## COURSE OUTLINE: MH5100

| Course Title | Advanced Investigations in Calculus I |  |
| :--- | :--- | :--- |
| Course Code | MH5100 |  |
| Offered | Study Year X, Semester 1 |  |
| Course Coordinator | Wang Huaxiong (Assoc Prof) | hxwang@ntu.edu.sg |
| 65137472 |  |  |
| Pre-requisites | MH1100 or Approval by Division of Mathematical Sciences |  |
| Co-requisites | MH1100 |  |
| AU | 1 |  |
| Contact hours | Tutorials: 26 |  |
| Approved for delivery from | AY 2022/23 semester 2 |  |
| Last revised | 10 Jan 2023, 11:39 |  |

## Course Aims

This course aims to introduce advanced techniques and skills for students to solve challenging problems in calculus learnt from MH110. Students are required to read alongside MH1100 and division approval for course is needed. It covers the following contents in calculus: the fundamental mathematical concepts (functions, limits, continuity, derivatives and integrals); computation of derivatives (sum, product, and quotient formulas, chain rule, and implicit differentiation); and application of derivatives to optimization problems and related rates of change problems. This course lays the foundation for more advanced studies in mathematics, physics, engineering, and other related subjects.

## Intended Learning Outcomes

Upon successfully completing this course, you should be able to:

1. Demonstrate mastery of the basic concepts in calculus, such as functions, limits, continuity, derivatives and integrals.
2. Employ advanced approaches to evaluate limits, rates of change, derivatives, extreme values, etc.
3. Solve challenging problems for the limit and continuity of a function at a certain point or infinity.
4. Use the advanced calculus techniques, such as chain rules, curve sketching, and optimization.
5. Use mean value theorem to prove challenging problems
6. Select the right mathematical concepts and complicated models for real problems, such as those related to velocity and curve properties.
7. Apply the calculus techniques to solve related rate and optimization challenging problems.

## Course Content

Problems in complicated functions
Advanced problems to the concept of limit, limit laws, squeeze theorem
Advanced problems related to precise definition of limit: the epsilon-delta definition
Challenging problems to limits: Advanced topics such as the uniqueness of the limit, one-sided limits, limits to positive or negative infinity.

Challenging problems to the concept of continuity, continuous functions and limits, intermediate value theorem

Challenging problems to the definition of derivative, continuous and differentiable properties of a function

Challenging problems to differentiation rules, the calculus of the trigonometric functions
Challenging problems to chain rule
Challenging problems to the theory of extreme values, mean value theorem
Challenging problems to limits at infinity, curve sketching, Optimization problem
Challenging problems to Newton's method
Challenging problems to antiderivatives

## Assessment

| Component | Course ILOs tested | SPMS-MAS Graduate Attributes tested | Weighting | Team / Individual | Assessment Rubrics |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Continuous Assessment |  |  |  |  |  |
| Tutorials |  |  |  |  |  |
| Participation | $\begin{aligned} & 1,2,3,4,5,6, \\ & 7 \end{aligned}$ | 1. $a, b, c$ <br> 2. $a, b, c$ <br> 3. a, b | 30 | individual | See Appendix for rubric |
| Presentation | $\begin{aligned} & 1,2,3,4,5,6, \\ & 7 \end{aligned}$ | $\begin{aligned} & \text { 1. } a, b \\ & \text { 2. } b \\ & \text { 3. } a, b \end{aligned}$ | 20 | both | See Appendix for rubric |
| Mid-semester Quiz |  |  |  |  |  |
| Test in week 13 | $\begin{aligned} & 1,2,3,4,5,6, \\ & 7 \end{aligned}$ | $\begin{aligned} & \text { 1. a, b, c } \\ & \text { 2. } a, b, c \end{aligned}$ | 50 | individual | See Appendix for rubric |
|  |  | Total | 100\% |  |  |

These are the relevant SPMS-MAS Graduate Attributes.

## 1. Competence

a. Independently process and interpret mathematical theories and methodologies, and apply them to solve problems
b. Formulate mathematical statements precisely using rigorous mathematical language
c. Discover patterns by abstraction from examples
2. Creativity
a. Critically assess the applicability of mathematical tools in the workplace
b. Build on the connection between subfields of mathematics to tackle new problems
c. Develop new applications of existing techniques
3. Communication
a. Present mathematics ideas logically and coherently at the appropriate level for the intended audience
b. Work in teams on complicated projects that require applications of mathematics, and communicate the results verbally and in written form

## Formative Feedback

Students are given questions and time to solve problems, and present their solutions in the class. Instructors will discuss the solutions and common mistakes in student work. Online discussions are also available.

## Learning and Teaching Approach

| Tutorials <br> (26 | 1. Qualifying test - week 1 |
| :--- | :--- |
| hours) | 2. Problem solving and presentation - week 2-12 <br> a) Problem solving (30\%): Each student is supposed to present the solutions of at least 3 <br> questions. Acceptable if you use a different approach to solve the same problem already taken by <br> others. <br> b) Booking reading and presentation (20\%): 2-4 students are formed a group to read and <br> present a given part of content from a reference book. <br> 3. Test (50\%) - week 13 |

## Reading and References

Reference book: Principle of Mathematical Analysis, Walter Rudin, The 3rd edition, McGraw-Hill, Inc. ISBN-13 : 978-0070542358.

## Course Policies and Student Responsibilities

Absence Due to Medical or Other Convincing Reasons
If you are unable to attend a common test with a valid reason, you have to:

1. Send an email to the instructor regarding the absence.
2. Submit the original Medical Certificate* or documents (if it is not due to medical reason) to administrator (in School Office).

* The medical certificate mentioned above should be issued in Singapore by a medical practitioner registered with the Singapore Medical Association.


## Academic Integrity

Good academic work depends on honesty and ethical behaviour. The quality of your work as a student relies on adhering to the principles of academic integrity and to the NTU Honour Code, a set of values shared by the whole university community. Truth, Trust and Justice are at the core of NTU's shared values.

As a student, it is important that you recognize your responsibilities in understanding and applying the principles of academic integrity in all the work you do at NTU. Not knowing what is involved in maintaining academic integrity does not excuse academic dishonesty. You need to actively equip yourself with strategies to avoid all forms of academic dishonesty, including plagiarism, academic fraud, collusion and cheating. If you are uncertain of the definitions of any of these terms, you should go to the Academic Integrity website for more information. Consult your instructor(s) if you need any clarification about the requirements of academic integrity in the course.

## Course Instructors

Instructor Office Location Phone Email

| Wang Huaxiong (Assoc Prof) | SPMS-MAS-05-26 | 65137472 | hxwang@ntu.edu.sg |
| :--- | :--- | :--- | :--- | :--- |

## Planned Weekly Schedule

| We | Topic | Course ILO | Readings/ Activities |
| :---: | :---: | :---: | :---: |
| 1 | Qualifying test | 1, 2, 3, 4 | 2 hour qualifying test for course enrollment |
| 2 | Problems in complicated functions. <br> Advanced problems to the concept of limit, limit laws, squeeze theorem | 1,2,3 | Lecture and tutorial |
| 3 | Advanced problems related to precise definition of limit: the epsilon-delta definition | 2, 3, 6 | Lecture and tutorial |
| 4 | Challenging problems to limits: Advanced topics such as the uniqueness of the limit, one-sided limits, limits to positive or negative infinity. | 2, 3, 4, 6 | Lecture, tutorial and student presentation |
| 5 | Challenging problems to the concept of continuity, continuous functions and limits, intermediate value theorem | 2, 3, 4, 6 | Lecture, tutorial and student presentation |
| 6 | Challenging problems to the definition of derivative, continuous and differentiable properties of a function | 3, 4, 5 | Lecture, tutorial and student presentation |
| 7 | Challenging problems to differentiation rules, the calculus of the trigonometric functions | 3, 4, 5, 6 | Lecture, tutorial and student presentation |
| 8 | Challenging problems to chain rule | 3, 4, 5, 6 | Lecture, tutorial and student presentation |
| 9 | Challenging problems to the theory of extreme values, mean value theorem | 4, 5, 6, 7 | Lecture, tutorial and student presentation |
| 10 | Challenging problems to limits at infinity, curve sketching, Optimization problem | 4, 5, 6, 7 | Lecture and tutorial |
| 11 | Challenging problems to Newton's method | 4, 5, 6, 7 | Lecture and tutorial |
| 12 | Challenging problems to antiderivatives | 4, 5, 6, 7 | Lecture and tutorial |
| 13 | Final Test | $\begin{aligned} & 1,2,3,4 \\ & 5,6,7 \end{aligned}$ | 2 hour test |

## Appendix 1: Assessment Rubrics

## Rubric for Tutorials: Participation (30\%)

Each student is supposed to present the solutions of at least 3 questions.
Acceptable if you use a different approach to solve the same problem already taken by others.

| Criteria | Standards |  |  |
| :--- | :--- | :--- | :--- |
|  | Fail standard (0-14) | Pass standard (15-23) | High standard (24-30) |
| Participation | Lack participation | Participation takes place | Active participation, including <br> voluntering on questions and helping <br> other presenters |
| Presentation | Poor explanation or <br> incoherent <br> presentation | Explanation can be followed <br> but may have a few vague <br> points | Explanation is clear |

## Rubric for Tutorials: Presentation (20\%)

Everyone should join one reading group. Each group has at most 4 members.
Every week one reading group presents some materials in the reference book sequentially. The next group starts where the previous one stops.

Every group member is required to present 10-20 minutes. No more than 60 minutes in total for each group.

Please note that in principle, you would get the same score as your team. However, your score may vary should there be evidence that you had not contributed to your team.

| Criteria | Standards |  |  |
| :--- | :--- | :--- | :--- |
|  | Fail standard (0-9) | Pass standard(10-15) | High standard (16-20) |
| Methods of <br> approach | Using methods that are <br> irrelevant or do not apply to <br> the given problem. <br> Invoking theorems whose <br> conditions are not satisfied. | Using relevant methods <br> that help solve the <br> problem. <br> Invoking theorems whose <br> conditions are satisfied. | Finding methods and utilizing <br> theorems that are both relevant <br> and effective |
| Validity of <br> reasoning | Reasoning is logically invalid. | Reasoning is logically <br> valid. | Reasoning is logically valid and <br> effective. |
| Clarity of <br> argument | Reasoning is poorly explained <br> or not explained at all. | Reasoning is clear but <br> may contain some gaps. | Reasoning is clear, precise with <br> no or insignificant gaps. |

## Rubric for Mid-semester Quiz: Test in week 13 (50\%)

This quiz aims to test all topics that reflect the learning outcomes.
To prepare for this quiz, the students should understand the learning objectives of each lecture and practise the tutorial questions/exercises in lecture slides.

