variety of yield curve shapes. Both the three-factors model and two-factors model (without curvature factor) fit yield curves across countries well, but three-factors model has higher explanatory power according to the average R^2 .

Table 2 About Here

In order to estimate the AFDNS model, we fix the λ_t at 0.0600. The Kalman filter estimation is initialized by using the unconditional covariance matrix of the state vector and mean vector from the DNS estimates. The starting transition matrix parameters are also from OLS regression of factors extracted by the DNS model. Yield adjustment term⁴ of the AFDNS model bridges the connection between the

⁴The last item on the right side of equation (49), chapter 2.

state variables dynamics under data-generating measure and the volatility matrix Σ under risk-neutral measure. This makes the AFDNS model theoretically consistent and hence differs from the DNS model. The conditional mean-reversion matrix ($\exp(-K^P \Delta t_i)$) determines the mean-reverting rate of state variables. We present the mean-reversion matrix and yield adjustment terms in Table 3. In CDR, the yield adjustment terms are trivial for all maturities. However, in our estimation, the yield adjustment terms is significant for long-term interest rates. This may come from the first item on the right hand side of the last equation in section 1.4.2.

Table 3 About Here

The extracted level, slope and curvature factors across countries are plotted in figure 2-4, respectively. For the purpose of comparison, factors from the DNS model and AFDNS model are depicted together in figure 5 and figure 6 for each country. The dynamics of factors from the AFDNS model mimic the dynamics of factors from the DNS model, the small difference may be induced by yield adjustment terms. The level factor dynamic is homogenous across countries. In contrast, the curvature evolution over time is heterogeneous across country. The principal component analysis in section 3.2.3 supports the conclusion.

Figure 2 About Here

Figure 3 About Here

Figure 4 About Here

Figure 5 About Here

Figure 6 About Here

The summary statistics of factors from AFDNS model is presented in Table 4. The level factor is more persistent than the slope and curvature factors. The ADF

tests show curvature factors across counties may be stationary except Germany, but level factors are nonstationary except U.S. level factor. As to the volatility feature, the level factor can be more or less volatile than the slope and curvature factors. We note that the slope factors are more correlated with curvature factors than level factors. Although the Pearson and likelihood ratio tests reject the independence of level factor and slope factor, in empirical macro-finance model (Diebold, Rudebusch and Aruoba (2006), Tam and Yu (2008)), one factor contains little extra information about other factors or macro-variables. Diebold and Li (2006) provides the empirical evidence of the interpretation of level, slope and curvature factors as long-term, short-term and medium term factors. Figure 7 plots the 10-year yield, 3-month yield minus 10-year yield, and two times 2-year yield minus 10-year and 3-month yeilds with the level, slope and curvature factors⁵.

Table 4 About Here

Figure 7 About Here

The fit of the AFDNS model is good. The error terms of the estimation is plotted in figure 8. To faciliate the comparison, the scale of the figure is set to be the same as in figure 1. Figure 9 plots the average fitted yield curves along with the obseved yield curves. The AFDNS model replicates the upward sloping yield curves of Germany, Japan and the U.S. and the S-shape of the U.K.. It is important to note that the model fits the middle region of the yield curves better than the end regions. This might be a matter of fact of the model, as pointed out in Diebold and Li (2006): ".....because the maturities are not equally spaced, we implicitly weight the most

⁵For other countries, the 3-month yield is not available.

"active" region of the yield curve most heavily when fitting the model".

Figure 8 About here

Figure 9 About Here

3 Global yield curve factors

3.1 Model specification

A number of studies have focused on the international linkages of bond markets. There seems to be a consensus that the bond yields and returns are highly correlated across countries. Hafer et. al. (1997) found that long-term yields seem to be cointegrated across countries, hence there is comovement of international bond markets. Later, Sutton (2000) tried to relate the cointegration in long-term yields with comovement of short-term yields by rational expectation hypothesis. The conclusion is that the comovement of long-term yields coming out of the comovement in term premia. Ilmanen (1995) found that a small set of global instruments can forecast a significant fraction of monthly yields variation, and the author concludes that the predictability of global bond returns come from a few global factors. The empirical study of Driessen et. al. (2003) find that world bond markets are correlated by using a linear factor model and principal component analysis, the driving force of the comovement is the level of yields in each country, this is consistent with the matter of fact that the level factor dominates the term structure of interest rates. Engsted and Tanggaard (2007) found that inflation news drive the comovement between the U.S. and Germany bond markets. Barr and Priestley (2004) applied the international CAPM model allowing time-varying market segmentation to investigate global mar-

ket integration, and they found almost 70% of the variation can be explained by world dynamic beta, and the degree of market integration is stable during the period covered by the sample.

Recently, DLY focused on the entire term structure of interest rates. They used the latent factor dynamic Nelson-Siegel model to fit the yield curve. For a set of country yield curves, they fit them by allowing common global factors and country-specific factors. There are interactions between global factors and country-specific factors, and the loading of country-specific factor on global factors is allowed to vary across countries. The finding is that global factors explain a big fraction of country yield curve. In this paper I use country-specific factors from the AFDNS model to extract the global yield curve factors. The specification is different with DLY. DLY use two-factors model, while the dynamic Nelson-Siegel model estimation⁶ shows that three-factors model significantly improve the goodness-of-fit according to the R-square. Secondly, the level, slope and curvature factors seem to be independent as presented in the macro-finance model of Tam and Yu (2008). CDR shows that the forecasting performance of the independent factors model is no worse than the correlated factors model. Taking into account above points, we use a three-factors model, but assume that the global level (slope, curvature) factor only depends on the domestic level (slope, curvature) factors. This simplifies the estimation and alleviate the local maximum problem associated with the numerical optimization.

For extracting the common factor, principal component analysis is a popular method, although it is difficult to interpret. In this paper, we use Kalman filter to extract global factors, and principal component is an interesting benchmark for comparison. First, we decompose the country-specific factors:

⁶The OLS estimation of Diebold and Li (2006) model is applied by fixing the λ equal to 0.0600.

$$\begin{aligned}
l_{it} &= L_t + u_{it}^l \\
s_{it} &= S_t + u_{it}^s \\
c_{it} &= C_t + u_{it}^c
\end{aligned} \tag{6}$$

where l_{it} , s_{it} , c_{it} are country-specific factors from the independent AFDNS estimation, L_t , S_t , C_t are global level, slope and curvature factors. The u_{it}^l , u_{it}^s , u_{it}^c are country idiosyncratic level, slope and curvature factors. The i denotes one of four countries: the U.S., the U.K., Germany and Japan. As aforementioned, the assumption of independent level, slope and curvature dynamics are reasonable, therefore, we extract three global factors independently. we assume country idiosyncratic factors follow an AR(1) process:

$$\begin{pmatrix} l_{1t} - L_t \\ l_{2t} - L_t \\ \vdots \\ l_{kt} - L_t \end{pmatrix} = \begin{pmatrix} \beta_1 & 0 & 0 & 0 \\ 0 & \beta_2 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \beta_k \end{pmatrix} \begin{pmatrix} l_{1t-1} - L_{t-1} \\ l_{2t-1} - L_{t-1} \\ \vdots \\ l_{kt-1} - L_{t-1} \end{pmatrix} + \begin{pmatrix} w_{1t} \\ w_{2t} \\ \vdots \\ w_{kt} \end{pmatrix} \tag{7}$$

k is number of countries. The specification assumes the country idiosyncratic factors are independent, with the diagonal variance-covariance matrix. This make sense economically if there are no regional factors in the hierachical model. It is more an empirical issue than an theoretical one. We can choose the specification with likelihood ratio test of the form

$$LR = 2[\log L(\text{correlated}) - \log L(\text{independent})]$$

The alternative specification is

$$\begin{pmatrix} l_{1t} - \varphi_1 L_t \\ l_{2t} - \varphi_2 L_t \\ \vdots \\ l_{kt} - \varphi_k L_t \end{pmatrix} = \begin{pmatrix} \beta_1 & 0 & 0 & 0 \\ 0 & \beta_2 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \beta_k \end{pmatrix} \begin{pmatrix} u_{1t}^l \\ u_{1t}^s \\ \vdots \\ u_{kt}^c \end{pmatrix} + \begin{pmatrix} w_{1t} \\ w_{2t} \\ \vdots \\ w_{kt} \end{pmatrix} \quad (8)$$

This specification allows different factor loadings of country-specific factors on global factors, while the fit is also an empirical problem, this specification nests the (7) specification. The global factor dynamics are also given by an AR(1) process:

$$L_t = \alpha + \rho L_{t-1} + \epsilon_t \quad (9)$$

if we replace the L_t and l_{it} with S_t , C_t and s_{it} , c_{it} , the model can be used to extract the global slope and curvature factors.

3.2 Global factors and idiosyncratic factors

The Kalman filter estimation of system equation (7) and (9) is applied to extract the global factors of yield curves. The unrestricted vector autoregression (VAR) estimation shows that one factor has little extra information about the dynamic of other factor, plus empirical evidence from Diebold and Li (2006), Tam and Yu (2008), therefore we extract the global level, slope and curvature factors by independently iterating the Kalman filter. This simplifies the extraction of global factors significantly. We initialize the Kalman filter with the unconditional covariance matrix and a mean vector from the average of country-specific factors. The estimated parameters are reported in the upper panel of Table 5.

Table 5 About Here

The global factors essentially are one common component of country-specific factors. Two interesting questions before we scrutinize the global factors are: what is the explanatory power of one common component at most? What is the relationship of the components extracted by the Kalman filter and components from principle component analysis? To answer questions, the principal component analysis results are presented in the lower panel of Table 5. As we mentioned, the cross-correlation of level factors is higher than slope and curvature factors. The first principle component can explain 91% of variation of the country-specific level factors. For slope and curvature factors, only 57% and 48% of variation can be interpreted by the first principal component. There is strong interactions between global factors from the Kalman filter and the first principal component. The adjusted first principal component and global factors are plotted in the figure 12. The correlations of level, slope and curvature factors and the corresponding first principle components are respectively 0.99, 0.87 and 0.75. Table 6 gives the descriptive statistics of global factors. As in the country model, the level factor is more persistent than the slope and curvature factors. The skewness and kurtosis of curvature factor is not in favor of the normality distribution.

Figure 12 About Here

Table 6 About Here

To investigate the correlation and explanatory power of global factors on country specific factors, we run the following regression:

$$f_{it} = \alpha + \beta f_{wt} + \varepsilon_{it}, i = GM, JP, UK, US;$$

$$f = level, slope, curvature; f_{wt} : global \text{ factor}$$

Table 7 presents the results. All country-specific factors have positive loadings on global factors. The global level factor has the most significant explanatory power judged by R^2 of the regression. The global slope and curvature factors have lower but still significant power of explanation. The regression implies that the level factor has the highest degree of integration.

Table 7 About Here

The previous analysis provides the static correlation of global factors and country-specific factors. The purpose of the paper is to investigate the probably time-varying bond markets integration, hence the dynamic correlation is our interest. Engle (2002) proposes the dynamic conditional correlation (DCC-GARCH) model, and it is appropriate for the purpose here. The model is extensively applied because it preserves the simplicity of univariate model in a multivariate setting. The DCC-GARCH model for factors is as follows:

$$\mathbf{f}_t | \Omega_t \sim N(0, H_t)$$

$$H_t = D_t R_t D_t$$

$\mathbf{f}_t$ is the vector of 5 level, slope or curvature factors (Germany, Japan, The U.K, the U.S., world). The maximum likelihood method can be used to estimate the DCC-GARCH model. Figures 13, 14 and 15 present the dynamic conditional correlation of

level, slope and curvature factors across countries. In general, the global level factor is positively correlated with the country-specific factors. The dynamic conditional correlations of slope and curvature factors shift more frequently. Anyway, all factors are highly correlated, although the correlation may be positive or negative.

Figure 13 About Here

Figure 14 About Here

Figure 15 About Here

4 Global market interactions and integration

4.1 The integration model

There are interactions and linkages of the government bond markets across countries. However, market integration is a stricter restriction in the sense that it implies the comovement or interaction in the bond markets across countries, but not vice versa. Interactions are empirical phenomena, the market integration should be theoretically consistent in addition to being empirically correlated. In the international CAPM model (Barr and Priestley (2004)), government bond markets are integrated if the world beta price the excess return. In contrast, the AFDNS model is for describing the yield curve level dynamics, therefore we define bond market integration as a situation in which the movement in global yield factors determines the movement in yields with different maturities in each country's market. Otherwise, the markets are segmented if the movement of yields is determined by the movement of idiosyncratic factors.

The notion of integration is challenging and controversial. The definition here requires bond markets fluctuate together. The accuracy of the measure relies on the performance of underlying yield curve models. Once the AFDNS is called into questions, so is the dynamic measure of integration. Empirically, the AFDNS model provides the necessary accuracy.

The definition focuses on the change of interest rates instead of the levels. Because the level factor dominates the yield curve dynamics, the change of factors eliminates the level-factor dominance effect. This allows a stable interest rate differential between two markets that is consistent with the efficient market hypothesis if there is also a stable inflation wedge. With this definition, more attention is directed to the interactions of three latent factors from the AFDNS model. These interactions have important information about the market integration. For example, Sutton (2000) finds that the comovement of long-term yields can't be explained by the interactions of short-term yields. This is the evidence of heterogeneous factor dynamics. The cross-section maturity structure of the term structure contains useful information for forecasting future interest rates and the macroeconomic dynamics (Ang, Piazzesi, and Wei (2006), Campbell and Shiller (1991), Diebold, Rudebusch, and Aruoba(2006), Estrella and Hardouvelis (1991), Hamilton and Kim (2002)). Tan and Yu (2008) offers the further evidence of heterogeneous factors dynamics using the dynamic conditional correlation analysis. Most of previous studies focus on the time series properties of comovement, with above definition, the properties of yield curves in cross-section are investigated. According to our decomposition of country-specific factors in (6), even the world factors are constant, they still play a role in explaining the yields in each market, while the change of factors circumvent the problem. This provides another reason.

In completely segmented markets, the change of yields in each market is determined by the country idiosyncratic factors

$$\begin{pmatrix} \Delta y_{it(\tau_1)} \\ \Delta y_{it(\tau_2)} \\ \vdots \\ \Delta y_{it(\tau_N)} \end{pmatrix} = A \begin{pmatrix} \Delta u_{it}^l \\ \Delta u_{it}^s \\ \Delta u_{it}^c \end{pmatrix} + \begin{pmatrix} w_{it(\tau_1)} \\ w_{it(\tau_2)} \\ \vdots \\ w_{it(\tau_N)} \end{pmatrix} \quad (10)$$

where i = Germany, Japan, UK, US, and N is the the observations in cross-section at each point of time, and

$$A = \begin{pmatrix} 1 & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} - e^{-\lambda\tau_1} \\ 1 & \frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2} & \frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2} - e^{-\lambda\tau_2} \\ \vdots & \vdots & \vdots \\ 1 & \frac{1-e^{-\lambda\tau_N}}{\lambda\tau_N} & \frac{1-e^{-\lambda\tau_N}}{\lambda\tau_N} - e^{-\lambda\tau_N} \end{pmatrix} \quad (11)$$

In a completely integrated global market, the change of yields in each market is determined by global factors

$$\begin{pmatrix} \Delta y_{1t(\tau_1)} \\ \Delta y_{1t(\tau_2)} \\ \vdots \\ \Delta y_{kt(\tau_N)} \end{pmatrix} = A \begin{pmatrix} \Delta L_t \\ \Delta S_t \\ \Delta C_t \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t(\tau_1)} \\ \varepsilon_{1t(\tau_2)} \\ \vdots \\ \varepsilon_{kt(\tau_N)} \end{pmatrix} \quad (12)$$

This equation is consistent with uncovered interest rate parity and the AFDNS model if the expected change of exchange rate is a martingale process. Given the uncovered interest rate parity is

$$E_t(ex_{t+1} - ex_t)/ex_t = y_{1t(\tau_j)} - y_{2t(\tau_j)} \quad (13)$$

where ex is the exchange rate, $y_{it(\tau_j)}$ is the yield for country 1 with maturity τ_j . Because the expected change of exchange rate is zero (martingale process), the yields differential should be zero for all maturities according to the law of one price. Since the yield curve is given by the AFDNS model, the zero differential holds in time series when

$$\begin{aligned} l_{1t} &= l_{2t} \\ s_{1t} &= s_{2t} \\ c_{1t} &= c_{2t} \end{aligned} \quad (14)$$

therefore the uncovered interest rate parity is a polar case of our model.

In the real world, government bond markets across countries are expected to be neither perfectly integrated nor completely segmented. The degree of segmentation might be time-varying, it is even expected there is a trend of increasing degree of integration due to deregulations. The switch of regimes may be caused by common shocks in both financial markets and real economy. It could be a surprise or partially expected. The Markov-switching Nelson-Siegel model allows time-varying dynamic evolution of market integration,

$$\begin{pmatrix} \Delta y_{1t(\tau_1)} \\ \Delta y_{1t(\tau_2)} \\ \vdots \\ \Delta y_{kt(\tau_N)} \end{pmatrix} = \phi_t A \begin{pmatrix} \Delta L_t \\ \Delta S_t \\ \Delta C_t \end{pmatrix} + (1 - \phi_t) A \begin{pmatrix} \Delta u_{it}^l \\ \Delta u_{it}^s \\ \Delta u_{it}^c \end{pmatrix} + \begin{pmatrix} \epsilon_{1t(\tau_1)} \\ \epsilon_{1t(\tau_2)} \\ \vdots \\ \epsilon_{kt(\tau_N)} \end{pmatrix} \quad (15)$$

where ϕ_t is the probability of the market integration. $\phi_t = 1$ implies the perfect market integration, $\phi_t = 0$ means markets are completely segmented. The regime probability ϕ_t follows a Markov chain process and the EM algorithm is an efficient estimator (Hamilton 1994), the optimal inference and forecast of regime is given by iterating the following equations

$$\hat{\xi}_{t|t} = \frac{(\hat{\xi}_{t|t-1} \odot \eta_t)}{1'(\hat{\xi}_{t|t-1} \odot \eta_t)}$$

and

$$\hat{\xi}_{t+1|t} = P\hat{\xi}_{t|t}$$

with P being the transition probability matrix. Moreover, the disturbance vector is given by

$$\begin{pmatrix} \epsilon_{1t(\tau_1)} \\ \epsilon_{1t(\tau_2)} \\ \vdots \\ \epsilon_{kt(\tau_N)} \end{pmatrix} = \phi_t \begin{pmatrix} \varepsilon_{1t(\tau_1)} \\ \varepsilon_{1t(\tau_2)} \\ \vdots \\ \varepsilon_{kt(\tau_N)} \end{pmatrix} + (1 - \phi_t) \begin{pmatrix} w_{it(\tau_1)} \\ w_{it(\tau_2)} \\ \vdots \\ w_{it(\tau_N)} \end{pmatrix}$$

This allows regime-dependent heteroskedasticity.

4.2 Integration measure and interpretations

The starting point of time of the sample used for integration analysis is January 1985. It is then that financial deregulation has become a global phenomenon. Based on prior knowledge, one might expect that the global bond markets are close to full integration. However, in the last two decades world financial market has gone through

a turbulent age, so bond market integration may be subject to the turbulences. It is reasonable to postulate ex ante that market integration is time-varying. We are also suspicious about the full integration due to the reasons enumerated in the introduction, such as, home bias, tax treatment difference, exchange rate risk, liquidity risk, among many others.

The Markov-switching Nelson-Siegel model allows us to measure the dynamic evolution of the government bond market integration. The estimated transition probabilities and the log likelihood are presented in Table 8. Here state 1 has natural interpretation of market integration and state 2 represents market segmentation. In this model, the transition probabilities are fixed to be constants. As the Markov-switching model is highly nonlinear, it may be subject to local maximum and corner solution. In estimation, we use the parameter vector from the regression of country-specific yields on global yields as the initial parameters for state 1. For state 2, the parameters vector from regression of country-specific yields on the idiosyncratic yields is used as the starting parameters. The global yields are defined as the first item on the right hand side of Equation (15), they are plotted in figure 16, and the idiosyncratic yields are the second item on the right hand side of the same equation. From the above definition, it is clear why we fix the $\lambda_t = 0.0600$ in equation system (7) and (9), it makes the same criterion when measuring the market integration.

Table 8 About Here

Figure 16 About Here

4.2.1 Germany

Two major changes may affect the government bond market in Germany for the period covered by the sample. One is the monetary union marked by the Deutsche mark becoming legal tender in East Germany, this rise the government funding needs. The other is the introduction of the Euro. There are also some structural changes in the Germany bond market, for example, issuing technique changes from the underwriting procedure to combined with auctions in July 1997, introduction of Bund futures and options on Bund futures in late 1980s. The estimated result for Germany is plotted in upper panel of Figure 17. The market integration is quite volatile by the Markov-switching Nelson-Siegel model. This is not surprising because we use the first moment to measure the integration, as long as world yields are rising (falling) while Germany yields are falling (rising), market is segmented. In contrast, the second moment (volatility) is applied in the capital asset pricing model. Instead of filtered probabilities, we may look at the predicted probabilities ($\xi_{t+1|t}$), that is the expected degree of market integration ex ante, the expected integration is stable and between the interval of 0.3 and 0.45.

Figure 17 About Here

4.2.2 Japan

The deregulation beginning in 1970 has reconstructed the Japanese financial markets, up to 1985, restrictions, for instance, interest rate ceiling, capital moving to and from overseas, have been removed. The financial markets are freed of strict regulations. After that, in 1997 the Bank of Japan Law was revised and the Bank of Japan acquired a more independent legal statue. The ongoing financial reform, "Japanese

Big Bang", has far-reaching consequences in financial markets. These events may change the state of integration of the Japanese government bond market with the world bond market. The middle panel of figure 17 presents the results for Japan. The market integration is also volatile. If we look at the expected degree of integration, it is relatively stable and in the interval 0.25 — 0.50.

4.2.3 U.K.

Our prior expectation is the UK would have high degree of integration because financial market in the UK are free of regulations. The results for the UK is in the lower panel of figure 17. The volatile integration is all the same as Germany and Japan. However, the dynamic degree of integration is in the interval 0.225 — 0.3, it is the lowest among four countries investigated.

4.2.4 U.S.

The US is the single biggest and most important market in the world. The dynamic expected degree of integration is in the interval 0.2 — 0.5, it is more volatile than other markets. Measured by the mean, it has the highest degree of integration. The filtered probabilities are still volatile due to the aforementioned reason. The upper panel of figure 18 plots the filtered and predicted probabilities of being in the integration state.

Figure 18 About Here

4.2.5 World

The integration of world bond market as a whole is main interest. The Markov-switching model allow us to measure the world market integration dynamically. We choose one long-term yield and one short-term yield for each country (Germany: 2-year, 9 year, Japan: 1-year, 10-year, the UK: 6-month, 8-year, the US: 3-month, 7-year), it consists of the dependent variables. The world factors and corresponding idiosyncratic factors are independent factors. We can't include all yields, otherwise, the coefficient matrix for the global factors is singular. The results are in the lower panel of figure 18. The expected dynamic integration is in the interval 0.1 — 0.4. This is not suprising given the degree of integration for each country. The world integration is a stricter restriction because it requires the global factors to explain yields across countries at the same time.

Figure 18 About Here

The stable dynamic predicted probabilities of integration implies that the market expectation of integration is stable. This is consistent with the finding in BP where the authors reject the time-varying bond market integration. However, our results imply a lower degree of integration. It is not suprising because we take into account the maturity structure of the yield curve in cross-section. Combined with the results of the principal component analysis, the idiosyncratic regression and DCC-GARCH analysis, the market segmentation stems mainly from low degree of integration on slope and curvature factors. Another reason is that the heterogeneous dynamics of the factors, the integration of one factor may accompany the segmentation of some other factor. Because the level factor represents the long-term factor, the long end of the term structure is more likely to be integrated than short end of the term structure

that are represented by the curvature factors. This finding is consistent with Sutton (2000). This may suggest that the short end of the bond market is the impediment to market integration.

Empirically, the level factor is highly correlated with inflation. The slope factor is associated with real activity (Diebold et. al (2006)), and curvature is associated with uncertainty (Zhu 2008). In previous macro-finance model (Ang and Piazzesi 2003), the macro factors are found to affect the short end of the yield curve, but leave the long end not accounted for. Therefore, the segmentation is more likely coming from the dynamics of real economy than from the nominal dynamics.

5 Conclusions

It is difficult to measure the integration of the government bond market into the world bond market. Some previous studies investigate the long end integration of yield curves, while others investigate the short end integration of yield curves. In this paper, we propose a measure that take into account the maturity structure of bond yields. This measure also allows for time-varying conditional market integration. We use a theoretically consistent latent factors model, the AFDNS model, to describe the dynamics of the yield curves in both time series and cross-section. Then the global factors are extracted from the country factors. Finally, the market integration is measured by the Markov-switching Nelson-Siegel model.

Some results are consistent with previous studies, but we shed new light on bond market integration. The new finding is that market segmentation are from two aspects, one is the integration asymmetry of bond markets. The level factor is more integrated than the global slope and curvature factors. The other reason is

the heterogeneous dynamics of latent factors. The integration of factors are not simultaneous. This tells us that market integration is from the short- and medium end of yield curves.

A number of extensions deserve further exploration. Why is the short end of the market more segmented than the long end of the market? It is widely believed that the central banks can take control of the short-term interest rates. However, can monetary policy shocks explain partially the short end segmentation of the market? This question is associated with the monetary policy transmission mechanism. The rational expectations hypothesis plays a pivotal role in the term structure of interest rates. Sutton (2000) hence tried to relate the market integration with the rational expectation hypothesis. In this framework of integration, what role do the rational expectations play?

From a financial economics perspective, does this imply arbitrage opportunities? If not, what is the risk of portfolio diversification. There are a number of risks, such as exchange rate risk, liquidity risk, but which one is the dominant one? Associated with the risk, one also has to explain the price of risk. These are some possible future extensions.

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Table 1: Summary Statistics for Bond Yields

Germany								
maturity (months)	Mean	Std. Dev.	skew- ness	kurto- sis	$\hat{\rho}_{(1)}$	$\hat{\rho}_{(12)}$	$\hat{\rho}_{(30)}$	ADF
12	4.6981	1.9934	0.8865	2.8322	0.9925	0.8239	0.4654	-1.482
24	4.8881	1.9001	0.8248	2.7951	0.9911	0.8091	0.5000	-1.623
60	5.4378	1.6805	0.4953	2.4366	0.9908	0.8245	0.6302	-1.476
120	5.9188	1.4821	0.1114	2.0354	0.9903	0.8484	0.7597	-1.126
Japan								
maturity (months)	mean	Std. Dev.	skew- ness	kurto- sis	$\hat{\rho}_{(1)}$	$\hat{\rho}_{(12)}$	$\hat{\rho}_{(30)}$	ADF
6	0.6474	0.9599	1.7709	4.9275	0.9636	0.5883	0.2093	-3.183
12	0.7299	0.9855	1.6646	4.5191	0.9639	0.6016	0.2462	-1.366
60	1.4694	1.1333	1.2654	3.4682	0.9648	0.6535	0.3677	-1.757
120	2.2334	1.1761	1.0684	2.9223	0.9692	0.7132	0.4459	-1.405
U.K.								
maturity (months)	mean	Std. Dev.	skew- ness	kurto- sis	$\hat{\rho}_{(1)}$	$\hat{\rho}_{(12)}$	$\hat{\rho}_{(30)}$	ADF
6	7.1674	2.9581	0.8249	2.4826	0.9635	0.5883	0.2093	-1.972
12	7.1004	2.7671	0.7402	2.3789	0.9878	0.8186	0.6054	-1.438
60	7.1984	2.3906	0.3329	1.7688	0.9878	0.8602	0.7724	-1.435
120	7.2003	2.3584	0.2000	1.5532	0.9893	0.8968	0.8223	-1.450
U.S.								
maturity (months)	mean	Std. Dev.	skew- ness	kurto- sis	$\hat{\rho}_{(1)}$	$\hat{\rho}_{(12)}$	$\hat{\rho}_{(30)}$	ADF
6	4.9810	2.0397	-0.1685	2.4790	0.9929	0.6960	0.2327	-1.711
12	5.1507	2.0483	-0.1063	2.4441	0.9911	0.7047	0.2952	-1.712
60	6.0391	1.9105	0.3038	2.5432	0.9811	0.7164	0.6114	-2.416
120	6.4194	1.8052	0.5292	2.6213	0.9907	0.8443	0.7225	-2.692

(1) The summary statistics for Japan is based on Bloomberg sample.

Notes: (2) $\hat{\rho}_{(\tau)}$ is the τ -th autocorrelation coefficient.

(3) The lag length of ADF test is selected by SIC.

Table 2: OLS Estimates of Dynamic Nelson-Siegel Model

Germany							
model	R^2	level	t-value	Std.Dev.	slope	t-value	Std.Dev
three factors	0.9823	6.4474	2396	1.4431	-1.7399	-433.7	1.6689
		curvature	t-value	Std.Dev.			
		-2.2329	-143.8	2.3655			
two factors	0.8426	6.1004	867.8	1.5138	-2.1903	-75.52	1.9845
Japan							
model	R^2	level	t-value	Std.Dev.	slope	t-value	Std.Dev
three factors	0.9804	2.9365	170.3	1.8958	-2.0608	-117.7	1.1887
		curvature	t-value	Std.Dev.			
		-3.5256	-45.8	1.8454			
two factors	0.8287	2.1938	80.63	1.2506	-2.1318	-34.63	0.7634
U.K							
model	R^2	level	t-value	Std.Dev.	slope	t-value	Std.Dev
three factors	0.9425	7.3024	1231	2.4339	-0.1741	-41.63	2.0108
		curvature	t-value	Std.Dev.			
		-0.3378	-4.226	1.9574			
two factors	0.7958	6.1004	1094	2.3130	-2.1903	-6.27	2.0090
U.S.							
model	R^2	level	t-value	Std.Dev.	slope	t-value	Std.Dev
three factors	0.9183	6.7331	488.5	1.7350	-2.1327	-193.8	1.6050
		curvature	t-value	Std.Dev.			
		-0.3017	-22.24	2.0716			
two factors	0.8387	6.6523	507.1	1.9768	-2.0859	-83.46	1.4816

Notes: (1) Statistics for Japan are based on the sample retrieved from Bloomberg.
(2) The two-factors model doesn't have curvature factor.
(3) The reported statistics are mean of cross-section OLS estimation.
(4) For regression details, refer to Diebold and Li (2006); Here $\lambda = 0.0600$.

Table 3: Estimates of the Diagonal AFDNS Model Parameters

Germany						
$\exp(-K^P \frac{1}{12}) = \begin{pmatrix} 0.9905 & 0 & 0 \\ 0 & 0.9681 & 0 \\ 0 & 0 & 0.9579 \end{pmatrix}$						
Yield-term	-0.00250	-0.00795	-0.01476	-0.02183	-0.02857	-0.03480
	-0.4054	-0.4590	-0.5101	-0.5600		
Japan						
$\exp(-K^P \frac{1}{12}) = \begin{pmatrix} 0.9796 & 0 & 0 \\ 0 & 0.9645 & 0 \\ 0 & 0 & 0.9164 \end{pmatrix}$						
Yield-term	-0.0020	-0.0076	-0.0293	-0.0619	-0.09976	-0.1390
	-0.1781	-0.2167	-0.2553	-0.3341		
U.K.						
$\exp(-K^P \frac{1}{12}) = \begin{pmatrix} 0.9887 & 0 & 0 \\ 0 & 0.9705 & 0 \\ 0 & 0 & 0.9213 \end{pmatrix}$						
Yield-term	-0.0086	-0.0180	-0.0218	-0.0259	-0.0303	-0.1032
	-0.2009	-0.3070	-0.4113	-0.5100	-0.6028	-0.6912
	-0.7769	-0.8617				
U.S.						
$\exp(-K^P \frac{1}{12}) = \begin{pmatrix} 0.9820 & 0 & 0 \\ 0 & 0.9828 & 0 \\ 0 & 0 & 0.9481 \end{pmatrix}$						
Yield-term	-0.0001	-0.0008	-0.0032	-0.0119	-0.0428	-0.0868
	-0.1885	-0.2908	-0.4467			

- Notes:
- (1) Reported parameters for Japan based on Bloomberg sample.
 - (2) Yield-term: yield-adjustment terms in equation (3) associated with corresponding maturities (CDR for details),
 - (3) $\exp(-K^P \frac{1}{12})$: one-month conditional mean-reversion matrix

Table 4: Summary Statistics for factors across countries (AFDNS Estimates)

Germany								
factor	mean	Std.Dev.	skewness	kurtosis	$\hat{\rho}_{(1)}$	$\hat{\rho}_{(12)}$	$\hat{\rho}_{(30)}$	ADF
level	7.2674	1.4346	-0.1706	1.8273	0.9886	0.8175	0.7692	-1.1929
slope	-2.139	1.6749	0.0154	2.2248	0.9694	0.5380	-0.087	-2.2305
curv	-3.974	2.6487	0.2730	2.3342	0.9516	0.3440	-0.034	-2.6392
Japan								
factor	mean	Std.Dev.	skewness	kurtosis	$\hat{\rho}_{(1)}$	$\hat{\rho}_{(12)}$	$\hat{\rho}_{(30)}$	ADF
level	4.6381	2.1103	0.2006	1.5152	0.9967	0.9101	0.8250	-1.2063
slope	-1.879	0.9093	0.0561	3.2731	0.9666	0.6060	0.1854	-2.1107
curv	-4.650	1.6599	0.2252	3.2175	0.9184	0.2012	-0.352	-3.6357
U.K.								
factor	mean	Std.Dev.	Skewness	Kurtosis	$\hat{\rho}_{(1)}$	$\hat{\rho}_{(12)}$	$\hat{\rho}_{(30)}$	ADF
level	8.5185	2.4518	0.1324	1.4582	0.9881	0.8995	0.8300	-3.0653
slope	-1.196	2.0116	0.0602	3.4697	0.9704	0.5371	0.0178	-3.0490
curv	-2.378	1.9584	0.2813	3.0086	0.9177	0.0313	0.3752	-3.3258
U.S.								
factor	Mean	Standard	Skewness	Kurtosis	$\hat{\rho}_{(1)}$	$\hat{\rho}_{(12)}$	$\hat{\rho}_{(30)}$	ADF
level	7.4470	1.7460	0.6574	2.9736	0.9817	0.8029	0.8607	-1.711
slope	-2.606	1.6344	-0.1147	1.8272	0.9828	0.4660	-0.270	-1.712
curv	-1.930	1.8439	-0.7676	2.9917	0.9545	0.4319	-0.079	-2.416
(1) Factors are from AFDNS estimation.								
Notes: (2) $\hat{\rho}_{(\tau)}$ is the autocorrelation coefficient with lag length τ periods.								
(3) The lag length of ADF test is selected by SIC.								

Table 5: Extraction of Global Yield Curve Factors

Kalman Filter*						
	$\hat{\alpha}$	$\hat{\rho}$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$
level	0.0190 (0.036)	0.9953 (0.005)	0.9408 (0.024)	0.9803 (0.008)	0.9606 (0.021)	0.9998 (0.003)
slope	-0.045 (0.063)	0.9735 (0.032)	0.9874 (0.010)	0.9694 (0.014)	0.9677 (0.016)	0.9643 (0.019)
curv	-0.303 (0.183)	0.9276 (0.046)	0.9822 (0.014)	0.9557 (0.024)	0.9510 (0.019)	0.9314 (0.023)
Principal Component Analysis						
level	eigenvalue		3.6536	0.2150	0.1025	0.0288
	variance Prop.		0.9134	0.0537	0.0256	0.0072
	cumulative Prop.		0.9134	0.9172	0.9928	1.0000
slope	eigenvalue		5.9135	2.6078	1.5672	0.2611
	variance Prop.		0.5714	0.2520	0.1514	0.0252
	cumulative Prop.		0.5714	0.8233	0.9748	1.0000
curv	eigenvalue		8.1583	4.4534	2.6453	1.7494
	variance Prop.		0.4797	0.2619	0.1555	0.1029
	cumulative Prop.		0.4797	0.7416	0.8971	1.0000

- Notes:
- (1) *: Equation system (7) and (9) in the text body.
 - (2) The statistic in the parentheses is Std. Error.
 - (3) The Japan factors used for extracting global factors consist of estimates of two samples from the PACAP and Bloomberg.

Table 6: Summary Statistics for Global Factors

	mean	Std.Dev.	skewness	kurtosis	$\hat{\rho}_{(1)}$	$\hat{\rho}_{(12)}$	$\hat{\rho}_{(30)}$
level	7.3025	1.4169	0.2342	1.9836	0.9901	0.8029	0.8607
slope	-1.9008	0.3899	0.1629	2.3144	0.9701	0.4762	-0.0159
curv	-3.9876	0.6066	1.3589	6.3131	0.9278	0.2208	-0.1473

Note: $\hat{\rho}_{(\tau)}$ is the autocorrelation coefficient with lag length τ periods

Table 7: Idiosyncratic Regression

$f_{it} = \alpha + \beta f_{wt} + \varepsilon_{it}, i = GM, JP, UK, US;$ $f = level, slope, curvature; f_{wt} : global \text{ factor}$			
Germany			
	$\hat{\beta}$	Std. Error	R^2
Level	0.5837	0.1936	0.8171
Slope	0.3348	0.1827	0.4990
Curvature	2.7673	0.2030	0.4016
Japan			
	$\hat{\beta}$	Std. Error	R^2
Level	1.4114	0.0286	0.8980
Slope	1.3795	0.1130	0.3499
Curvature	1.9063	0.1180	0.4853
U.K.			
	$\hat{\beta}$	Std. Error	R^2
Level	1.6501	0.0313	0.9093
Slope	3.7154	0.2151	0.5186
Curvature	0.7813	0.1882	0.0586
U.S.			
	$\hat{\beta}$	Std. Error	R^2
Level	1.1945	0.0182	0.9395
Slope	2.4012	0.2065	0.3281
Curvature	1.2116	0.1675	0.1588

Table 8: Bond Market Integration:
Estimates of Markov-switching Nelson-Siegel Model

Germany		
transition matrix $\begin{pmatrix} 0.4202 & 0.3321 \\ 0.5798 & 0.6679 \end{pmatrix}$	t-values of transition matrix $\begin{pmatrix} 1.3662 & \\ & -3.5387 \end{pmatrix}$	Log Likelihood 2024.08
Japan		
transition matrix $\begin{pmatrix} 0.4689 & 0.2630 \\ 0.5311 & 0.7364 \end{pmatrix}$	t-values of transition matrix $\begin{pmatrix} 0.3332 & \\ & -3.4716 \end{pmatrix}$	636.70
U.K.		
transition matrix $\begin{pmatrix} 0.2977 & 0.2149 \\ 0.7023 & 0.7851 \end{pmatrix}$	t-values of transition matrix $\begin{pmatrix} 2.6776 & \\ & -6.5729 \end{pmatrix}$	1697.70
U.S.		
transition matrix $\begin{pmatrix} 0.4956 & 0.2141 \\ 0.5044 & 0.7859 \end{pmatrix}$	t-values of transition matrix $\begin{pmatrix} 0.9600 & \\ & -6.2117 \end{pmatrix}$	1524.11
World		
transition matrix $\begin{pmatrix} 0.4178 & 0.0896 \\ 0.5822 & 0.9104 \end{pmatrix}$	t-values of transition matrix $\begin{pmatrix} 0.8397 & \\ & -8.3693 \end{pmatrix}$	1059.62
Notes: (1) The model is given in equation (9) in the text body (2) The State 1 represents the market integration.		

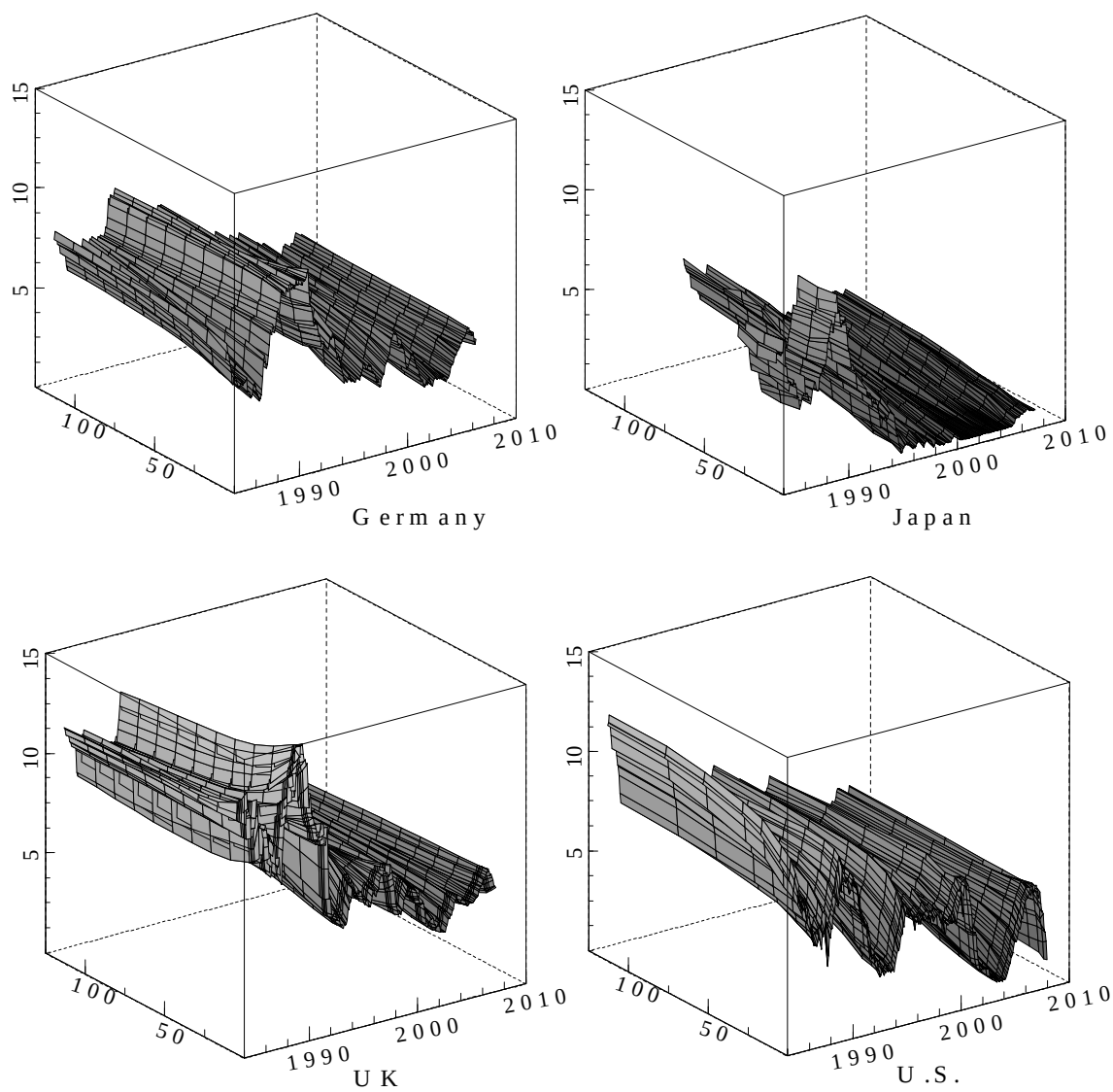


Figure 1: Yield Curves Across Countries

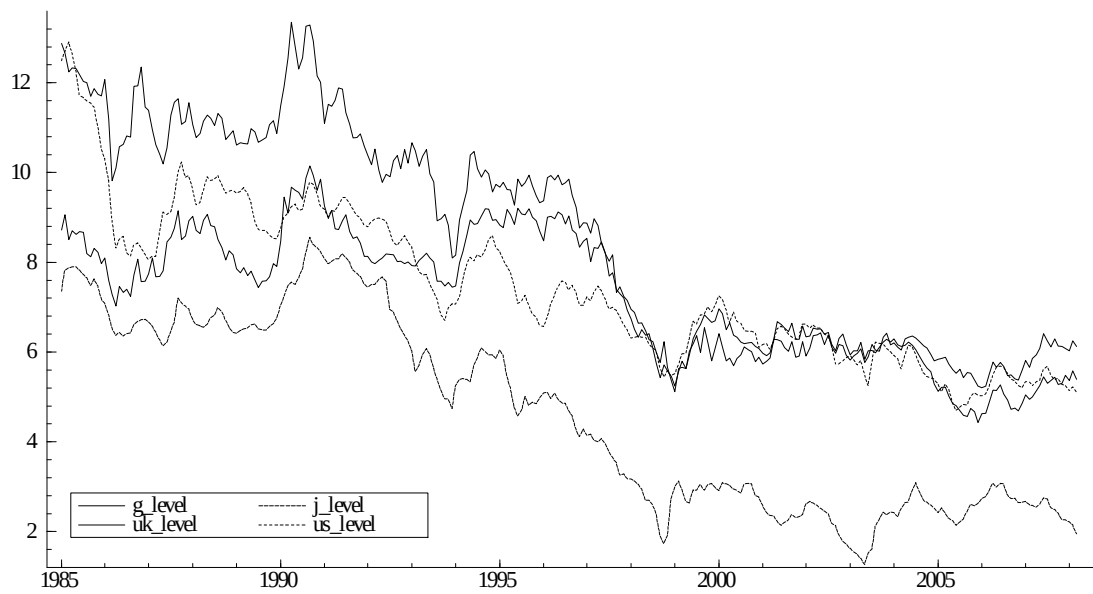


Figure 2: AFDNS Model Estimates of Level Factor Across Countries

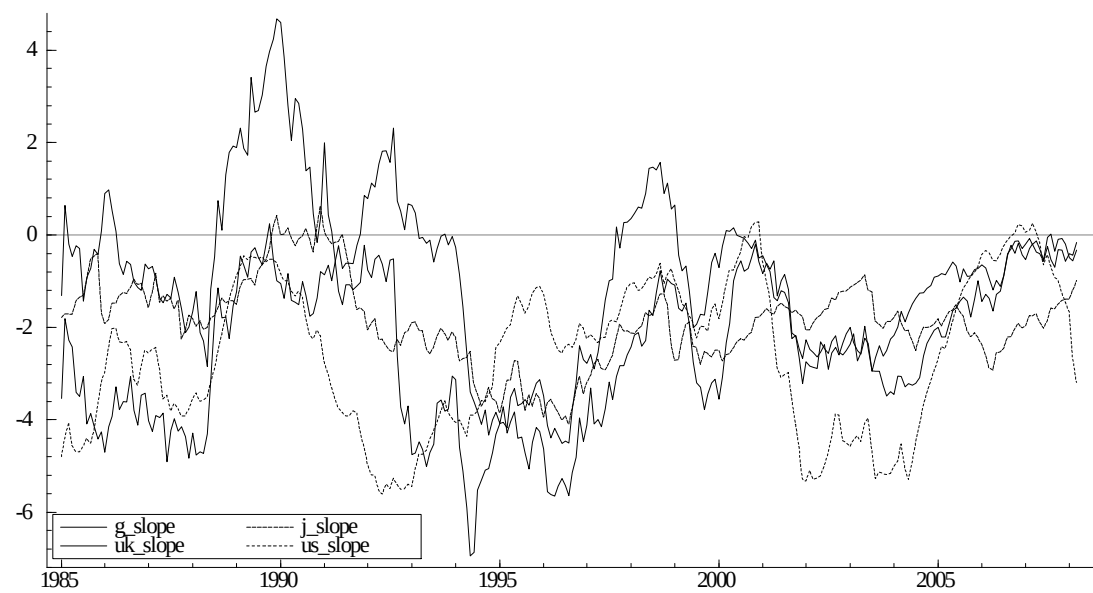


Figure 3: AFDNS Model Estimates of Slope Factor Across Countries

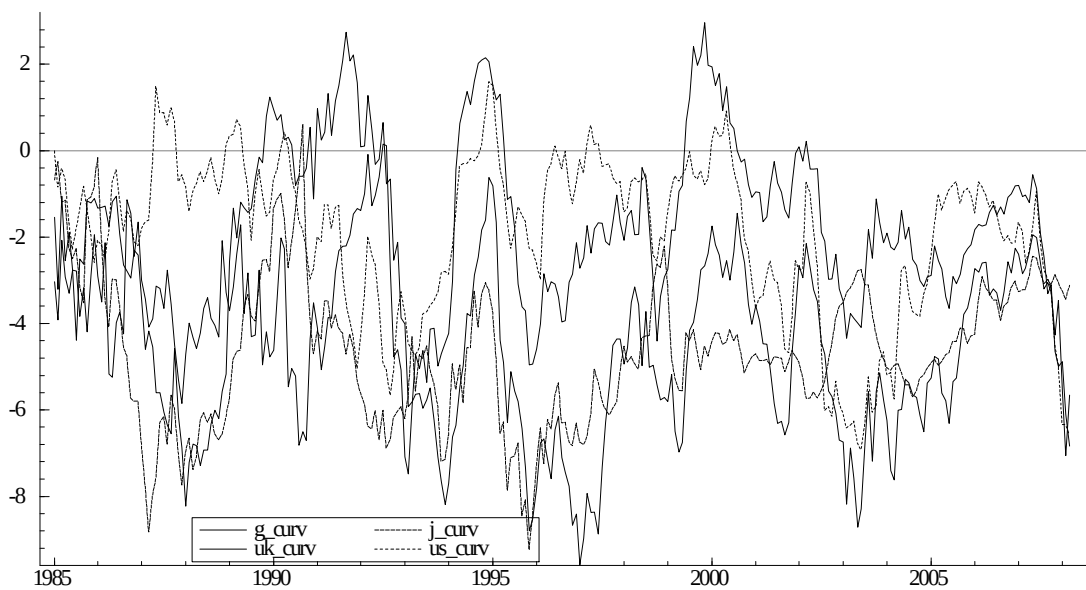


Figure 4: AFDNS Model Estimates of Curvature Factor Across Countries

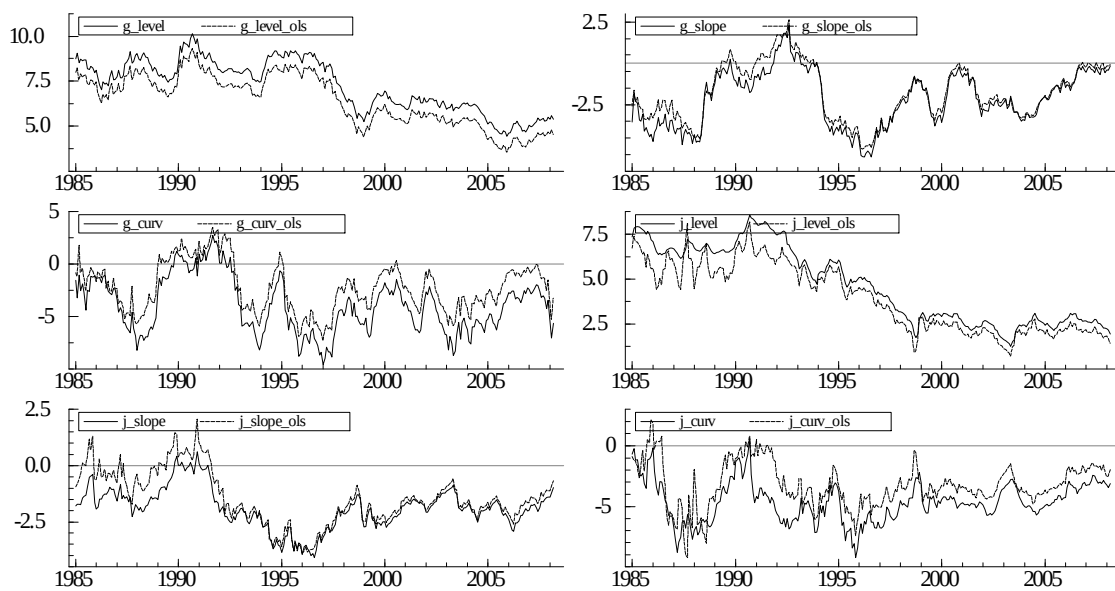


Figure 5: Factors from AFDNS Estimates and DNS Estimates: Germany and Japan

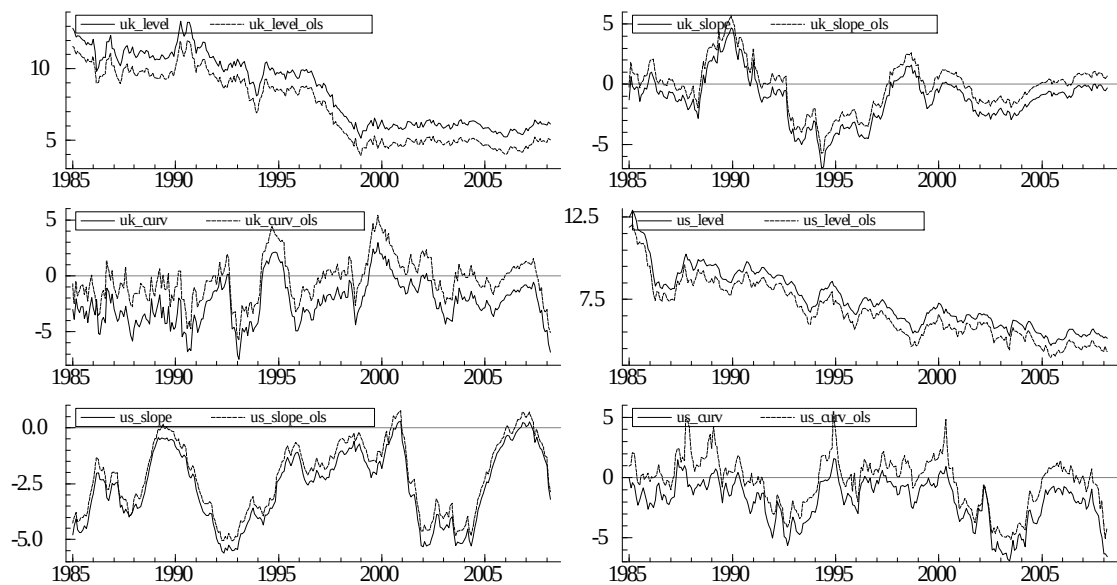


Figure 6: Factors from AFDNS Estimates and DNS Estimates: U.K. and U.S.

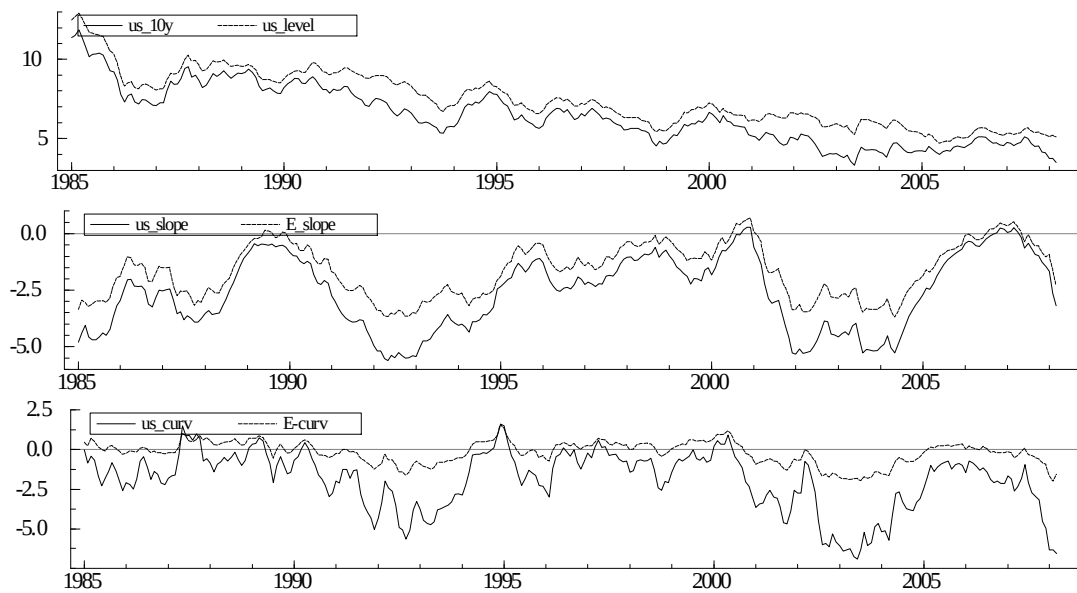


Figure 7: US level, slope and curvature factors and empirical factors

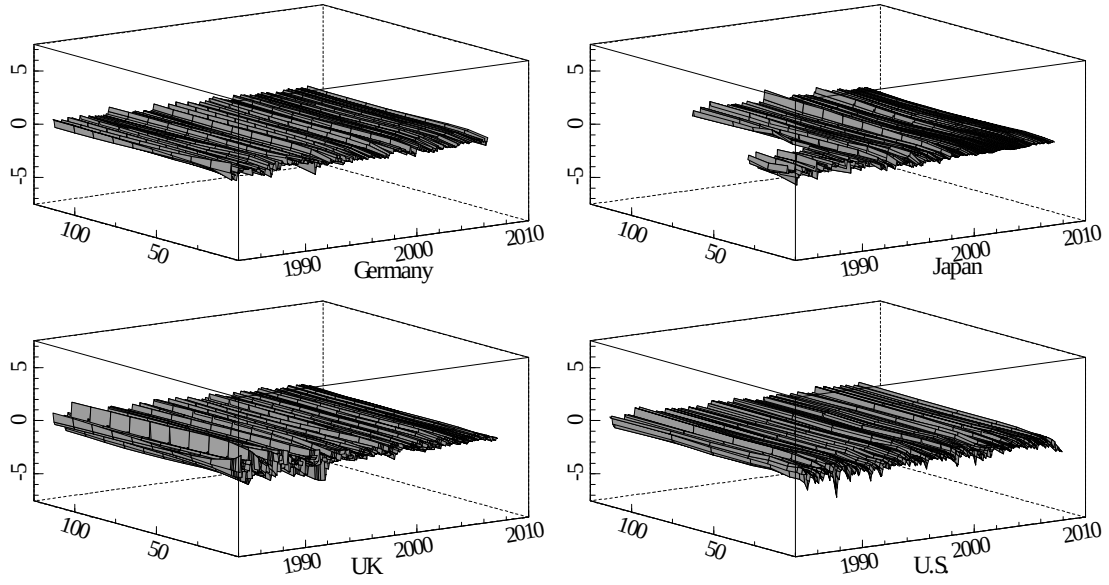


Figure 8: Error Terms of AFDNS Estimates across Countries

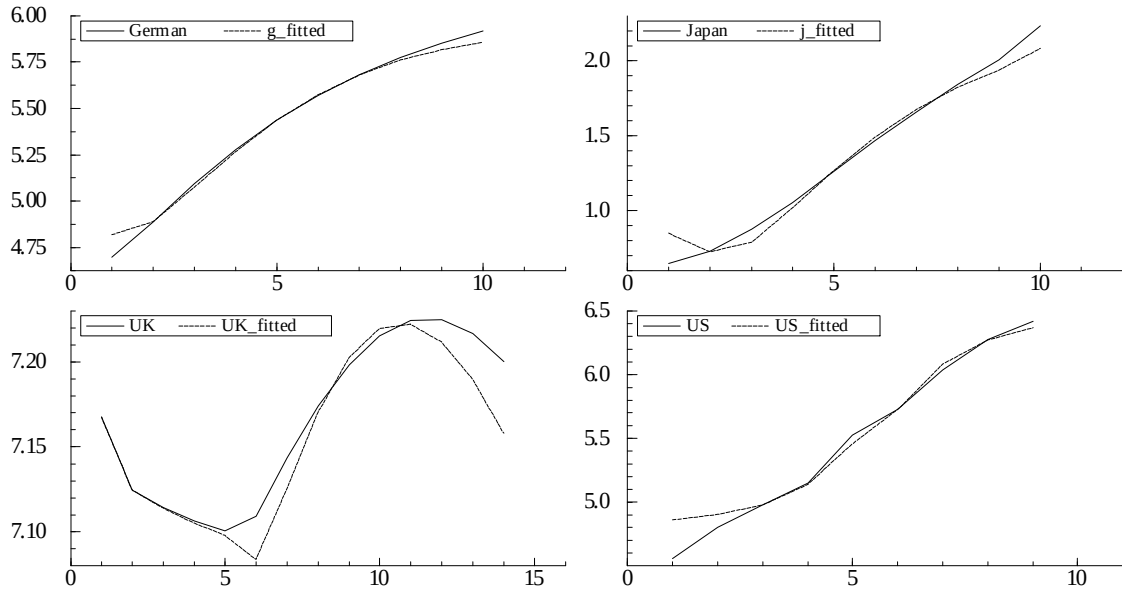


Figure 9: Average and AFDNS-fitted Yield Curves across Countries and Time:

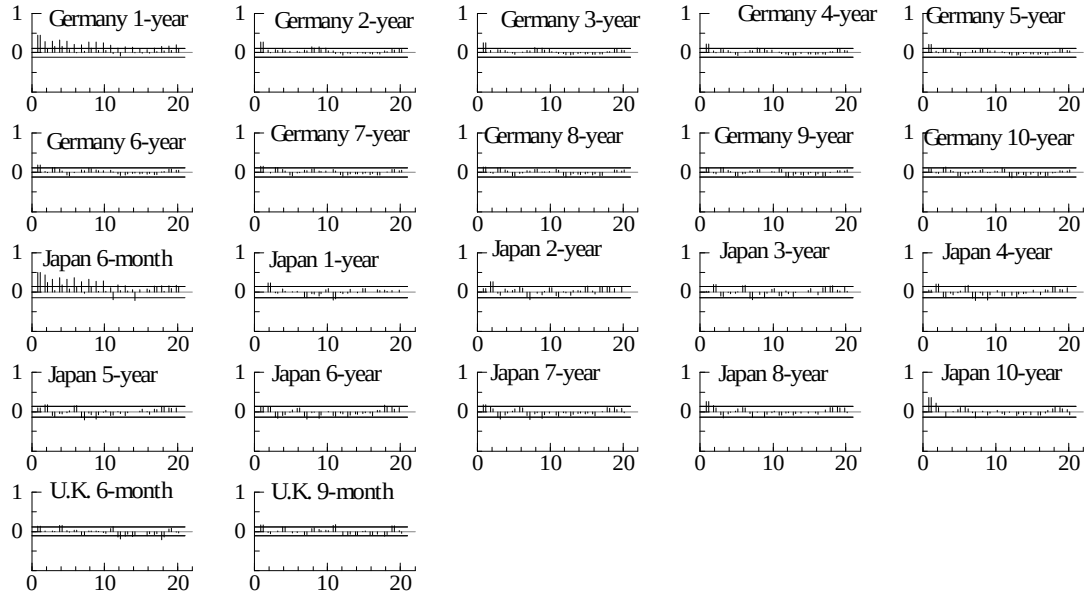


Figure 10: The ACF and PACF of Errors from the AFDNS Estimation: Part 1

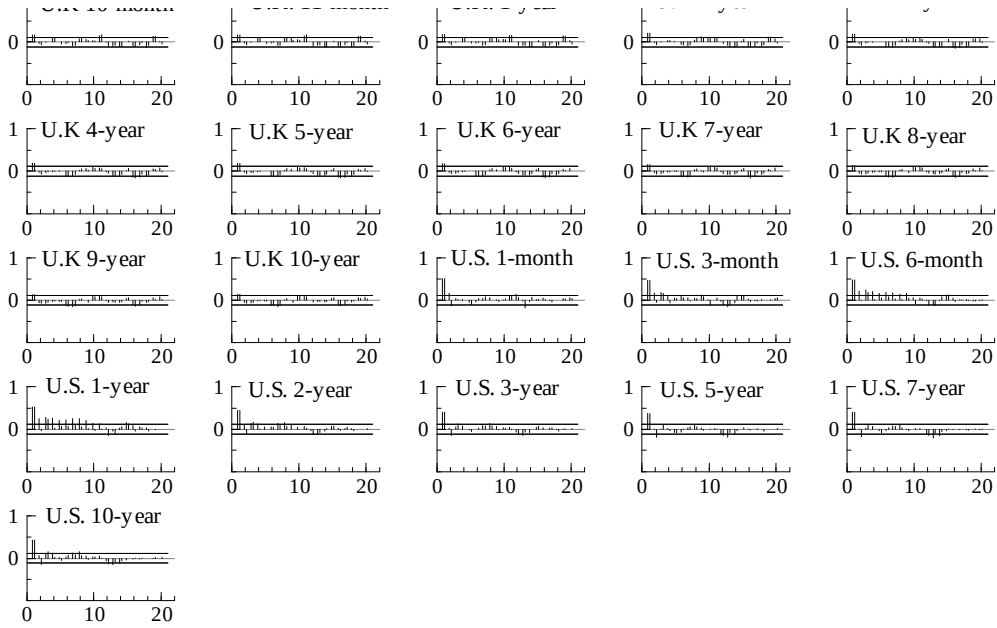


Figure 11: The ACF and PACF of Errors from the AFDNS Estimation: Part 2

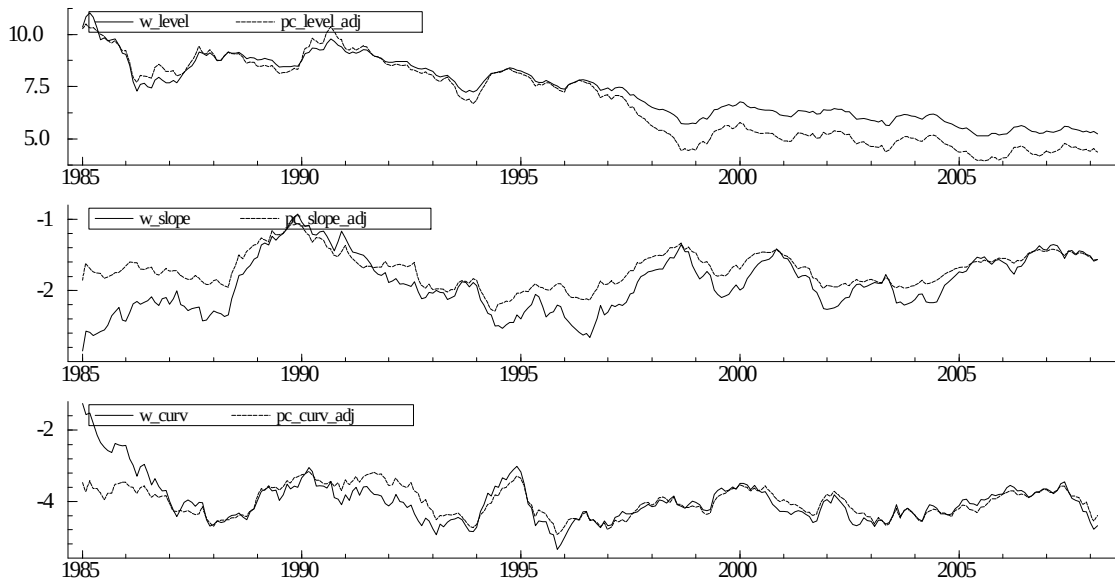


Figure 12: Global Factors from Kalman Filter Estimates and Principle Component Analysis(with appropriate adjustment)

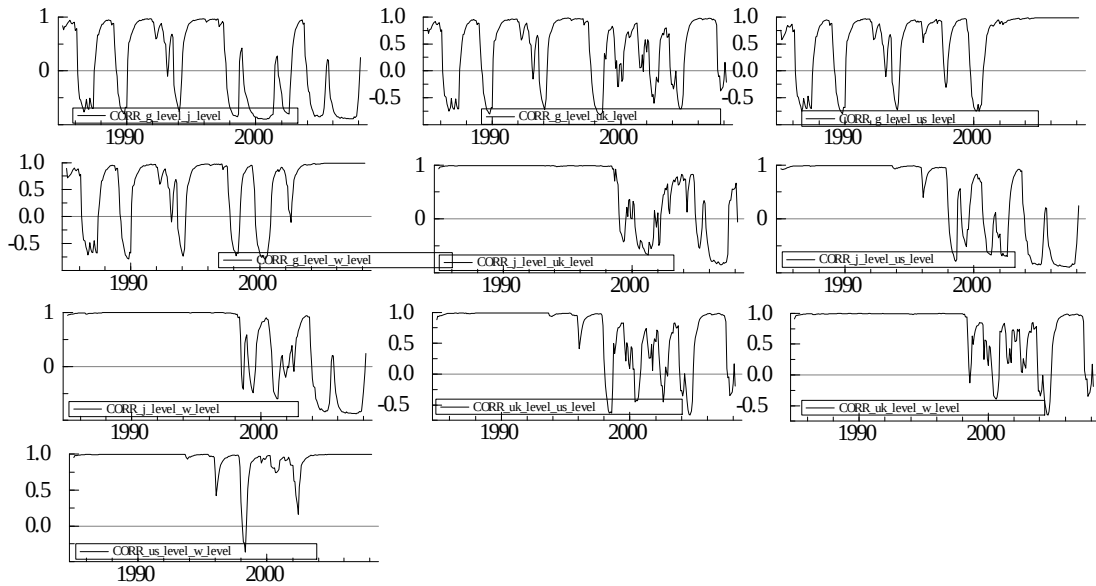


Figure 13: Dynamic Conditional Correlations: Level Factor

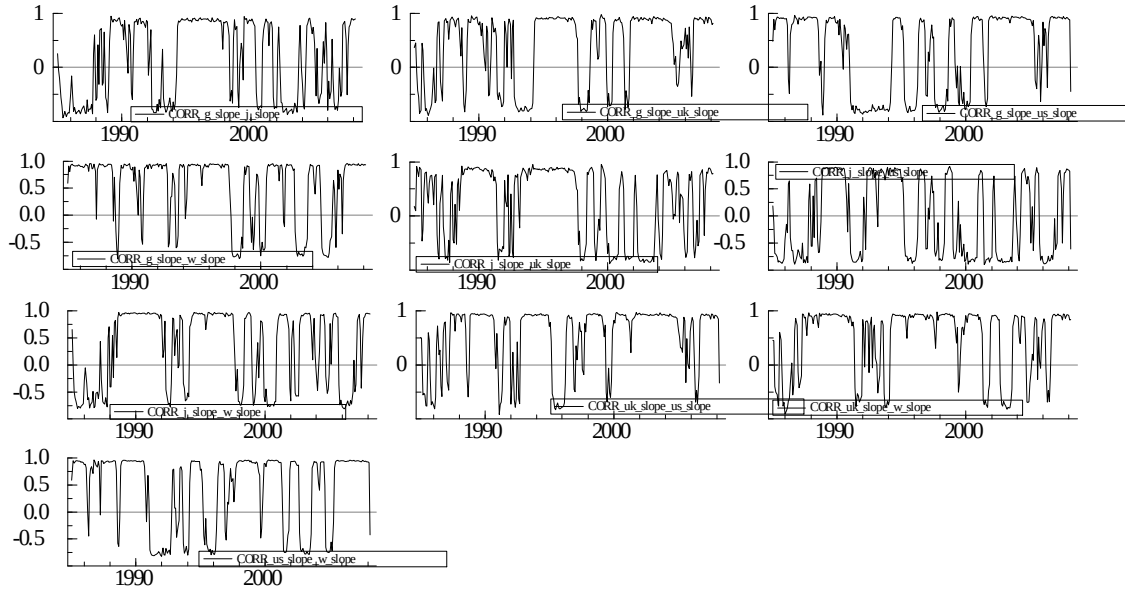


Figure 14: Dynamic Conditional Correlations: Slope Factor

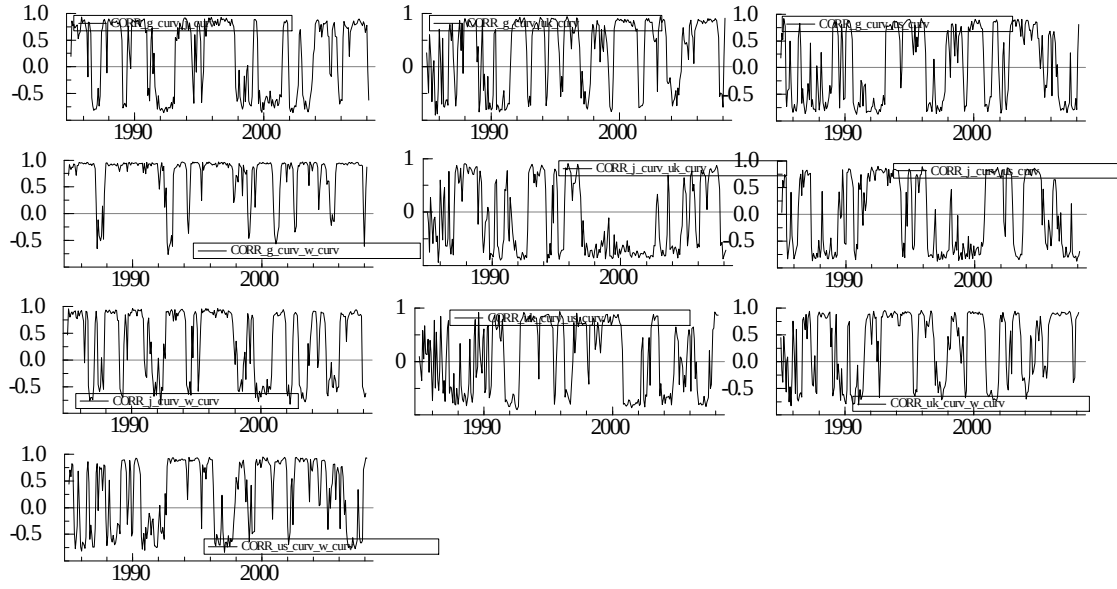


Figure 15: Dynamic Conditional Correlations: Curvature Factor

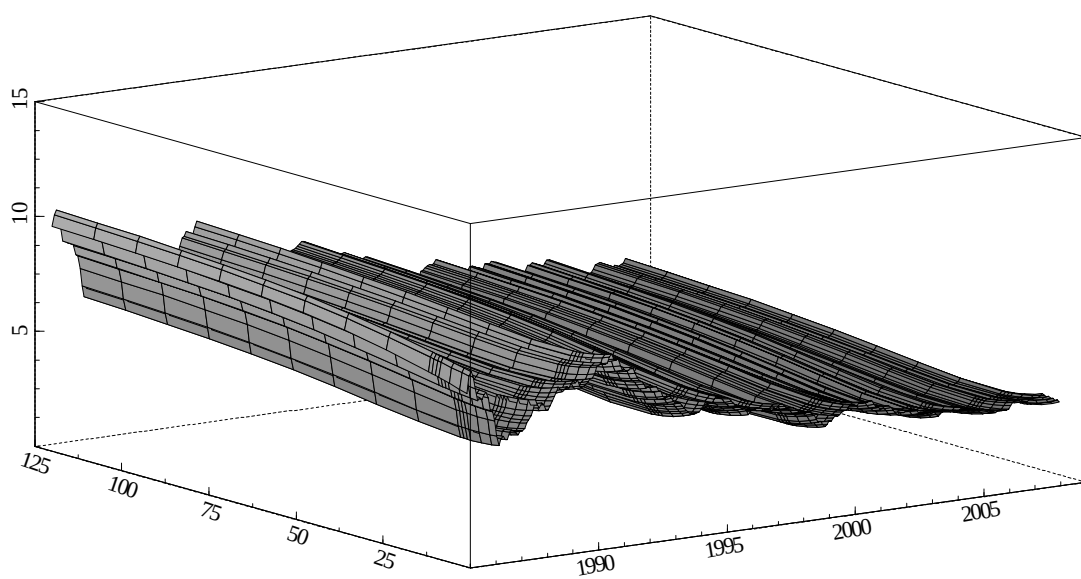


Figure 16: Global Yield Curves

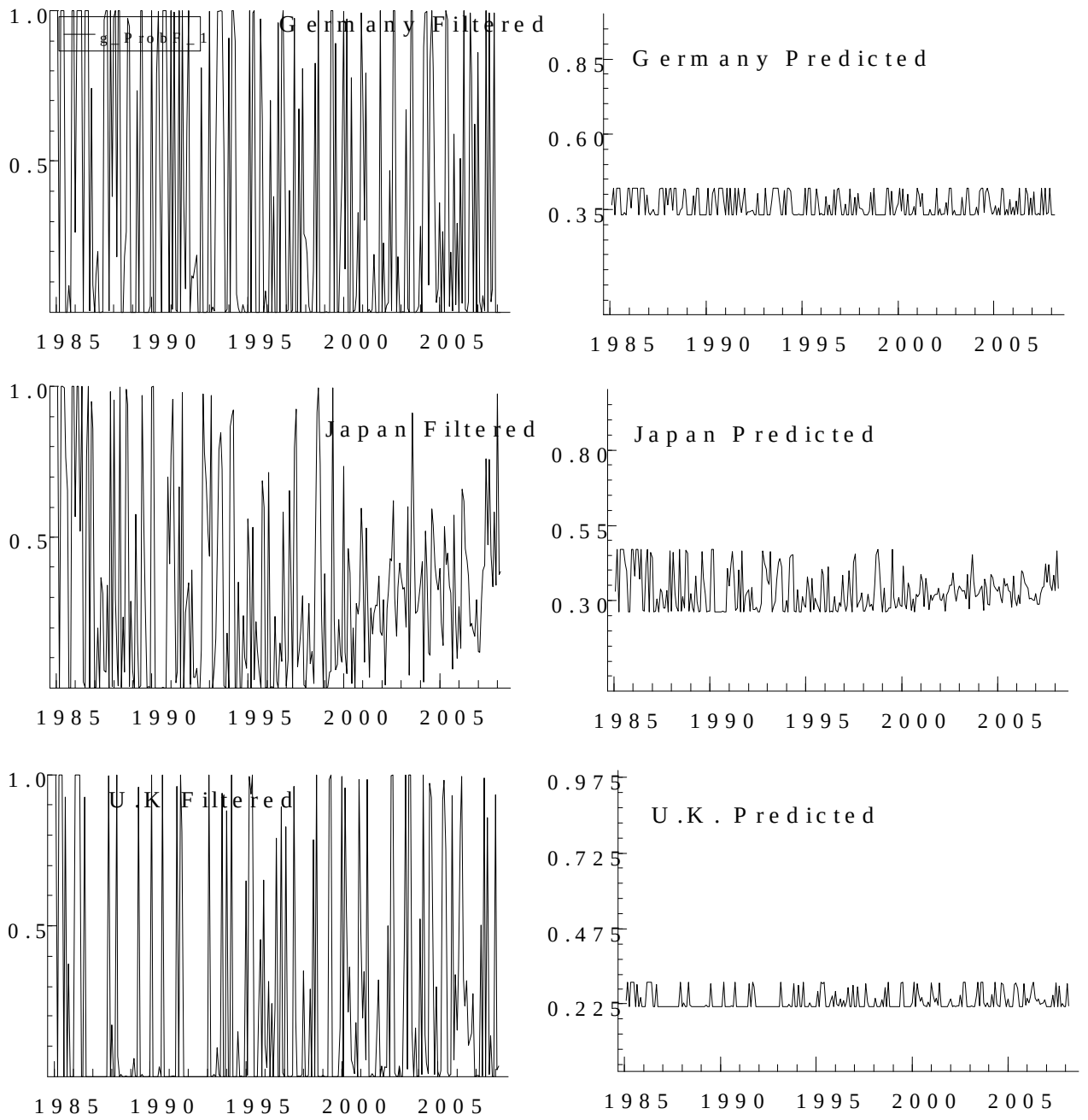


Figure 17: Filtered and Predicted Probabilities of Integration for Germany, Japan and the UK

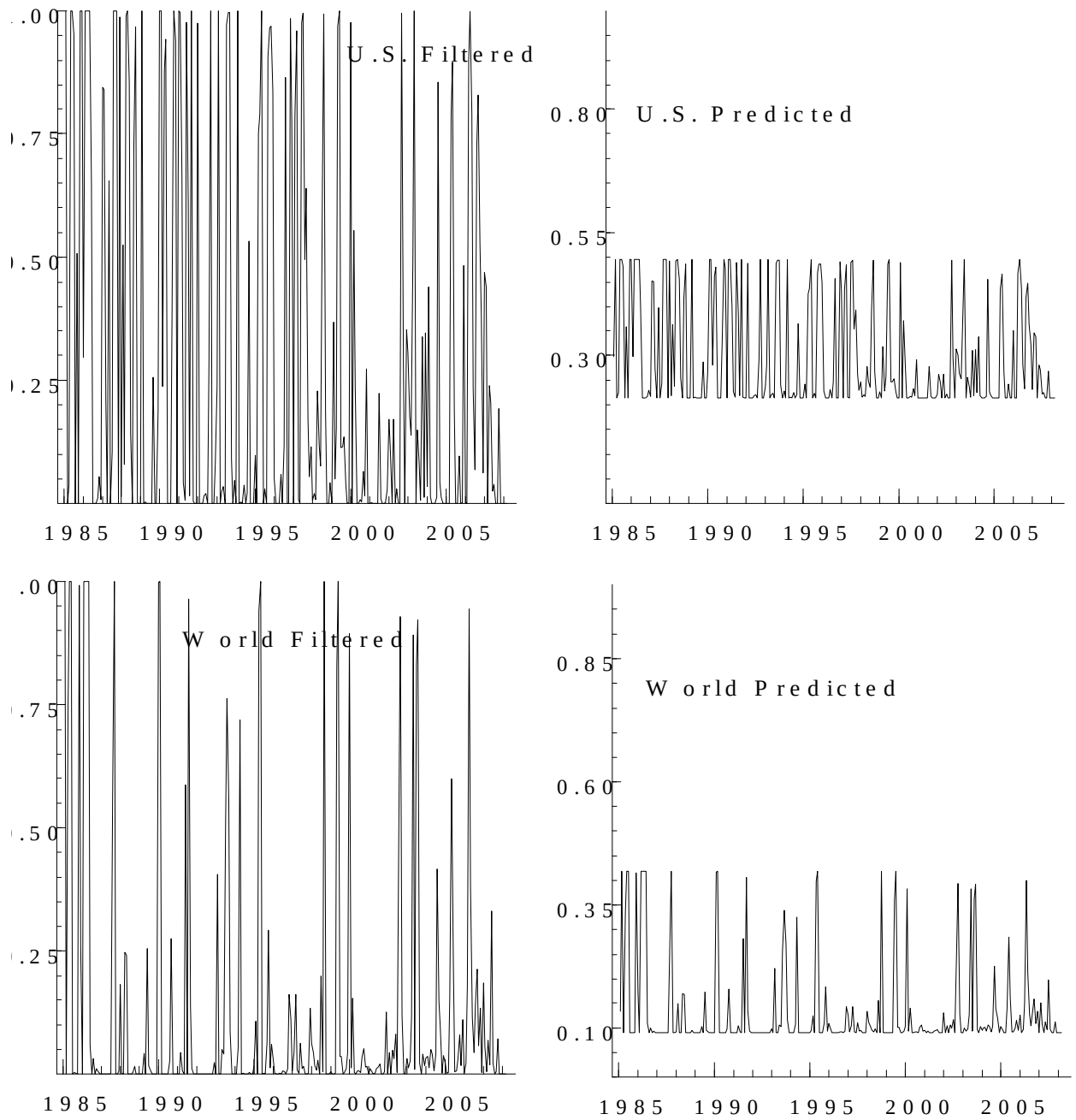


Figure 18: Filtered and Predicted Probabilities of Integration: the US and World