GE REVISED COURSE CONTENT

Academic Year	2023	**Semester**	1
School/Programme	SPMS		
Course Coordinator	Assoc Prof Chew Lock Yue		
Course Code	PH7011		
Course Title	Nonlinear Dynamics		
Pre-requisites	For graduate students: No pre-requisites For undergraduates: PH3502/MH3320 Dynamical System Theory with Chaos and Fractals or equivalent		
No of AUs	4		
Contact Hours	52 contact hours (3 hours lecture each week; 1 hour tutorial each week)		
Proposal Date ***i.e. date proposal was drafted***	Oct 2023		
Expected Implementation date of new/revised course	Aug 2024		
Suggested Class Size	30		
Any cross-listing? ***Is course opened to all Postgraduate students (including IGP) or specific program (please indicate)?***	Yes, SPMS Master by Research		

Course Aims

This course aims to equip you with the basic concepts of determinism and randomness in the physical world. You will develop a basic understanding of dynamical system theory which is an essential component in physics, engineering, chemistry, biology, and the social sciences. You will also gain basic computational and analytical skills to solve and understand real-world problems involving chaotic and non-linear systems.

Intended Learning Outcomes (ILO)

By the end of this course, you should be able to:

Mathematical Formalism (MAT):

1. Analyse and solve problems in linear system and linear dynamical system in N-dimensions mathematically and computationally.
2. Analyse and solve problems in nonlinear system and nonlinear dynamical system in N-dimensions mathematically and computationally.
3. Formulate first-order differential equations to model the evolution of diverse continuous-time dynamical phenomena and solve the equations mathematically and computationally.
4. Formulate recurrence equations to model the evolution of diverse discrete-time dynamical phenomena and solve the equations mathematically and computationally.
5. Perform geometric analysis on the phase portrait of linear and nonlinear dynamical systems.
6. Perform mathematical definition and analysis of self-similar and fractal sets.

Dynamical System Theory (DYN):

7. Determine the fixed points, limit cycles/periodic orbits, strange and non-strange attractors of the dynamical system under-study analytically and numerically.

8. Determine the stability properties of the fixed points and limit cycles/periodic orbits of the dynamical system under-study analytically and numerically.
9. Explain the concept of stable manifold and unstable manifold as geometric structures in phase space that guide the flow of the dynamical trajectories.
10. Demonstrate mastery in the use of the phase portrait (as a geometric picture of phase space) that contains the set of fixed points, limit cycles, strange attractors, stable and unstable manifolds, as the solution to dynamical system problems.
11. Analyse dynamical systems that are chaotic, compute its trajectories, and yield the level of chaos by evaluating its Lyapunov exponents numerically or analytically.
12. Account for the different type of bifurcations that occur in nonlinear dynamical systems.

Fractals (FRA):

13. Explain the concepts of countable and uncountable sets.
14. Identify sets that have fractional dimension.
15. Construct and analyse fractal sets that are self-similar and non self-similar.
16. Determine diverse fractal dimensions of fractal sets that are self-similar and non self-similar.

Course Content

Introduction

- A dynamical view of the world
- What is nonlinear dynamics?

Linear Dynamical System

- Examples
- General formulation

Ingredients of a Dynamical System

- Phase space
- Evolution equations
- Initial conditions

Stability Properties of Linear Dynamical System

- Two-dimensional linear dynamical system
- N-dimensional linear dynamical system

Phase Portraits from the Stability Properties of Manifolds of Fixed Points in Continuous-Time Nonlinear Dynamical System

- Phase portraits
- Existence and uniqueness, no-intersection theorem
- Stability properties of fixed points in nonlinear dynamical systems
- Stable and unstable manifolds

Bifurcations

- Saddle-node bifurcation
- Transcritical bifurcation
- Pitchfork bifurcation
- Hopf bifurcation
- Global bifurcation of cycles

Lorenz Equations

- Linear stability of Poincaŕe orbits via Poincaŕe map
- Homoclinic and Heteroclinic orbits
- Bifurcations, chaos, and strange attractors

Nonlinear Mapping and their Dynamical Properties

- Fixed points and cobwebs
- Periodic points
- Logistic map
- The fully chaotic logistic map at A = 4
- Symbolic dynamics and the Bernoulli shift map

Fractals

- Countable and uncountable sets
- Cantor set
- Dimension of self-similar fractals
- Box dimension
- Hausdorff dimension

Assessment (includes both continuous and summative assessment)

Note: It is advised that Group component and class participation should not be more than 40% and 20% respectively, unless with good justification.

Component	**ILO Tested**	**Weighting**	**Team/Individual**	**Assessment Rubrics**
CA1: Problem Sets	All	20%	Individual	Point-based marking (not rubric-based)
CA2: Project	All	40%	Individual	Rubric-based marking, see Appendix 1
CA3: Project	All	40%	Individual	Rubric-based marking, see Appendix 1
Total		100%		

Description of Assessment Components:

CA1 consists of problem sets for students to practice the application of theory and mathematical formulation learnt in class by solving problems in dynamical systems. The purpose is to develop understanding, competence, and intuition on the topic, as well as attaining both analytical and computational skills. CA2 is known as a *Theme* project where each student works on a project of the same theme as defined by the course instructor. CA3 is known as a *Term* project where each student defines his/her own project related to nonlinear science and the project is encouraged to be relevant to his/her PhD research topic.

Formative feedback

You will receive formative feedback which will be given through discussion within tutorial lessons.

Feedback will also be provided for each marked problem set, where any particularly problematic areas will be identified in the marked scripts.

Finally, feedback will be given constantly during lectures and tutorials on the common mistakes and level of difficulty of the course materials and applied examples/problems. Past exam questions and examiner's report are also made available for you. They will be discussed near the end of the course.

Learning and Teaching Approach

Note: Please include and indicate TEL component.

Approach	**How does this approach support you in achieving the learning outcomes?**
Problem solving (tutorial and lecture)	This is to develop your competence and perseverance in solving physics problems
Problem sets (homework)	This enables you to apply the theory and mathematical formulation learnt in class to solve problems in nonlinear dynamics in order to develop the understanding, competence, and intuition on the topic, as well as to develop both analytical and computational skills.
Projects (homework, lecture and tutorial)	This will sharpen your knowledge in nonlinear dynamics through creatively working on a project in a team of two/three persons. It will also enhance your analytical and computational skills as you work to deliver the requirement of the project. Furthermore, your presentation and communication skills will be developed through project presentation and answering critical questions from your peers and seniors during the Question and Answer session.
Technology-Enhanced Learning (TEL)	This will test your understanding and address any misconception you have by providing instant feedback with correct solution. It will be conducted via online quizzes using Wooclap.

Reading and References

1. Nonlinear dynamics and Chaos, 2nd Edition, Steven Strogatz, (2014). ISBN-13: 978-0738204536; ISBN-10: 0738204536

Course Policies and Student Responsibilities

<u>Absence Due to Medical or Other Reasons</u>

If you are sick and unable to attend your class, you must:
1. Send an email to the instructor regarding the absence and request for a replacement class
2. Submit the original Medical Certificate* or official letter of excuse to administrator.
3. Attend the assigned replacement class (subject to availability)

*The medical certificate mentioned above should be issued in Singapore by a medical practitioner registered with the Singapore Medical Association.

Academic Integrity

Good academic work depends on honesty and ethical behaviour. The quality of your work as a student relies on adhering to the principles of academic integrity and to the NTU Honour Code, a set of values shared by the whole university community. Truth, Trust and Justice are at the core of NTU's shared values.

As a student, it is important that you recognize your responsibilities in understanding and applying the principles of academic integrity in all the work you do at NTU. Not knowing what is involved in maintaining academic integrity does not excuse academic dishonesty. You need to actively equip yourself with strategies to avoid all forms of academic dishonesty, including plagiarism, academic fraud, collusion and cheating. If you are uncertain of the definitions of any of these terms, you should go to the academic integrity website for more information. Consult your instructor(s) if you need any clarification about the requirements of academic integrity in the course.

Course Instructors

Instructor	Office Location	Phone	Email
Assoc Prof Chew Lock Yue	SPMS-PAP-04-04	6316 2968	lockyue@ntu.edu.sg

Industry Participation

Nil. This is an upstream science-based academic course without any relevant industry partner available for engagement.

Planned Weekly Schedule

Week	Topic	ILO	Readings/ Activities
1	Introduction	MAT1	Lecture
2	Linear Dynamical System and Ingredients of Dynamical System	MAT1	Lecture, Tutorial
3	Stabilities of Linear Dynamical System	MAT1, MAT3, MAT5, DYN 7-8	Lecture, Tutorial
4	Phase Portraits	MAT 2-3, MAT5, DYN 7-8	Lecture, Tutorial
5	Bifurcations	DYN 12	Lecture, Tutorial
6	Poincaŕe map, Homoclinic and Heteroclinic orbits	DYN 9	Lecture, Tutorial
7	Lorenz equations	MAT 2-3, 5, DYN 9-11	Lecture, Tutorial
8	Lorenz equations	MAT 2-3, 5, DYN 9-11	Lecture, Tutorial
9	Nonlinear Mappings	MAT 2, 4-5, DYN 7-8, 10-11	Lecture, Tutorial
10	Nonlinear Mappings	MAT 2, 4-5, DYN 7-8, 10-11	Lecture, Tutorial
11	Fractals	MAT6, FRA 13-14	Lecture, Tutorial
12	Fractals, Revision	MAT6, FRA 15-16	Lecture, Tutorial

13	Project Presentation	All	Project

Other information(s)

Nil

Appendix 1: Assessment Criteria for Project

The assessment criteria will be based on a score of 4:

- Excellently demonstrated – 4
- Mostly demonstrated – 3
- Moderately demonstrated – 2
- Somewhat demonstrated – 1
- Never demonstrated – 0

Criteria Description	Assessment					Score
	Poor (0)	**Somewhat (1)**	**Moderately (2)**	**Good (3)**	**Excellent (4)**	
CREATIVITY AND RESOURCENESS (10%) Ability to make new connections under constraint.	Never Demonstrated	Somewhat Demonstrated	Moderately Demonstrated	Mostly Demonstrated	Excellently Demonstrated	Max 4
KNOWLEDGE (7%) Ability to correctly apply the knowledge in the course.	Never Demonstrated	Somewhat Demonstrated	Moderately Demonstrated	Mostly Demonstrated	Excellently Demonstrated	Max 4
ANALYTICAL SKILL (7%) Ability to conduct rigorous mathematical and numerical analysis as well as perform computational simulation.	Never Demonstrated	Somewhat Demonstrated	Moderately Demonstrated	Mostly Demonstrated	Excellently Demonstrated	Max 4
PREPARATION (8%) Ability to systematically organize the idea, concepts, approach, analysis, results and conclusion in the presentation materials.	Never Demonstrated	Somewhat Demonstrated	Moderately Demonstrated	Mostly Demonstrated	Excellently Demonstrated	Max 4

CLASS PRESENTATION (8%) Ability to communicate clearly and effectively, and to give appropriate response and answers to questions raised on the project	Never Demonstrated	Somewhat Demonstrated	Moderately Demonstrated	Mostly Demonstrated	Excellently Demonstrated	Max 4
					Total	Max 20