Nanyang Technological University
Division of Physics and Applied Physics

COURSE CONTENT FOR PH3502/MH3320

Academic Year	2023/24	**Semester**	2
Course Coordinator	Chew Lock Yue		
Course Code	PH3502/MH3320[1]		
Course Title	Dynamical System Theory with Chaos and Fractals		
Pre-requisites	PH2104 Analytical Mechanics (for PHY) MH1201 Linear Algebra II and MH2100 Calculus III (for MAS)		
Mutually Exclusive	MH3320/PH3502		
No of AUs	4 AU		
Contact Hours	Total 52 contact hours per semester (40 lecture hours + 12 tutorial hours)		
Proposal Date	10 October 2023		

Course Aims

This course aims to equip you with the basic concepts of determinism and randomness in the physical world. You will develop a basic understanding of dynamical system theory which is an essential component in physics, engineering, chemistry, biology, finance, and also the social sciences. You will also gain basic computational and analytical skills to solve and understand real-world problems involving chaotic and non-linear systems.

Intended Learning Outcomes (ILO)

Upon the successful completion of this course, you (as a student) would be **able to**:

Mathematical Formalism (MAT):

1. analyze and solve problems in linear system and linear dynamical system in N-dimensions mathematically and computationally
2. analyze and solve problems in nonlinear system and nonlinear dynamical system in N-dimensions mathematically and computationally
3. formulate first-order differential equations to model the evolution of diverse continuous-time dynamical phenomena and solve the equations mathematically and computationally
4. formulate recurrence equations to model the evolution of diverse discrete-time dynamical phenomena and solve the equations mathematically and computationally
5. Perform geometric analysis on the phase portrait of linear and nonlinear dynamical systems.
6. Perform mathematical definition and analysis of self-similar and fractal sets

Dynamical System Theory (DYN):

7. determine the fixed points, limit cycles/periodic orbits, strange and non-strange attractors of the dynamical system under-study analytically and numerically
8. determine the stability properties of the fixed points and limit cycles/periodic orbits of the dynamical system under-study analytically and numerically
9. gain the concept of stable manifold and unstable manifold as geometric structures in phase space that guide the flow of the dynamical trajectories
10. master the use of the phase portrait (as a geometric picture of phase space) that contains the set of fixed points, limit cycles, strange attractors, stable and unstable manifolds, as the solution to dynamical system problems
11. analyze dynamical systems that are chaotic, compute its trajectories, and yield the level

[1] Common course with PH3502

of chaos by evaluating its Lyapunov exponents numerically or analytically
12. account for the different type of bifurcations that occur in nonlinear dynamical systems

Fractals (FRA):
13. develop a clear understanding on the concepts of countable and uncountable sets
14. develop a good understanding on sets that have fractional dimension
15. construct and analyze fractal sets that are self-similar and non-self-similar
16. determine diverse fractal dimensions of fractal sets that are self-similar and non-self-similar

Course Content

The course consists of the following topics:

Introduction
- A dynamical view of the world
- What is nonlinear dynamics?

Linear Dynamical System
- Examples
- General formulation

Ingredients of a Dynamical System
- Phase space
- Evolution equations
- Initial conditions

Stability Properties of Linear Dynamical System
- Two-dimensional linear dynamical system
- N-dimensional linear dynamical system

Phase Portraits from the Stability Properties of Manifolds of Fixed Points in Continuous-Time Nonlinear Dynamical System
- Phase portraits
- Existence and uniqueness, no-intersection theorem
- Stability properties of fixed points in nonlinear dynamical systems
- Stable and unstable manifolds

Bifurcations
- Saddle-node bifurcation
- Transcritical bifurcation
- Pitchfork bifurcation
- Hopf bifurcation
- Global bifurcation of cycles

Lorenz Equations
- Linear stability of Poincaŕe orbits via Poincaŕe map
- Homoclinic and Heteroclinic orbits
- Bifurcations, chaos, and strange attractors

Nonlinear Mapping and their Dynamical Properties
- Fixed points and cobwebs

- Periodic points
- Logistic map
- The fully chaotic logistic map at A = 4
- Symbolic dynamics and the Bernoulli shift map

Fractals

- Countable and uncountable sets
- Cantor set
- Dimension of self-similar fractals
- Box dimension
- Hausdorff dimension

Assessment (includes both continuous and summative assessment)

Component	ILO Tested	Weighting	Team/Individual	Assessment Rubrics
1. Final Examination	All	50%	Individual	Point-based marking (not rubric based)
2. Continuous Assessment 1 (CA1): Problem Sets	All	10%	Individual	Point-based marking (not rubric based)
3. CA2: Project	All	40%	Team	Rubric based marking (see Appendix 1)
Total		100%		

For CA2 on project, the rubric based assessment is done on individual student while taking into account the teamwork and collaboration between the students who work in a team of 2 or at most 3. Each student in the team needs to indicate and submit their role in the project, perform a peer evaluation, and the rubric based marking is done on each student and not on each team.

Formative feedback

You will receive formative feedback which will be given through discussion within tutorial lessons.

Feedback will also be provided for each marked problem set, where any particularly problematic areas will be identified in the marked scripts.

Finally, feedback will be given constantly during lectures and tutorials on the common mistakes and level of difficulty of the course materials and applied examples/problems. Past exam questions and examiner's report are also made available for you, and will be discussed near the end of the course.

Learning and Teaching approach

Approach	How does this approach support students in achieving the learning outcomes?
Problem formulation and solution (lecture	Develop your competence in mathematical formulation in diverse inter-disciplinary contexts and the application of dynamical system theory to

and tutorial)	analyze and solve mathematical problems
Problem sets (tutorial)	Apply the theory and mathematical formulation learnt in class to solve problems in dynamical systems which develops your understanding, competence, and intuition on the topic, as well as attaining both analytical and computational skills at the same time.
Projects (lecture and tutorial)	Sharpen your knowledge in dynamical system theory through creatively working on a project in a team of two/three persons. The project enhances your analytical and computational skillsets as you work to deliver the requirements of the project. Furthermore, it trains you on your presentation and communication skills through project presentation and the answering of probing questions from peers and seniors during the question-and-answer session.

Reading and References

1. Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering, 2nd Edition, Steven Strogatz (2014). ISBN: 978-0738204536

NOTE: The above listing comprises the foundational readings for the course and more up-to-date relevant readings will be provided when they become available.

Course Policies and Student Responsibilities

Absence Due to Medical or Other Reasons

If you are sick and unable to attend your class, you have to:
1. Send an email to the instructor regarding the absence and request for a replacement class.
2. Submit the Medical Certificate* or official letter of excuse to administrator.
3. Attend the assigned replacement class (subject to availability).

* The medical certificate mentioned above should be issued in Singapore by a medical practitioner registered with the Singapore Medical Association.

Academic Integrity

Good academic work depends on honesty and ethical behaviour. The quality of your work as a student relies on adhering to the principles of academic integrity and to the NTU Honour Code, a set of values shared by the whole university community. Truth, Trust and Justice are at the core of NTU's shared values.

As a student, it is important that you recognize your responsibilities in understanding and applying the principles of academic integrity in all the work you do at NTU. Not knowing what is involved in maintaining academic integrity does not excuse academic dishonesty. You need to actively equip yourself with strategies to avoid all forms of academic dishonesty, including plagiarism, academic fraud, collusion and cheating. If you are uncertain of the definitions of any of these terms, you should go to the academic integrity website for more information.

On the use of technological tools (such as Generative AI tools), different courses / assignments have different intended learning outcomes. Students should refer to the specific assignment instructions on their use and requirements and/or consult your instructors on how you can use these tools to help your learning.

Consult your instructor(s) if you need any clarification about the requirements of academic integrity in the course.

Course Instructors

Instructor	Office Location	Phone	Email
Chew Lock Yue	SPMS-PAP-04-04	6316 2968	lockyue@ntu.edu.sg

Planned Weekly Schedule

Week	Topics (Refer to the above listed lecture)	Course ILO	Readings/ Activities
1	Introduction	MAT1	Lecture
2	Linear Dynamical System and Ingredients of Dynamical System,	MAT1	Lecture, Tutorial
3	Stabilities of Linear Dynamical System	MAT1, MAT3, MAT5, DYN 7-8	Lecture, Tutorial
4	Phase Portraits	MAT 2-3, MAT5, DYN 7-8	Lecture, Tutorial
5	Bifurcations	DYN 12	Lecture, Tutorial
6	Poincaŕe map, Homoclinic and Heteroclinic orbits	DYN 9	Lecture, Tutorial
7	Lorenz equations	MAT 2-3, 5, DYN 9-11	Lecture, Tutorial
8	Lorenz equations	MAT 2-3, 5, DYN 9-11	Lecture, Tutorial
9	Nonlinear Mappings	MAT 2, 4-5, DYN 7-8, 10-11	Lecture, Tutorial
10	Nonlinear Mappings	MAT 2, 4-5, DYN 7-8, 10-11	Lecture, Tutorial
11	Fractals	MAT6, FRA 13-14	Lecture, Tutorial
12	Fractals, Revision	MAT6, FRA 15-16	Lecture, Tutorial
13	Project Presentation	All	Project

Appendix 1: Rubrics for Project

The assessment criteria will be based on a score of 4:

- Excellently demonstrated – 4
- Mostly demonstrated – 3
- Moderately demonstrated – 2
- Somewhat demonstrated – 1
- Never demonstrated – 0

Criteria Description	**Assessment**					**Score**
	Poor (0)	**Somewhat (1)**	**Moderately (2)**	**Good (3)**	**Excellent (4)**	
CREATIVITY AND RESOURCEFULNESS (10%) Ability to make new connections under constraint.	Never Demonstrated	Somewhat Demonstrated	Moderately Demonstrated	Mostly Demonstrated	Excellently Demonstrated	Max 4
KNOWLEDGE (7%) Ability to correctly apply the knowledge in the course.	Never Demonstrated	Somewhat Demonstrated	Moderately Demonstrated	Mostly Demonstrated	Excellently Demonstrated	Max 4
ANALYTICAL SKILL (7%) Ability to conduct rigorous mathematical and numerical analysis as well as perform computational simulation.	Never Demonstrated	Somewhat Demonstrated	Moderately Demonstrated	Mostly Demonstrated	Excellently Demonstrated	Max 4
PREPARATION (8%) Ability to systematically organize the idea, concepts, approach, analysis, results and conclusion in the presentation materials.	Never Demonstrated	Somewhat Demonstrated	Moderately Demonstrated	Mostly Demonstrated	Excellently Demonstrated	Max 4
CLASS PRESENTATION (8%) Ability to communicate clearly and effectively, and to give appropriate response and answers to questions raised on the project	Never Demonstrated	Somewhat Demonstrated	Moderately Demonstrated	Mostly Demonstrated	Excellently Demonstrated	Max 4
					Total	Max 20

Note: This rubric is for the assessment of individual student. Although the students work in a team of 2 or at most 3, each student in the team needs to indicate and submit their role in the project, and the rubric based marking is done on each student and not on each team.

Peer Review for teamwork evaluation

Criteria (Weighs)	**Score from 1 to 9*** (**1**: Never; **3**: Rarely; **5**: Occasionally; **7**: Frequently; **9**: Always)				
(For 6-members team)	*Member A*	*Member B*	*Member C*	*Member D*	*Member E*
Member name					
a. Fulfilling one's responsibilities duly (15%)					
Behaved responsibly--such as attend meetings punctually and regularly; participate in discussion; complete assigned tasks/roles punctually.	*Score from 1 to 9*				
	Qualitative comments/reasons				
b. Fulfilling one's responsibilities effectively (25%)					
Behaved and contributed effectively--such as quality of work produced; creativity of ideas; extensiveness of research and thinking.	*Score from 1 to 9*				
	Qualitative comments/reasons				
c. Managing interpersonal relationships (30%)					
Listened attentively to and sought inputs from others; helped team resolve conflicts and achieved common understanding to function effectively; promoted respect for others and differences; fostered camaraderie.	*Score from 1 to 9*				
	Qualitative comments/reasons				
d. Providing support to others to achieve goals (30%)					
Behaved fairly and ethically—such as sharing responsibilities and giving credits. Exhibited group citizenship behavior--such as helping others to learn and complete their work through guidance and encouragement; standing up for others when needed.	*Score from 1 to 9*				
	Qualitative comments/reasons				

* Score of 1 should be given only when a team member does not really deserve to be awarded any mark for the team assignment (i.e., zero mark) because the member either has not or has barely participated and/or contributed to the team assignment in any meaningful manner.