Annexe A: New/Revised Course Content in OBTL+ Format

Course Overview

Expected Implementation in Academic Year	AY2024-2025
Semester/Trimester/Others (specify approx. Start/End date)	Semester 1
Course Author * Faculty proposing/revising the course	Francois Joachim Marcel Gay-Balmaz
Course Author Email	francois.gb@ntu.edu.sg
Course Title	Geometric methods in mathematical physics
Course Code	MH3702/PH3409
Academic Units	4
Contact Hours	51
Research Experience Components	Not Applicable

Course Requisites (if applicable)

Pre-requisites	MH2100 or MH1803
Co-requisites	Nil
Pre-requisite to	Nil
Mutually exclusive to	MH3702 mutually exclusive to PH3409 and vice versa
Replacement course to	Nil
Remarks (if any)	

Course Aims

This course is an introduction to the mathematical structures which underlie the description of physical systems, with a special emphasis on geometric concepts. You will acquire basic notions on manifolds, differential forms, curvatures, Lie groups, symmetries. You will also learn about the direct use of these concepts for the description of various physical phenomena and for the construction of a geometrically consistent formulation of Lagrangian and Hamiltonian systems. The understanding of such concepts leads to crucial insight into topics ranging from electromagnetism and quantum mechanics to plasmas physics and general relativity. Both the mathematical and intuitive understanding of the introduced objects will be emphasized.

This course is relevant for mathematics and physics students interested to later work in areas such as robotics, quantum computing, electromagnetic and acoustic sensing, control and learning. Students who want to pursue postgraduate degrees in pure or applied mathematics and theoretical physics will find this course relevant.

Course's Intended Learning Outcomes (ILOs)

Upon the successful completion of this course, you (student) would be able to:

ILO 1	Describe the basic notions of Lagrangian and Hamiltonian mechanics on vector spaces and understand the limitations of this approach.
ILO 2	Develop a mathematical and intuitive understanding of the notion of manifold and its need for the intrinsic description of physical systems.
ILO 3	Perform computations with vector fields, differential forms, and Lie derivatives, in the context of examples from electromagnetism, elasticity, and fluid dynamics.
ILO 4	Develop a mathematical and intuitive understanding of the notions of pseudo-Riemannian metrics, covariant derivatives, curvature, and their use in the context of special relativity and elasticity.
ILO 5	Integrate basic notions on Lie groups and Lie algebra and their role as configuration space and symmetry groups in dynamical systems.
ILO 6	Formulate the geometric formulation of Lagrangian and Hamiltonian systems on manifolds, their symmetries, and conservation laws.

Course Content

A first approach to Lagrangian and Hamiltonian mechanics

Approach in coordinates, Hamilton's principle, Legendre transforms, canonical transformations, Hamilton-Jacobi equations, Noether theorem, Poisson brackets,

limitation of the approach with coordinates.

Calculus on manifolds

Definition of manifolds, tangent space, vector fields, tensor fields, Lie derivative, differential forms, use of these concepts in electromagnetism, elasticity, and fluid dynamics.

Pseudo-Riemannian geometry and applications

Basic definition, covariant derivatives, various notions of curvature, use of these concepts in special relativity and elasticity.

Lie groups and Lie algebras

Basic definitions, classical Lie groups (orthogonal, unitary, Poincaré, Lorentz groups, etc.), action of Lie groups, symmetries.

Dynamical systems of manifolds and symmetries

Symplectic and Poisson structures, canonical and noncanonical Hamiltonian systems, symmetries, Noether theorem, rigid body dynamics, infinite dimensional examples from field theory, elasticity, fluid dynamics.

Reading and References

Readings:

J. Jost, Geometry and Physics, Springer, 2009. ISBN: 9783642005404

J.E. Marsden and T.S. Ratiu, Mechanics and Symmetry, Texts in Applied Mathematics, 1999. ISBN: 978-0-387-98643-2

G.L. Naber, The Geometry of Minkowski Spacetime, Applied Mathematical Sciences, 2011. ISBN: 978-1-4419-7837-0

S. C. Newman, Semi-Riemannian geometry : the mathematical language of general relativity, Hoboken, New Jersey : Wiley, 2019. ISBN: 9781119517535

A. Marsh, Mathematics for Physics, An Illustrated Handbook, World Scientific Publishing Co. Pte. Ltd., 2018. ISBN: 978-981-3233-91-1

NOTE: The above readings comprise the foundational readings for the course and more up-to-date relevant readings will be provided when they are available.

Planned Schedule

Week or Session	Topics or Themes	ILO	Readings	Delivery Mode	Activities
1	A first approach to Lagrangian and Hamiltonian mechanics: Approach in coordinates, Hamilton's principle, Legendre transforms, canonical transformations, Hamilton-Jacobi equations, Noether theorem, Poisson brackets, limitation of the approach with coordinates.	1	Marsden and Ratiu	In-person	Lecture
2	Calculus on manifolds: Definition of manifolds, tangent space, main properties, examples	2	Marsden and Ratiu Marsh	In-person	Lecture and tutorial
3	Calculus on manifolds: Vector fields, tensor fields, Lie derivative, examples	2	Marsden and Ratiu Marsh	In-person	Lecture and tutorial
4	Calculus on manifolds: Differential forms and exterior derivative, examples	2, 3	Marsden and Ratiu Marsh	In-person	Lecture and tutorial
5	Calculus on manifolds: Use of these concepts in electromagnetism, elasticity, and fluid dynamics	3	Marsden and Ratiu Naber	In-person	Lecture and tutorial
6	Pseudo-Riemannian geometry: Basic definition and covariant derivatives.	4	Newman	In-person	Lecture and tutorial
7	Pseudo-Riemannian geometry: Various notions of curvature, examples	4	Newman	In-person	Lecture and tutorial
8	Pseudo-Riemannian geometry: Use of these concepts in special relativity and elasticity	4	Marsden and Ratiu Naber Newman	In-person	Lecture and tutorial
9	Lie groups and Lie algebras Basic definitions, exponential map, classical Lie groups (orthogonal, unitary, Poincaré, Lorentz groups, etc.)	5	Marsden and Ratiu Marsh	In-person	Lecture and tutorial
10	Lie groups and Lie algebras Action of Lie groups, symmetries.	5	Marsden and Ratiu Marsh	In-person	Lecture and tutorial

Week or Session	Topics or Themes	ILO	Readings	Delivery Mode	Activities
11	Dynamical systems on manifolds and symmetries Symplectic and Poisson structures, canonical and noncanonical Hamiltonian systems, symmetries, Noether theorem.	6	Marsden and Ratiu	In-person	Lecture and tutorial
12	Dynamical systems on manifolds and symmetries Examples	6	Marsden and Ratiu Jost	In-person	Lecture and tutorial
13	Project presentations and revision			In-person	Lecture and tutorial

Learning and Teaching Approach

Approach	How does this approach support you in achieving the learning outcomes?
Lectures (39 hours)	The lectures consist in the presentation of the mathematical concepts as well as the intuitive justification behind them and their need in physical theories. Several fundamental examples are considered throughout the course.
Tutorials (12 hours)	The tutorials help the student to develop his/her intuition on the topic and proficiency in problem solving skills. They play a main role in reinforcing the concepts already covered in the lectures and in giving an opportunity for weaker or more reserved students to clarify doubts. The students lead the tutorial sessions by presenting their solutions to the problems they have selected for discussion by the group.

Assessment Structure

Assessment Components (includes both continuous and summative assessment)

No.	Component	ILO	Related PLO or Accreditation	Weightage	Team/Individual	Rubrics	Level of Understanding
1	Continuous Assessment (CA): Assignment(Point-based marking (non rubric-based))	1-4	Not Applicable	20	Individual	Analytic	Extended Abstract
2	Continuous Assessment (CA): Presentation(See Appendix 1)	1-6	Not Applicable	10	Individual	Analytic	Extended Abstract
3	Continuous Assessment (CA): Project(See Appendix 1)	1-6	Not Applicable	10	Individual	Holistic	Extended Abstract
4	Summative Assessment (EXAM): Final exam(Point-based marking (non rubric-based))	1-6	Not Applicable	60	Individual	Analytic	Extended Abstract

Description of Assessment Components (if applicable)

Appendix 1: Assessment Rubrics

Rubric for Presentation (10%)

Each student has to present a certain number of times during the semester (to be determined according to the number of course participants) and is graded individually in the style of an oral examination.

Rubric for Project (10%)
Each student has to give a final presentation in the last week of class. Presentations may be related to research or reading projects. Presentations are graded individually in the style of an oral examination.

Formative Feedback

There is a formal formative feedback designed as follows: 20% of the course assessment is for presenting homework (10%) as well as the results of a reading/research project at the end of the course (10%). Students will get informal but individual feedback from the peers and from the course instructor for each of these presentations.

NTU Graduate Attributes/Competency Mapping

This course intends to develop the following graduate attributes and competencies (maximum 5 most relevant)

Attributes/Competency	Level
Communication	Basic
Curiosity	Basic
Problem Solving	Basic
Transdisciplinarity	Basic

Course Policy

Policy (Academic Integrity)

Good academic work depends on honesty and ethical behaviour. The quality of your work as a student relies on adhering to the principles of academic integrity and to the NTU Honour Code, a set of values shared by the whole university community. Truth, Trust and Justice are at the core of NTU's shared values. As a student, it is important that you recognize your responsibilities in understanding and applying the principles of academic integrity in all the work you do at NTU. Not knowing what is involved in maintaining academic integrity does not excuse academic dishonesty. You need to actively equip yourself with strategies to avoid all forms of academic dishonesty, including plagiarism, academic fraud, collusion and cheating. If you are uncertain of the definitions of any of these terms, you should go to the academic integrity website for more information. On the use of technological tools (such as Generative AI tools), different courses / assignments have different intended learning outcomes. Students should refer to the specific assignment instructions on their use and requirements and/or consult your instructors on how you can use these tools to help your learning. Consult your instructor(s) if you need any clarification about the requirements of academic integrity in the course.

Policy (General)

You are expected to attend all lectures punctually and take all scheduled assignments, presentation by due dates. You are expected to take responsibility to follow up with course notes, assignments and course related announcements for lectures you have missed. You are expected to participate in all tutorial sessions.

Policy (Absenteeism)

If you are sick and unable to attend your class, you have to:
1. Send an email to the instructor regarding the absence.
2. Submit the Medical Certificate* or official letter of excuse to your Home school.

* The medical certificate mentioned above should be issued in Singapore by a medical practitioner registered with the Singapore Medical Association.

Policy (Others, if applicable)

Assessments:
1. A late submission for an assignment is not accepted.
2. No make-up assignments or presentations will be arranged.
3. A student who is late for an assignment or presentation without valid Leave of Absence will be given zero mark.
4. In case of a valid reason for absence, the total course marks would subsequently be rescaled to a base of 100%.

Appendix with Rubric (Assessment Criteria)

Appendix 1: Assessment Rubrics

Rubric for Presentation (10%)

Each student has to present a certain number of times during the semester (to be determined according to the number of course participants) and is graded individually in the style of an oral examination.

Grading Criteria	Exceptional (9-10)	Effective (7-8)	Acceptable (5-6)	Developing (0-4)
Accuracy	The interpretation is highly accurate, concise and precise.	The interpretation is mostly accurate. Some parts can be better explained or more succinct.	The interpretation is somewhat accurate. However, it contains some inaccuracies, missing points or ideas that are not related to the interpretation.	The interpretation is mostly inaccurate.
Visual and Oral delivery	Slides are informative, good clear voice and constant eye contact	Slides are adequate, oral delivery is satisfactory and frequent eye contact	Slides are bare and somewhat disorganized, voice is sometimes inaudible and little eye contact	Slides are disorganized, voice it too soft, and no eye contact
Presentation	Very clear and organized. It is easy to follow your train of thought	Mostly clear and organized. Some parts can have better transitions.	Somewhat clear. It requires some careful reading to understand what you are writing.	Mostly unclear and messy. It is difficult to understand what you are writing as there is no clear flow of ideas.
Question and Answer (for each individual student)	Very clear and precise answers to all problems. Explain the problems from various different perspectives logically.	Correct answers to most of the problems. Explain the problems in an organized way.	Partially-correct answers to most of the problems. Explain the some of the problems.	Unclear and messy answers. Difficult to understand.

Rubric for Project (10%)

Each student has to give a final presentation in the last week of class. Presentations may be related to research or reading projects. Presentations are graded individually in the style of an oral examination.

Grading Criteria	Exceptional (9-10)	Effective (7-8)	Acceptable (5-6)	Developing (0-4)
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