COURSE OUTLINE: MH4601

Course Title	**Differential Geometry**		
Course Code	**MH4601**		
Offered	Study Year 4, Semester 1		
Course Coordinator	Andrew James Kricker (Assoc Prof)	ajkricker@ntu.edu.sg	6513 7458
Pre-requisites	MH3100 and MH3600		
AU	4		
Contact hours	Lectures: 39, Tutorials: 12		
Approved for delivery from	AY 2023/24 semester 1		
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Course Aims

This course will introduce the mathematical framework and most important fundamental concepts of classical Differential Geometry. Differential Geometry is the mathematical study of geometric shapes using tools from analysis such as calculus and differential forms. The ideas and tools are important not just in pure mathematics but play a fundamental role in many areas of physics such as general relativity and gauge theory, and in diverse parts of Engineering such as robotics and computational geometry.

The course aims to give you a foundational understanding in the most important topics within Differential Geometry.

1. The mathematical machinery of differential forms.
2. The classical theory of curves and embedded surfaces.
3. First steps towards abstract Riemannian geometry.

The aim is for you to be sufficiently prepared to continue deeper study in this topic and also to equip you so that when you encounter these ideas in different topics (such as in physics or in Engineering) then you have the ability to bring in an expert understanding of the theory and the ability to deepen your learning as is needed in the context.

Intended Learning Outcomes

Upon successfully completing this course, you should be able to:

1. Perform computations using calculus of differential forms.
2. Calculate the Frenet apparatus of a regular curve in Euclidean space.
3. State and prove the fundamental theorems of Frenet theory. In particular, construct the isometry relating two curves with isomorphic curvature and torsion functions.
4. Calculate the apparatus associated to a frame field, such as dual 1-forms, connection forms, and covariant derivatives. Develop the differential equations governing this apparatus.
5. Verify whether a given subset of Euclidean 3-space is a surface, either by using the definitions directly or by deploying the level set theorem.
6. Derive formula for doing calculus on a surface, such as curve parameterizations, change of co-ordinates formulae, and bases for tangent spaces.
7. Represent the push-forward map associated to a differentiable mapping via a Jacobian matrix.
8. Define the shape operator associated to a surface and develop its fundamental properties.
9. Calculate and interpret the various curvatures associated to a surface: the principal curvatures, the mean curvature and the Gaussian curvature.
10. Define the geodesics of a surface and set up the system of differential equations that determine them.

11. State and prove Gauss's Theorem Egregium.
12. State the Gauss-Bonnet theorem and explain some of the underlying concepts.

Course Content

The motivating problems and main themes of differential geometry.

Calculus of differential forms - tangent vector, tangent space, vector field, frame field, directional derivatives, 1-forms, general differential forms, algebraic structures forming the complex of differential forms.

The Frenet theory of curves - length of a curve segment, unit speed parametrization of curves, vector fields on curves, the curvature vector field and the curvature function, the Frenet apparatus and the system of differential equations that governs it.

The Calculus of frame fields - covariant derivatives of vector fields, connection forms, dual 1-forms, the system of differential equations governing the apparatus associated to a frame field.

Calculus of isometries of Euclidean space - orientations, the fundamental theorem of Frenet theory.

Surfaces in 3-space - The definition of a surface in Euclidean 3-space, examples of surfaces, the level set theorem, surfaces of revolution, calculus on surfaces, coordinate representations of curves and functions, change-of-coordinates maps.

The Geometry of surfaces in 3-space - The shape operator of a surface, normal curvature, diagonalization of the shape operator, mean curvature and Gaussian curvature.

Advanced topics - Adapted frame fields on surfaces, Gauss's Theorem Egregium, Riemannian Geometry, Geodesics, Gauss-Bonnet Theorem.

Assessment

Component	Course ILOs tested	SPMS-MAS Graduate Attributes tested	Weighting	Team / Individual	Assessment Rubrics
Continuous Assessment					
Tutorials					
Assignment	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	**1.** a, b, c **2.** b, c **3.** a	20	individual	See Appendix for rubric
Presentation	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	**1.** a, b, c **2.** b, c **3.** a	10	individual	See Appendix for rubric
Project	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	**1.** a, b, c **2.** b, c **3.** a	10	individual	See Appendix for rubric
Examination (2 hours)					
Short Answer Questions	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	**1.** a, b, c **2.** b, c **3.** a	60	individual	See Appendix for rubric
		Total	**100%**		

These are the relevant SPMS-MAS Graduate Attributes.

1. Competence

a. Independently process and interpret mathematical theories and methodologies, and apply them to solve problems

b. Formulate mathematical statements precisely using rigorous mathematical language

c. Discover patterns by abstraction from examples

2. Creativity

b. Build on the connection between subfields of mathematics to tackle new problems

c. Develop new applications of existing techniques

3. Communication

a. Present mathematics ideas logically and coherently at the appropriate level for the intended audience

Formative Feedback

Every solution to a problem submitted by each student every week will be considered by the instructor and detailed feedback about the correctness and presentation will be given to the student.

During their regular presentations questions and comments from the instructor and other students will provide feedback to the students about their areas of misunderstanding and presentation skills.

Learning and Teaching Approach

Lectures (39 hours)	Present the key ideas behind mathematical concepts. Illustrate the key ideas with examples. Present important steps used to solve different types of problems.
Tutorials (12 hours)	Develop proficiency in problem solving skills. Reinforce concepts already covered in the lectures. Give an opportunity for weaker or more reserved students to clarify doubts. Students will lead the tutorial sessions - presenting their solutions to problems they have selected for discussion by the group.

Reading and References

Core textbok:
O'Neill, B., Elementary Differential Geometry, Revised 2nd Edition, Academic Press, Elsevier. ISBN-13: 978-0-12-088735-4

Supplementary reading:
Pressley, A., Elementary Differential Geometry, Springer Undergraduate Mathematics Series. ISBN: 1-85233-152-6
Spivak, M., Calculus on Manifolds, Addison-Wesley, ISBN 0-8053-9021-9.
Do Carmo, M., Differential forms and applications, Springer Universitext. ISBN-13: 978-3-540-57618-1

Course Policies and Student Responsibilities

Absence Due to Medical or Other Reasons

If you are sick and not able to attend a quiz or midterm, you have to submit the original Medical Certificate (or another relevant document) to the administration to obtain official leave. In this case, the missed assessment component will not be counted towards the final grade. There are no make-up quiz or make-up midterm.

Academic Integrity

Good academic work depends on honesty and ethical behaviour. The quality of your work as a student relies on adhering to the principles of academic integrity and to the NTU Honour Code, a set of values shared by the whole university community. Truth, Trust and Justice are at the core of NTU's shared values.

As a student, it is important that you recognize your responsibilities in understanding and applying the principles of academic integrity in all the work you do at NTU. Not knowing what is involved in maintaining academic integrity does not excuse academic dishonesty. You need to

actively equip yourself with strategies to avoid all forms of academic dishonesty, including plagiarism, academic fraud, collusion and cheating. If you are uncertain of the definitions of any of these terms, you should go to the Academic Integrity website for more information. Consult your instructor(s) if you need any clarification about the requirements of academic integrity in the course.

Course Instructors

Instructor	Office Location	Phone	Email
Andrew James Kricker (Assoc Prof)	SPMS-MAS-04-18	6513 7458	ajkricker@ntu.edu.sg

Planned Weekly Schedule

Week	Topic	Course ILO	Readings/ Activities
1	The motivating problems and main themes of differential geometry.	1	
2	Calculus of differential forms - tangent vector, tangent space, vector field, frame field, directional derivatives, 1-forms, general differential forms, algebraic structures forming the complex of differential forms.	1	
3	The Frenet theory of curves - length of a curve segment, unit speed parametrization of curves, vector fields on curves, the curvature vector field and the curvature function, the Frenet apparatus.	2	
4	The Frenet theory of curves - the Frenet apparatus and the system of differential equations that governs it.	2, 3	
5	The Calculus of frame fields - covariant derivatives of vector fields, connection forms, dual 1-forms, the system of differential equations governing the apparatus associated to a frame field.	4	
6	Calculus of isometries of Euclidean space - orientations, the fundamental theorem of Frenet theory.	4	
7	Surfaces in 3-space - The definition of a surface in Euclidean 3-space, examples of surfaces, the level set theorem, surfaces of revolution.	5	
8	Surfaces in 3-space - Calculus on surfaces, coordinate representations of curves and functions, change-of-coordinates maps.	6, 7	
9	The Geometry of surfaces in 3-space - Normal curvature, The definition of the shape operator of a surface.	8	
10	The Geometry of surfaces in 3-space - Properties and diagonalization of the shape operator, definition of mean curvature and Gaussian curvature.	8, 9	
11	The Geometry of surfaces in 3-space - Computation of curvatures, examples of computations, and significance of curvature functions.	8, 9	
12	Advanced topics - Adapted frame fields on surfaces, Gauss's Theorem Egregium.	11	
13	Advanced topics - Riemannian Geometry, Geodesics, Gauss-Bonnet Theorem.	10, 12	

Appendix 1: Assessment Rubrics

Rubric for Tutorials: Assignment (20%)

Point-based marking (not rubrics based)

Rubric for Tutorials: Presentation (10%)

Each student has to present 5 times during the semester, and is graded individually in the style of an oral examination.

Grading Criteria	Exceptional (9-10)	Effective (7-8)	Acceptable (5-6)	Developing (0-4)
Accuracy	The interpretation is highly accurate, concise and precise.	The interpretation is mostly accurate. Some parts can be better explained or more succinct.	The interpretation is somewhat accurate. However, it contains some inaccuracies, missing points or ideas that are not related to the interpretation.	The interpretation are mostly inaccurate.
Visual and Oral delivery	Slides are informative, good clear voice and constant eye contact	Slides are adequate, oral delivery is satisfactory and frequent eye contact	Slides are bare and somewhat disorganized, voice is sometimes inaudible and little eye contact	Slides are disorganized, voice it too soft, and no eye contact
Presentation	Very clear and organized. It is easy to follow your train of thought	Mostly clear and organized. Some parts can have better transitions.	Somewhat clear. It requires some careful reading to understand what you are writing.	Mostly unclear and messy. It is difficult to understand what you are writing as there is no clear flow of ideas.
Question and Answer (for each individual student)	Very clear and precise answers to all problems. Explain the problems from various different perspectives logically.	Correct answers to most of the problems. Explain the problems in an organized way.	Partially-correct answers to most of the problems. Explain the some of the problems .	Unclear and messy answers. Difficult to understand.

Rubric for Tutorials: Project (10%)

Students are graded individually and will have to give a final presentation in the last

week of class.

Grading Criteria	Exceptional (9-10)	Effective (7-8)	Acceptable (5-6)	Developing (0-4)
Accuracy	The interpretation is highly accurate, concise and precise.	The interpretation is mostly accurate. Some parts can be better explained or more succinct.	The interpretation is somewhat accurate. However, it contains some inaccuracies, missing points or ideas that are not related to the interpretation.	The interpretation are mostly inaccurate.
Thoroughness	The literature review was comprehensive and rigorous. It includes several different perspectives, including a good spread of the first and latest ideas on the topic.	The literature review was mostly comprehensive and rigorous. It can improve in terms of the selection of the works relating to the topic.	The literature review was adequate. It covers some of the major works relating to the topic. References to primary source is largely missing.	The literature review was not thorough. It is based on a single source of information and/or inaccurate or unreliable secondary sources.
Presentation	Very clear and organized. It is easy to follow your train of thought	Mostly clear and organized. Some parts can have better transitions.	Somewhat clear. It requires some careful reading to understand what you are writing.	Mostly unclear and messy. It is difficult to understand what you are writing as there is no clear flow of ideas.
Originality	Evidence of extensive synthesis of ideas from different perspectives such that there is a very convincing original interpretation and that goes beyond what is already discussed in literature.	Evidence of some synthesis of ideas which lead to an original interpretation. The interpretation is good original summary of what is discussed in literature.	Evidence of an attempt to synthesise ideas. However, the attempt contains some misunderstandings.	No synthesis of ideas or originality. It is a repetition of what people have said or a laundry list of ideas with little interpretation.

Rubric for Examination: Short Answer Questions (60%)

Point-based marking (not rubrics based)