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Government Spending, Money Seigniorage and Macroeconomic Instability

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Abstract

This paper addresses the issue of the macroeconomic instability of the output effects of government spending financed by money seigniorage. The contribution of the paper is to show that these output effects are dependent on where the economy is in relation to certain inflation thresholds and that these thresholds are affected by the degree of ‘substitutability’ between government spending and private consumption. When government spending has no intertemporal effect on private consumption, there exists a single inflation threshold. When government spending has an intertemporal effect on private consumption, there exist two inflation thresholds. As the economy crosses each inflation threshold, it will suffer a reversal of output effects.

JEL Classification: E62, E52

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1. Introduction

In a classic paper, Cagan (1956) shows the actual rate of monetary growth and inflation in economies suffering from hyperinflation to far exceed the rate of seigniorage-maximizing monetary growth and inflation. For example, in the Hungarian hyperinflation of 1945-1946, the rate of seigniorage-maximizing monetary growth was estimated to be 32%, but the actual average monthly inflation rate was 12,200%. The reason that central banks keep increasing the rate of monetary growth and inflation to finance fiscal imbalances is simple. Imagine the central bank increases monetary growth from zero to some positive value in the first round to attain the seigniorage desired by the government. The rising inflation raises expected inflation and reduces the real demand for money, hence reducing real money seigniorage. As a result of the fall in seigniorage, the central bank would have to increase monetary growth in the second round to attain the desired seigniorage. This second-round increase in monetary growth increases inflation and reduces real money balances, lowering real seigniorage once more. This would then trigger a third-round of monetary growth, and so on. The result is high inflation and macroeconomic instability.

In the above scenario, the central bank does not hold rational expectations. Because it holds adaptive expectations, it systematically makes mistakes in attaining the desired seigniorage. Suppose the central bank holds rational expectations and is able to attain the seigniorage it desires.¹ This would then eliminate the macroeconomic instability pointed out above.

The objective of my paper is to show that, even with the assumption of rational expectations so that the macroeconomic instability mentioned above does not arise, there is another form of macroeconomic instability when government spending is financed by money seigniorage. This instability is that the output effects of such government spending can suffer reversals as

the inflation rate crosses certain threshold rates of monetary growth and inflation. The source of the macroeconomic instability of the output effects of government spending here arises from the interaction of two effects of government spending on savings. The first effect, called the intertemporal allocation effect, is due to intertemporal substitution between private consumption and government spending. An increase in government spending may increase private consumption and reduce saving if consumption is complementary with government spending, or decrease private consumption and increase saving if consumption is substitutable with government spending. The second effect, called the inflation effect, is due to the financing requirement of government spending. As government spending is increased, money seigniorage has to be increased; however, increasing money seigniorage is dependent upon where the economy is on the money-seigniorage Laffer curve. On the upward-sloping portion of the curve, increasing money seigniorage entails increasing monetary growth and inflation; on the downward-sloping portion of the curve, increasing money seigniorage entails decreasing monetary growth and inflation. Changes in inflation in turn changes savings. The interaction of the two effects causes an increase in government spending to affect savings, capital accumulation and output and result in expansionary or contractionary outcomes that depend on where the economy is in relation to the inflation thresholds mentioned earlier. This paper will show how and why the long-run effects of government spending financed by money seigniorage are dependent on the inflation thresholds and the degree of substitutability between government spending and private consumption.

2. Model Formulation

To achieve the objective of the paper, the model used is based on a deterministic version of the overlapping generations (OLG) model,² in which both money and government spending

enters into the utility function for consumers. All economic agents, including the government, hold rational expectations. However, I do not assume uncertainty.³

2.1 Consumers

Consumers live for two periods and form overlapping generations of constant size. In period t , a member of generation t supplies a unit of labor, earning a real wage rate of $\omega_t \equiv w_t - \tau_t$, where w_t is the pre-tax real wage rate and τ_t the labor-income tax. He consumes c_t^t of a private good and divides his income after consumption between saving s^t and holding a stock m_{t+1} of money at the end of period t , in real terms. Money earns no interest, while saving earns a real interest rate of r_{t+1} in period $t+1$. The consumer's first-period budget constraint is $P_t c_t^t + P_t s^t + P_t m_{t+1} \leq P_t \omega_t$, where P_t is the money price of the private good in period t . In period $t+1$, he retires and consumes c_{t+1}^t . His second period budget constraint is $P_{t+1} c_{t+1}^t \leq P_t m_{t+1} + (1 + i_{t+1}) P_t s^t$, where i_{t+1} is the nominal interest rate. Eliminating $P_t s^t$ from the budget constraints and using the exact relationship between the nominal and real interest rates, $(1 + i_{t+1}) = (1 + r_{t+1})(1 + \pi_{t+1})$, where π_{t+1} is the inflation rate in period $t+1$, the consumer's intertemporal budget constraint is $c_t^t + (1 + r_{t+1})^{-1} c_{t+1}^t + i_{t+1} (1 + i_{t+1})^{-1} m_{t+1} \leq \omega_t$. Each member of generation t also consumes a public good of amount g_t and g_{t+1} per capita in periods t and $t+1$ respectively.

Consumer preferences are represented by a continuous, strictly quasi-concave and increasing utility function $U(c^t, g^t, m_{t+1}) \equiv U(c_t^t, c_{t+1}^t, g_t, g_{t+1}, m_{t+1})$. Consumers maximize utility subject to their budget constraints. Under the consumer regularity condition assumed, consumption, saving and money holdings are uniquely determined:

$$c^t = c^t(\omega_t, r_{t+1}, \pi_{t+1}, g^t), s^t = s^t(\omega_t, r_{t+1}, \pi_{t+1}, g^t), m_{t+1} = m_{t+1}(\omega_t, r_{t+1}, \pi_{t+1}, g^t). \quad (1)$$

Assuming c_t^t and m_{t+1} are normal, satisfying $0 < \partial c_t^t / \partial \omega_t < 1$ and $0 < m_\omega \equiv \partial m_{t+1} / \partial \omega_t < 1$, respectively, the derivatives of $s^t(\omega_t, r_{t+1}, \pi_{t+1}, g^t)$ and $m_{t+1}(\omega_t, r_{t+1}, \pi_{t+1}, g^t)$ are summarized in:

Lemma 1: (a) $0 < s_\omega \equiv \partial s^t / \partial \omega_t < 1$; $s_r \equiv \partial s^t / \partial r_{t+1}$ is ambiguous in sign; $s_\pi \equiv \partial s^t / \partial \pi_{t+1} > 0$; $s_{g1} \equiv \partial s^t / \partial g_t, s_{g2} \equiv \partial s^t / \partial g_{t+1} \leq$ or > 0 according as $c_{g1} \equiv \partial c_t^t / \partial g_t, c_{g2} \equiv \partial c_t^t / \partial g_{t+1} \geq$ or < 0 . (b) $m_r \equiv \partial m_{t+1} / \partial r_{t+1} < 0$; $m_\pi \equiv \partial m_{t+1} / \partial \pi_{t+1} < 0$.

In Lemma 1, $c_{g1}, c_{g2} \geq 0$ is interpreted to mean that consumption of the private good in period t is complementary with, or independent of, consumption of the public good in periods t and $t+1$, and $c_{g1}, c_{g2} < 0$ to mean that consumption of the private good in period t is substitutable for consumption of the public good in periods t and $t+1$. It is assumed that money holdings are independent of the consumption of the public good. It is clear that inflation reduces the real return to money holdings, decreases real money holdings and increases saving.

2.2 Producers

Producers are perfectly competitive, and produce a single good using labor and capital. Capital is simply production of the good that is not consumed in the previous period. On a per-capita basis, k_t units of capital installed at the beginning of period t are employed together with one

unit of labor in period t to produce y_t units of output. Production is subject to constant returns to scale. The per-capita production function, $F(k_t)$, is continuous, strictly concave and increasing.

Following Tan (1995a), define a real unit-labor pre-wage profit function:

$$\Pi(r_t) \equiv \max\{y_t - r_t k_t : F(k_t) \geq y_t; (y_t, k_t) \geq 0\} \text{ for } r_t > 0. \quad (2)$$

Assume the profit function is twice differentiable. Under constant returns to scale, profits are assumed to be zero. Hence, the wage rate is:

$$w(r_t) = \Pi(r_t). \quad (3)$$

By Hotelling's (1932) Lemma, the demand for capital is

$$k(r_t) = -\partial \Pi(r_t) / \partial r_t. \quad (4)$$

Under the monotonicity assumption of $F(k_t)$, it is well known that $k_r \equiv \partial k_t / \partial r_t < 0$.

2.3 Government

At the beginning of period t , the government has a per-capita stock M_t of nominal money supply. During the period, the government issues new money and collects labor-income taxes of $P_t \tau_t$ to finance its expenditure on a public good of $P_t g_t$ per capita. The per-capita stock of

money will accumulate to M_{t+1} by the end of period t . The per-capita government-budget constraint for period t is

$$M_{t+1} - M_t + P_t \tau_t = P_t g_t . \quad (5a)$$

Dividing (5a) by P_t yields the real per-capita government budget constraint:

$$(1 + \pi_{t+1})m_{t+1} - m_t + \tau_t = g_t . \quad (5b)$$

Assuming equilibrium in the money market and substituting the money demand function in (1) into (5b) yields⁴

$$(1 + \pi_{t+1})m_{t+1}[w(r_t) - \tau_t, r_{t+1}, \pi_{t+1}] - m_t[w(r_{t-1}) - \tau_{t-1}, r_t, \pi_t] + \tau_t = g_t . \quad (5c)$$

Let g_t be the exogenous policy variable and τ_t be fixed at $\tau_t = \tau$. Then the policy variable, π_{t+1} , is endogenous:

$$\pi_{t+1} = \pi_{t+1}(r_{t+1}, r_t, \pi_t, g_t) . \quad (6a)$$

It follows that

$$\pi_t = \pi_t(r_t, r_{t-1}, \pi_{t-1}, g_{t-1}) . \quad (6b)$$

Since $(r_{t-1}, \pi_{t-1}, g_{t-1})$ are predetermined, they can be omitted from (6b):

$$\pi_t = \pi_t(r_t). \quad (6c)$$

Substituting (6c) into (6a) yields

$$\pi_{t+1} = \pi_{t+1}(r_{t+1}, r_t, g_t). \quad (6d)$$

In the steady state, (5c) simplifies to

$$\pi m[w(r) - \tau, r, \pi] + \tau = g. \quad (5')$$

Then (6d) becomes simply

$$\pi = \pi(r, g), \quad (6')$$

where

$$\begin{aligned} \pi_r &\equiv \partial \pi / \partial r = \pi(km_\omega - m_r) / (m + \pi m_\pi), \\ \pi_g &\equiv \partial \pi / \partial g = 1 / (m + \pi m_\pi). \end{aligned} \quad (7')$$

In general, π_r and π_g are ambiguous in sign. (7') will be used to determine the signs of the derivatives in the next two subsections.

2.4 Capital Market

The capital market is in equilibrium at the end of period t when

$$s^t(\omega_t, r_{t+1}, \pi_{t+1}, g^t) = k(r_{t+1}). \quad (8a)$$

The equilibrium in (8a) is partial equilibrium. We are, however, interested in general equilibrium. To convert condition (8a) to one that expresses general equilibrium, use (8a) in conjunction with (6d) to define a (reduced-form) per-capita excess-supply-of-saving function:

$$\alpha_{t+1}(r_{t+1}, r_t, g^t) \equiv s^t[w(r_t) - \tau, r_{t+1}, \pi_{t+1}(r_{t+1}, r_t, \pi_t(r_t), g_t), g^t] - k(r_{t+1}). \quad (9)$$

This summary function embodies utility maximization by consumers, profit maximization by producers, and compliance with the government budget constraint, and is introduced to simplify the analysis for the rest of the paper.

In the steady state, function (9) reduces to

$$\alpha(r, g) \equiv s[w(r) - \tau, r, \pi(r, g), g] - k(r). \quad (9')$$

The partial derivatives of $\alpha(r, g)$ are, respectively,

$$\begin{aligned} \alpha_r &\equiv \partial \alpha / \partial r = s_r - k_r - s_\omega k + s_\pi \pi_r, \\ \alpha_g &\equiv \partial \alpha / \partial g = s_\pi \pi_g + s_g. \end{aligned} \quad (10')$$

The sign of α_r will be determined in the next subsection. Using (7') and (10'), the sign of α_g is summarized in:

Lemma 2: (a) For $s_g = 0$, $\alpha_g > \text{or} < 0$ according as $\pi < \text{or} > \pi^* = -(m/m_\pi)$. (b) For $s_g < 0$, $\alpha_g > 0$ if $\pi_1 = -(m/m_\pi) - (s_\pi/s_g m_\pi) < \pi < \pi^* = -(m/m_\pi)$ and $\alpha_g < 0$ if $\pi < \pi_1$ or $\pi > \pi^*$. (c) For $s_g > 0$, $\alpha_g < 0$ if $\pi^* < \pi < \pi_2 = -(m/m_\pi) - (s_\pi/s_g m_\pi)$ and $\alpha_g > 0$ if $\pi < \pi^*$ or $\pi > \pi_2$.

Lemma 2 determines the sign of α_g for each of the three cases: public spending is independent of, complementary with, or substitutable for, private consumption.

2.5 General Equilibrium

Making use of the excess-supply-of-savings function defined by (9), the evolution of the economy is written simply as

$$\alpha_{t+1}(r_{t+1}, r_t, g_t) = 0. \quad (11)$$

A temporary (general) equilibrium is an $r_{t+1} > 0$ satisfying the capital-market equilibrium condition (11) for given (r_t, g_t) . In the steady state, the economy settles down to

$$\alpha(r, g) = 0. \quad (11')$$

A steady-state equilibrium is an $r > 0$ satisfying equilibrium condition (11') for a given g .

The condition for stability of the steady-state equilibrium is summarized in

Lemma 3: Under consumption normality and local Walrasian stability at a temporary equilibrium, the steady-state equilibrium is locally dynamically stable only if $\alpha_r > 0$.⁵

2.6 Model

For the purpose of this paper, the model comprises

$$\alpha(r, g) = 0, \tag{11'}$$

$$k = k(r). \tag{12'}$$

Equation (11') determines the real interest rate for a given g . With r determined, (12') determines the capital-labor ratio.

The issue to be addressed is the effect of a change in g on k .

3. Results

Suppose that government spending is increased by a small amount $dg > 0$. The model comprising (11')-(12') can be differentiated at the initial equilibrium to yield:

$$\alpha_r dr = -\alpha_g dg, \tag{13'}$$

$$dk = k_r dr. \tag{14'}$$

Using (13'), as a result of increasing government spending by $dg > 0$, the real interest rate in the steady state will change by

$$dr = -(\alpha_r)^{-1} \alpha_g dg . \quad (15')$$

Using (15') to eliminate dr from (14'), the capital-labor ratio in the steady state will change by

$$dk = -k_r (\alpha_r)^{-1} \alpha_g dg . \quad (16')$$

Since $k_r < 0$, $\alpha_r > 0$ by Lemma 3, and $dg > 0$ by assumption, we have $dk > \text{or} < 0$ according as $\alpha_g > \text{or} < 0$. Using Lemma 2, we have the following propositions:

Proposition 1: Assume $s_g = 0$. Let government spending be increased by a small amount $dg > 0$. Then, subject to the assumptions of the model, the capital-labor ratio satisfies $dk > \text{or} < 0$ according as $\pi < \text{or} > \pi^* = -(m/m_\pi)$.

Proposition 2: Assume $s_g < 0$. Let government spending be increased by a small amount $dg > 0$. Then, subject to the assumptions of the model, the capital-labor ratio satisfies $dk > 0$ if $\pi_1 = -(m/m_\pi) - (s_\pi/s_g m_\pi) < \pi < \pi^* = -(m/m_\pi)$ and $dk < 0$ if $\pi < \pi_1$ or $\pi > \pi^*$.

Proposition 3: Assume $s_g > 0$. Let government spending be increased by a small amount $dg > 0$. Then, subject to the assumptions of the model, the capital-labor ratio satisfies $dk > 0$ if $\pi < \pi^*$ or $\pi > \pi_2 = -(m/m_\pi) - (s_\pi/s_g m_\pi)$ and $dk < 0$ if $\pi^* < \pi < \pi_2$.

What do Propositions 1-3 say about macroeconomic instability? Proposition 1 shows that, when public and private spending are independent, there is a single inflation threshold of $\pi^* = -(m/m_\pi)$, the crossing of which leads to a reversal of the effects of fiscal policy. Increasing government spending is expansionary below π^* but contractionary above it. Proposition 2 shows that, when public and private spending are complementary, there are two inflation thresholds, $\pi_1 = -(m/m_\pi) - (s_\pi/s_g m_\pi)$ and π^* . Crossing each of these thresholds again leads to a reversal of the effects of fiscal policy. Below π_1 , increasing government spending is contractionary. Between π_1 and π^* , increasing government spending is expansionary. Crossing the threshold of π^* , increasing government spending is once more contractionary. Proposition 3 shows that, when public and private spending are substitutable, there are again two inflation thresholds, π^* and $\pi_2 = -(m/m_\pi) - (s_\pi/s_g m_\pi)$. Below π^* , increasing government spending is expansionary. Between π^* and π_2 , increasing government spending is contractionary. Crossing the threshold of π_2 , increasing government spending becomes once more expansionary. If governments have no knowledge of these inflation thresholds, do not care about the degree of substitutability between public and private spending but do care about the output effects of government spending, then they are in for some surprise.

To explain the economic rationale of the propositions, note from the identity of α_g in (10') that government spending affects the supply of savings through two channels. First, government spending affects savings through its effect on monetary growth and inflation. Second, government spending affects savings through its effect on private consumption. Call the first

channel, the inflation effect of government spending, and the second channel, the intertemporal allocation effect of government spending.

When public and private consumption are independent ($s_g = 0$), government spending has a positive effect on the supply of savings for initial inflation rates below the seigniorage-maximizing inflation rate of π^* and a negative effect for rates above π^* .⁶ The seigniorage-maximizing inflation rate is the inflation rate corresponding to the turning point of the money-seigniorage Laffer curve. In this case, only the first channel of government spending is operative. When the initial inflation rate is below π^* , the economy is on the upward-sloping portion of the money-seigniorage Laffer curve, along which money seigniorage increases with inflation, so an increase in government spending, which has to be financed by an increase in money seigniorage, entails raising monetary growth and inflation, hence increasing savings and capital accumulation. However, when the initial inflation rate exceeds π^* , the economy is on the downward-sloping portion of the money-seigniorage Laffer curve, along which money seigniorage decreases with increasing inflation, in which case an increase in government spending entails lowering monetary growth and inflation, so decreasing savings and capital accumulation.

When government spending and private consumption are complementary ($s_g < 0$), the second channel of government spending becomes operative in addition to the first. In this case, through the second channel, an increase in government spending increases private consumption and decreases savings. Superimposing this intertemporal allocation effect onto the inflation effect of government spending, it is clear that the intertemporal allocation effect reinforces the inflation effect above π^* but works against it below π^* . Below π^* , therefore, the net effect of government spending on savings depends on which effect

dominates. At low inflation rates (below $\pi_1 = -(m/m_\pi) - (s_\pi/s_g m_\pi)$),⁷ the inflation effect is weak and the intertemporal allocation effect dominates; hence, an increase in government spending decreases savings and capital accumulation. At higher inflation rates, between π_1 and π^* , the inflation effect dominates, so an increase in government spending increases savings and capital accumulation.

The rationale for the case where government spending and private consumption ($s_g > 0$) are substitutable can be similarly explained.

4. Concluding Remarks

This paper addresses the issue of the macroeconomic instability of the long-run output effects of government spending financed by money seigniorage. The contribution of the paper is to show that the output effects are dependent on where the economy is in relation to certain inflation thresholds and that these thresholds are affected by the degree of ‘substitutability’ between government spending and private consumption. When government spending has no intertemporal effect on private consumption, there exists a single inflation threshold. When government spending has an intertemporal effect on private consumption, there exist two inflation thresholds. As the economy crosses each inflation threshold, the economy suffers a reversal of the output effects. While the macroeconomic instability of hyperinflations based on adaptive expectations, as in the work of Cagan (1956), is well known, the macroeconomic instability identified in this paper, based on rational expectations, appears not to have been documented in the literature.

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Notes

¹ We are not assuming that the central bank is necessarily aiming for the seigniorage-maximizing rate of monetary growth.

² Samuelson (1958) and Allais are pioneers of the OLG model. See Malinvaud (1987) on Allais' publication of the OLG model in 1947. Most deterministic versions of the OLG model are usually descendants of Diamond's (1965) version of the OLG model.

³ In the absence of uncertainty, assuming rational expectations is equivalent to assuming perfect foresight.

⁴ Bear in mind that money holdings are independent of consumption of the public good.

⁵ See Tan (1995b) for a proof of the stability condition. The existence and uniqueness of the steady-state equilibrium are also considered in Tan (1995b).

⁶ The sign of $\pi^* = -(m/m_\pi)$ is positive since m_π is negative.

⁷ Since m_π is negative, s_π is positive and s_g is negative when public and private consumption are complementary, $\pi_1 = -(m/m_\pi) - (s_\pi/s_g m_\pi)$ is less than $\pi^* = -(m/m_\pi)$.