Nanyang Technological University
Division of Physics and Applied Physics

Academic Year	2019/20	**Semester**	1
Course Coordinator	Assoc. Prof. Chew Lock Yue		
Course Code	PH3502		
Course Title	Chaotic Dynamical Systems		
Pre-requisites	PH2104		
No of AUs	4 AU		
Contact Hours	PH3502 (3 hr – lecture; 1 hr – tutorial)		
Proposal Date	01/2019		

Course Aims

This course aims to equip you with the basic concepts of determinism and randomness in the physical world. You will develop a basic understanding of dynamical system theory which is an essential component in physics, engineering, chemistry, biology, and also the social sciences. You will also gain basic computational and analytical skills to solve and understand real-world problems involving chaotic and non-linear systems.

Intended Learning Outcomes (ILO)

Upon the successful completion of this course, you (as a student) would be **able to**:

Mathematical Formalism (MAT) :

1. analyze and solve problems in linear system and linear dynamical system in N-dimensions mathematically and computationally
2. analyze and solve problems in nonlinear system and nonlinear dynamical system in N-dimensions mathematically and computationally
3. formulate first-order differential equations to model the evolution of diverse continuous-time dynamical phenomena and solve the equations mathematically and computationally
4. formulate recurrence equations to model the evolution of diverse discrete-time dynamical phenomena and solve the equations mathematically and computationally
5. perform geometric analysis on the phase portrait of linear and nonlinear dynamical systems.
6. perform mathematical definition and analysis of self-similar and fractal sets

Dynamical System Theory (DYN) :

7. determine the fixed points, limit cycles/periodic orbits, strange and non-strange attractors of the dynamical system under-study analytically and numerically
8. determine the stability properties of the fixed points and limit cycles/periodic orbits of the dynamical system under-study analytically and numerically
9. interpret dynamical trajectories and phase diagrams of stable manifold and unstable manifold as geometric structures in phase space
10. interpret and deduce the phase portrait (as a geometric picture of phase space) that contains the set of fixed points, limit cycles, strange attractors, stable and unstable manifolds, as the solution to dynamical system problems
11. analyze dynamical systems that are chaotic, compute its trajectories, and yield the level of chaos by evaluating its Lyapunov exponents numerically or analytically
12. analyze and account for the different type of bifurcations that occur in nonlinear dynamical systems

Fractals (FRA) :

13. define and derive of countable and uncountable sets and fractional dimension on sets
14. construct and analyze fractal sets that are self-similar and non self-similar
15. determine diverse fractal dimensions of fractal sets that are self-similar and non self-similar

Course Content

Introduction
A dynamical view of the world
What is nonlinear dynamics?
General formulation of linear dynamics

Ingredients of a Dynamical System
Phase space
Evolution equations
Initial conditions

Stability Properties of Linear Dynamical System
Two-dimensional linear dynamical system
N-dimensional linear dynamical system

Phase Portraits from the Stability Properties of Manifolds of Fixed Points in Continuous-Time Nonlinear Dynamical System
Phase portraits
Existence and uniqueness, no-intersection theorem
Stability properties of fixed points in nonlinear dynamical systems
Stable and unstable manifolds

Bifurcations
Saddle-node bifurcation
Transcritical bifurcation
Pitchfork bifurcation
Hopf bifurcation
Global bifurcation of cycles

Lorenz Equations
Linear stability of Poincaŕe orbits via Poincaŕe map
Homoclinic and Heteroclinic orbits
Bifurcations, chaos, and strange attractors

Nonlinear Mapping and their Dynamical Properties
Fixed points and cobwebs
Periodic points
Logistic map
The fully chaotic logistic map at A = 4
Symbolic dynamics and the Bernoulli shift map

Fractals
Countable and uncountable sets
Cantor set
Dimension of self-similar fractals
Box dimension
Hausdorff dimension

Assessment (includes both continuous and summative assessment)

Component	Course LO Tested	Related Programme LO or Graduate Attributes	Weighting	Team / Individual	Assessment Rubrics
1. Final Examination	All	Competence (1, 2, 3, 4, 5, 6) Creativity (1, 2) Communication (1, 2)	50%	Individual	Point-based marking (not rubric-based)
2. CA1: Problem Sets	All	Competence (1, 2, 3, 4, 5, 6) Creativity (1, 2) Communication (1, 2)	10%	Individual	Point-based marking (not rubric-based)
3. CA2: Project	All	Competence (1, 2, 4, 5, 6) Creativity (1, 2) Communication (1, 2, 3) Character (1, 2, 3) Civic Mindedness (1)	40%	Team	Rubric-based marking* Creativity and Resourcefulness (10%), Knowledge (7%), Analytical Skills (7%), Preparation (8%), Class Presentation (8%)
Total			100%		

*Refer to the assessment rubric

Formative feedback

You will receive formative feedback which will be given through discussion within tutorial lessons.

Feedback will also be provided for each marked problem set, where any particularly problematic areas will be identified in the marked scripts.

Finally, feedback will be given constantly during lectures and tutorials on the common mistakes and level of difficulty of the course materials and applied examples/problems. Past exam questions and examiner's report are also made available for you, and will be discussed near the end of the course.

Learning and Teaching approach

Approach	**How does this approach support students in achieving the learning outcomes?**
Problem solving (tutorial and lecture)	Develop competence and perseverance in solving physics problems
Problem sets (homework)	Apply the theory and mathematical formulation learnt in class to solve problems in nonlinear dynamics in order to develop the understanding, competence, and intuition on the topic, as well as to develop both analytical and computational skills.
Projects (homework, lecture and tutorial)	Sharpen your knowledge in nonlinear dynamics through creatively working on a project in a team of two/three persons. It will also enhance your analytical and computational skills as you work to deliver the requirement of the project. Furthermore, your presentation and communication skills will be developed through project presentation, and answering critical questions from their peers and seniors during the Question and Answer session.

Reading and References

1. Nonlinear dynamics and Chaos, 2nd Edition, Steven Strogatz, (2014). ISBN 13: 9780813349107

Course Policies and Student Responsibilities

Absence Due to Medical or Other Reasons

If you are sick and unable to attend your class / Mid-terms, you have to:

1. Send an email to the instructor regarding the absence and request for a replacement class and make-up mid-terms.
2. Submit the original Medical Certificate* or official letter of excuse to administrator.
3. Attend the assigned replacement class (*subject to availability*) and make-up mid-terms.

* The medical certificate mentioned above should be issued in Singapore by a medical practitioner registered with the Singapore Medical Association.

Academic Integrity

Good academic work depends on honesty and ethical behaviour. The quality of your work as a student relies on adhering to the principles of academic integrity and to the NTU Honour Code, a set of values shared by the whole university community. Truth, Trust and Justice are at the core of NTU's shared values.

As a student, it is important that you recognize your responsibilities in understanding and applying the principles of academic integrity in all the work you do at NTU. Not knowing what is involved in maintaining academic integrity does not excuse academic dishonesty. You need to actively equip yourself with strategies to avoid all forms of academic dishonesty, including plagiarism, academic fraud, collusion and cheating. If you are uncertain of the definitions of any of these terms, you should go to the academic integrity website for more information. Consult your instructor(s) if you need any clarification about the requirements of academic integrity in the course.

Course Instructors

Instructor	Office Location	Phone	Email
Assoc. Prof. Chew Lock Yue	SPMS-PAP-04-04	6316 2968	lockyue@ntu.edu.sg

Planned Weekly Schedule

Week	Topic	Course LO	Readings/ Activities
1	Introduction	MAT1	Lecture
2	Linear Dynamical System and Ingredients of Dynamical System,	MAT1	Lecture, Tutorial
3	Stabilities of Linear Dynamical System	MAT1, MAT3, MAT5, DYN 7-8	Lecture, Tutorial
4	Phase Portraits	MAT 2-3, MAT5, DYN 7-8	Lecture, Tutorial
5	Bifurcations	DYN 12	Lecture, Tutorial
6	Poincaŕe map, Homoclinic and Heteroclinic orbits	DYN 9	Lecture, Tutorial
7	Lorenz equations	MAT 2-3, 5, DYN 9-11	Lecture, Tutorial
8	Lorenz equations	MAT 2-3, 5, DYN 9-11	Lecture, Tutorial
9	Nonlinear Mappings	MAT 2, 4-5, DYN 7-8, 10-11	Lecture, Tutorial
10	Nonlinear Mappings	MAT 2, 4-5, DYN 7-8, 10-11	Lecture, Tutorial
11	Fractals	MAT6, FRA 13-14	Lecture, Tutorial
12	Fractals, Revision	MAT6, FRA 15-16	Lecture, Tutorial
13	Project Presentation	All	Project

Assessment Rubric for Project

Category	**Description**	**Points**	**Remarks**
Creativity and Resourcefulness (10%)	Did the student come up with innovative ideas to scope the project or just adopt current available ideas?		
	Did the student implement new approaches or simply apply standard procedures?		
Knowledge (7%)	Did the student have a firm grasp of the physical theories behind the project?		
	Did the student have a clear appreciation on the purpose and motivation behind the project?		
Analytical Skills (7%)	Did the student perform analytical work mathematically or computationally?		
	Is the student able to ask good questions and present relevant answers to these questions in clear quantitative terms?		
Preparation (8%)	Did the student put in good effort to produce a piece of work of sufficient quantity and quality?		
	Did the student think both broadly and deeply in realizing the project to fruition?		
Class presentation (8%)	Are the slides informative? Are review slides included? Are the slides too cluttered or too sparse?		
	Was the presentation audible? Was it monotonous or did the student inject emphasis at key points?		
	Was the relevant ideas brought across clearly and concisely?		
	Did the student employ sound arguments in his/her reasoning?		
	Was the student able to make educated guesses in open ended questions?		

Graduate Attributes

What we want our graduates from Physics and Applied Physics to be able to do:

Upon the successful completion of the PHY, APHY and PHMA programs, graduates should be able to:

Competency	1	demonstrate a rigorous understanding of the core theories and principles of physics involving (but not limited to) areas such as classical mechanics, electromagnetism, thermal physics and quantum mechanics [PHMA only] demonstrate a rigorous understanding of the core theories and principles of mathematical sciences involving (but not limited to) areas such as analysis, algebra and statistical analysis
	2	read and understand undergraduate level physics content independently;
	3	make educated guesses / estimations of physical quantities in general;
	4	apply fundamental physics knowledge, logical reasoning, mathematical and computational skills to analyse, model and solve problems;
	5	develop theoretical descriptions of physical phenomena with an understanding of the underlying assumptions and limitations;
	6	critically evaluate and distinguish sources of scientific/non-scientific information and to recommend appropriate decisions and choices when needed;
	7	demonstrate the ability to design and conduct experiments in a Physics laboratory, to make measurements, analyse and interpret data to draw valid conclusions.

Creativity	1	propose valid approaches to tackle open-ended problems in unexplored domains;
	2	offer valid alternative perspectives/approaches to a given situation or problem.

Communication	1	describe physical phenomena with scientifically sound principles;
	2	communicate (in writing and speaking) scientific and non-scientific ideas effectively to professional scientists and to the general public;
	3	communicate effectively with team members when working in a group.

Character	1	uphold absolute integrity when conducting scientific experiments, reporting and using the scientific results;
	2	readily pick up new skills, particularly technology related ones, to tackle new problems;
	3	contribute as a valued team member when working in a group.

Civic Mindedness	1	put together the skills and knowledge into their work in an effective, responsible and ethical manner for the benefits of society.