Nanyang Technological University
Division of Physics and Applied Physics

# COURSE CONTENT FOR PAP777 for Undergraduates

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| **Academic Year** | 2021/22 | **Semester** | 2 |
| **Course Coordinator** | Assoc Prof. Chong Yidong | | |
| **Course Code** | PAP777 | | |
| **Course Title** | Graduate Quantum Mechanics | | |
| **Pre-requisites** | PH3101 Quantum Mechanics II or equivalent and<br>PHY (PPHY) or PHY (APHY) programme and CGPA 4.0 or higher | | |
| **No of AUs** | 4 | | |
| **Contact Hours** | Lectures: 39, Tutorial: 12<br>(3 hr lecture each week; 1 hr tutorial each week) | | |
| **Proposal Date** | 14 Feb 2022 | | |

**Course Aims**

The course covers advanced concepts in quantum mechanics, with a focus on multi-particle systems. You will learn how to quantitatively describe the scattering of quantum mechanical particles, the effects of scattering resonances, entanglement between quantum subsystems, and the quantization of fields such as the electromagnetic field. Key analytical and numerical methods taught in this course include quantum Green's functions, Fermi's golden rule, density matrices, and second quantization. This course provides the theoretical background for advanced courses in quantum field theory, high energy physics, and condensed matter physics.

**Intended Learning Outcomes (ILO)**

Upon the successful completion of this course, you (as a student) will be able to do the following:

**A. Scattering Theory:**

1. formulate the quantum mechanical scattering problem in terms of scattering amplitudes and cross sections.
2. describe the scattering of non-relativistic quantum particles using quantum mechanical Green's functions.
3. use the first Born approximation to calculate scattering amplitudes for a given scattering potential.
4. discuss the physical origin of scattering resonances.
5. apply Fermi's Golden Rule to calculate resonance lifetimes.

**B. Entanglement:**

1. apply the postulates of quantum mechanics to systems of multiple particles.
2. discuss the Einstein-Podolsky-Rosen paradox and the implications of Bell's theorem.
3. calculate the entanglement entropy of a two-particle quantum state.
4. calculate the density matrix of a quantum subsystem, and use it to calculate measurement probabilities.
5. describe the many worlds interpretation of quantum mechanics, and how the measurement postulate emerges within this interpretation.

**C. Many-body Quantum Mechanics:**

1. express boson and fermion states using tensor product notation as well as second-quantized notation, and translate between the two representations.
2. formulate the second-quantized many-body Hamiltonian corresponding to a given first-quantized Hamiltonian with up to two-body interactions.
3. identify the ground state, and elementary excitations, of a given second quantized Hamiltonian describing non-interacting particles.

**D. Quantum Electrodynamics:**

1. derive the Hamiltonian of a non-relativistic charged particle in a classical

electromagnetic field.

2. determine the Aharonov-Bohm phase shift induced by a magnetic vector potential.
3. apply quantization rules to Maxwell's equations for a free electromagnetic field, and identify the excitations corresponding to photons.
4. calculate the rate of spontaneous emission by an excited state of an atom.

**Course Content**

**A. Scattering Theory**
Definition of scattering amplitudes, the differential scattering cross section, and the total scattering cross section. Special forms for incoming and outgoing waves in one, two, and three dimensions (e.g. Hankel functions). Definition and applications of the quantum mechanical Green's function. The Born series method for approximating scattering amplitudes. Definition of bound states, free states, and quasi-bound states (resonances). Derivation and applications of Fermi's Golden Rule.

**B. Entanglement**
How the postulates of quantum mechanics apply to multi-particle systems. Linear algebraic rules for tensor products. Calculations of partial measurement probabilities. The concept of quantum entanglement. Formulation of the Einstein-Podolsky-Rosen thought experiment, and Bell's Theorem. Definition of quantum mechanical entropy, and how to calculate it. Density matrices. The many worlds interpretation and its philosophical implications.

**C. Many-body Quantum Mechanics**
Definition of particle-exchange symmetry. Boson states and fermion states, expressed via tensor product notation as well as creation/annihilation operator notation. Use of creation/annihilation operators for expressing many-body Hamiltonians, and their eigenstates. Second quantization of single-particle quantum theories. Quantization of classical field theories.

**D. Quantum Electrodynamics**
Quantization of the non-relativistic Lorentz force law. The Aharonov-Bohm effect. The Dirac equation and its solutions in the presence of an electromagnetic field. Quantization of the source-free Maxwell equations, and the concept of the photon. Formulation of the electron-photon interaction. Calculation of the rate of spontaneous emission.

**Assessment (includes both continuous and summative assessment)**

| Component | Course LO Tested | Related Programme LO or Graduate Attributes | Weighting | Team / Individual | Assessment Rubrics |
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| 1. Final Examination | All | Competence 1,2,3,4,5<br>Creativity 2<br>Communication 1 | 40% | Individual | Point-based marking. Open-ended marking scheme. |
| 2. CA1: In-class quizzes | All | Competence 1,2,3,4,5<br>Creativity 2<br>Communication 1 | 20% | Individual | Point-based marking. Open-ended marking scheme. |
| 3. CA2: Project Report | All | Competence 1,2,3,4,5<br>Creativity 1,2<br>Communication 1,2,3 | 20% | Team | Rubrics marking (see Appendix 1 for the rubric) Open-ended |

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| | | | | | marking scheme. |
| 4. CA4: Mid-term Test | Topics A-C | Competence 1,2,3,4,5 Communication 1 | 20% | Individual | Point-based marking. |
| Total | | | 100% | | |

**Formative feedback**

Formative feedback is given through discussion within tutorial lessons, a discussion after the midterm, a written response to each project report, and an examiner's report for the final exam.

**Learning and Teaching approach**

| Approach | How does this approach support students in achieving the learning outcomes? |
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| Lectures | The lectures build up the central concepts of the course, and provide concrete examples for calculations of the type and difficulty students are expected to be able to do. |
| In-class quizzes | The in-class quizzes provide practice problems for the topics covered during the lectures in the current week, allowing students to apply and practice their newly-acquired knowledge. Feedback is provided for the quizzes of the previous week, to correct mistakes and misunderstandings. |
| Project | The students select a research topic of interest, and perform an in-depth study of the topic, culminating in a project report. This develops the ability to apply the principles developed in the lectures to new topics and scenarios outside the scope of the lectures. |

**Reading and References**

1. Online course notes.

## Course Policies and Student Responsibilities

*Absence Due to Medical or Other Reasons*

If you are sick and unable to attend your class / Mid-terms, you have to:

1. Send an email to the instructor regarding the absence and request for a replacement class and make-up mid-terms.
2. Submit the original Medical Certificate* or official letter of excuse to administrator.
3. Attend the assigned replacement class (*subject to availability*) and make-up mid-terms.

* The medical certificate mentioned above should be issued in Singapore by a medical practitioner registered with the Singapore Medical Association.

## Academic Integrity

Good academic work depends on honesty and ethical behaviour. The quality of your work as a student relies on adhering to the principles of academic integrity and to the NTU Honour Code, a set of values shared by the whole university community. Truth, Trust and Justice are at the core of NTU's shared values.

As a student, it is important that you recognize your responsibilities in understanding and applying the principles of academic integrity in all the work you do at NTU. Not knowing what is involved in maintaining academic integrity does not excuse academic dishonesty. You need to actively equip yourself with strategies to avoid all forms of academic dishonesty, including plagiarism, academic fraud, collusion and cheating. If you are uncertain of the definitions of any of these terms, you should go to the academic integrity website for more information. Consult your instructor(s) if you need any clarification about the requirements of academic integrity in the course.

## Course Instructors

| Instructor | Office Location | Phone | Email |
|---|---|---|---|
| Assoc Prof. Chong Yidong | SPMS-PAP-04-02 | +65 6316 2967 | Yidong@ntu.edu.sg |
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## Planned Weekly Schedule

| Week | Topic | Course LO | Readings/ Activities |
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| 1 | Scattering Theory | Formulate the quantum mechanical scattering problem in terms of scattering amplitudes and cross sections (A1). | Lecture |
| 2 | Scattering Theory | Describe the scattering of non-relativistic quantum particles using quantum mechanical Green's functions. Use the first Born approximation to calculate the scattering amplitudes for a given scattering potential (A2, A3). | Lecture and in-class quiz |

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| 3 | Scattering Theory | Discuss the physical origin of scattering resonances (A4). | Lecture and in-class quiz |
| 4 | Scattering Theory | Apply Fermi's Golden Rule to calculate resonance lifetimes (A5). | Lecture and in-class quiz |
| 5 | Entanglement | Apply the postulates of quantum mechanics to systems of multiple particles. Discuss the Einstein-Podolsky-Rosen paradox and the implications of Bell's theorem (B1, B2). | Lecture and in-class quiz |
| 6 | Entanglement | Calculate the entanglement entropy and reduced density matrix given a two-particle quantum state. Describe the many worlds interpretation of quantum mechanics, and how the measurement postulate emerges within this interpretation (B3, B4). | Lecture and Midterm Test |
| 7 | Many-body Quantum Mechanics | Express boson and fermion states using tensor product notation as well as second-quantized notation, and translate between the two representations (C1). | Lecture and in-class quiz |
| 8 | Many-body Quantum Mechanics | Formulate the second-quantized many-body Hamiltonian corresponding to a given first-quantized Hamiltonian with up to two-body interactions (C2). | Lecture and in-class quiz |
| 9 | Many-body Quantum Mechanics | Identify the ground state, and elementary excitations, of a given second quantized Hamiltonian describing non-interacting particles (C3). | Lecture, in-class quiz, and final project |
| 10 | Quantum Electrodynamics | Derive the Hamiltonian of a non-relativistic charged particle in a classical electromagnetic field (D1). | Lecture and final project |
| 11 | Quantum Electrodynamics | Determine the Aharonov-Bohm phase shift induced by a magnetic vector potential (D2). | Lecture, in-class quiz, and final project |
| 12 | Quantum Electrodynamics | Apply quantization rules to Maxwell's equations | Lecture and final project |

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| | | for a free electromagnetic field, and identify the excitations corresponding to photons. Calculate the rate of spontaneous emission by an excited state of an atom (D3, D4). | |
| 13 | All | Revision of course topics, and discussion of further advanced topics. | Lecture and final project |

**Appendix 1: Assessment Rubrics**

Assessment Component: CA 2: Project Report (20%)

Instructions: You are asked to do a short project to explore one of the course topics in greater detail. You may work either alone, or in pairs. If you want a partner, it is up to you to find one, and to figure out how to distribute the work. Both members of a team will get the same grade for the project. You must prepare a short project report (not more than 10 pages). The grade for the project will be entirely determined by the quality of the project report.

| Grading Criteria | Exceptional (18-20%) | Effective (15-17%) | Acceptable (10-14%) | Developing (<10%) |
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| **Learning outcomes A, B, C, or D (Depending on project topic.)** | Clearly-written and extensive discussion of project topic, containing an original take on the topic that does not simply follow existing sources. Clear formulation of theoretical topics, including concrete calculations and not just theoretical derivations. Project report is well-formatted according to the standards of the scientific literature. | Clearly-written and extensive discussion of project topic, roughly following the presentation of the topic in other sources, but including some original element(s). For example, the report may include an in-depth explanation of the details of a numerical calculation not presented in textbooks, or the application of a theoretical method to a specific circumstance not treated elsewhere Project report largely follows the formatting quality standards of the scientific literature. | Clearly-written and extensive discussion of project topic, mostly following the presentation in other sources, The discussion includes an attempt to utilize the discussed theoretical technique(s) to perform a concrete numerical calculation of some sort, along similar lines to examples presented elsewhere. Project report largely follows the formatting quality standards of the scientific literature. | Discussion of the project topic that is conceptually incomplete or lacks clear presentation. Derivative of other sources, lacking an attempt to perform an original calculation, or to apply the theoretical technique(s) in any non-standard direction. Project report fails to follow the formatting quality standards of the scientific literature. |

***Graduate Attributes***

***What we want our graduates from Physics and Applied Physics to be able to do:***

Upon the successful completion of the PHY, APHY, PHME and PHMS programs, graduates should be able to:

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| ***Competency*** | 1 | demonstrate a rigorous understanding of the core theories and principles of physics involving (but not limited to) areas such as classical mechanics, electromagnetism, thermal physics and quantum mechanics<br><br>[PHMS only] demonstrate a rigorous understanding of the core theories and principles of mathematical sciences involving (but not limited to) areas such as analysis, algebra and statistical analysis |
| | 2 | read and understand undergraduate level physics content independently; |
| | 3 | make educated guesses / estimations of physical quantities in general; |
| | 4 | apply fundamental physics knowledge, logical reasoning, mathematical and computational skills to analyse, model and solve problems; |
| | 5 | develop theoretical descriptions of physical phenomena with an understanding of the underlying assumptions and limitations; |
| | 6 | critically evaluate and distinguish sources of scientific/non-scientific information and to recommend appropriate decisions and choices when needed; |
| | 7 | demonstrate the ability to design and conduct experiments in a Physics laboratory, to make measurements, analyse and interpret data to draw valid conclusions. |

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| ***Creativity*** | 1 | propose valid approaches to tackle open-ended problems in unexplored domains; |
| | 2 | offer valid alternative perspectives/approaches to a given situation or problem. |

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| ***Communication*** | 1 | describe physical phenomena with scientifically sound principles; |
| | 2 | communicate (in writing and speaking) scientific and non-scientific ideas effectively to professional scientists and to the general public; |
| | 3 | communicate effectively with team members when working in a group. |

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| ***Character*** | 1 | uphold absolute integrity when conducting scientific experiments, reporting and using the scientific results; |
| | 2 | readily pick up new skills, particularly technology related ones, to tackle new problems; |
| | 3 | contribute as a valued team member when working in a group. |

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| ***Civic Mindedness*** | 1 | put together the skills and knowledge into their work in an effective, responsible and ethical manner for the benefits of society. |