COURSE OUTLINE: MH4600

Course Title	**Algebraic Topology**		
Course Code	**MH4600**		
Offered	Study Year 4, Semester 2		
Course Coordinator	Andrew James Kricker (Assoc Prof)	ajkricker@ntu.edu.sg	6513 7458
Pre-requisites	MH3200 and MH3600		
AU	4		
Contact hours	Lectures: 39, Tutorials: 12		
Approved for delivery from	AY 2020/21 semester 2		
Last revised	21 Dec 2020, 15:12		

Course Aims

This course will introduce the point of view, framework and most important tools of Algebraic Topology. Algebraic Topology is the mathematical theory whose fundamental problem is the investigation of topological spaces and related concepts using tools from abstract algebra. The ideas and tools from Algebraic Topology are important in many parts of pure mathematics and are becoming increasingly important in physics and in data science.

The course aims to give students a foundational understanding in the two most important topics within Algebraic Topology.

1. The theory of homology.
2. The theory of fundamental group and covering spaces.

The aim is for students to be sufficiently prepared to continue deeper study in this topic, perhaps at the graduate level. The aim is also to equip students so that when they encounter these ideas in different topics (such as in physics or in data science) then they have the ability to bring in an expert understanding of the theory and the ability to deepen their learning as is needed in the context.

Intended Learning Outcomes

Upon successfully completing this course, you should be able to:

1. Construct spaces using simplicial or CW complexes, determine the corresponding chain complexes, and compute corresponding algebraic topological quantities such as homology and cohomology groups.
2. Prove algebraic properties about homology groups by using tools from the theory of homological algebra, such as exact sequences and diagram chasing techniques.
3. Define the fundamental group of a space and develop its fundamental properties.
4. Classify the system of covering spaces of a topological space in terms of the system of subgroups of the fundamental group of the space.
5. Formulate mathematical ideas and relationships using the language of category theory.
6. Apply your expert background in algebraic topology to be able to engage with studies in other fields, such as physics and data science, which exploit concepts from algebraic topology.

Course Content

Introduction - The motivations for the theory of algebraic topology, the language of category theory and some fundamental examples, such as the Brouwer fixed point theorem.

Simplices and simplicial complexes - The definition of an n-simplex and related concepts such as the faces of a simplex, barycentric coordinates and barycentric subdivision, the definition of a simplicial complex and examples of simplicial complexes.

Chain complexes and their homology groups - The chain complex associated to a simplicial complex and its associated boundary map, the definitions of cycles, boundaries, and homology groups of chain complexes.

Homological algebra - Chain homotopies, short and long exact sequences, axioms of a homology theory and alternative theories of homology.

Fundamental group - Concept of homotopy, and the definition and fundamental properties of the fundamental group functor.

Covering spaces - The fundamental group of the circle, the definition of a covering map, lifting paths and homotopies to covering spaces.

Covering spaces and the fundamental group - The fundamental group of a covering space embeds in the fundamental group of the base space, and lifting maps to covering spaces.

The fundamental theorem of covering spaces - Isometries of covering maps, spaces constructed as quotients by group actions, construction of the universal cover of a space, normal subgroups and normal covers.

Assessment

Component	Course ILOs tested	SPMS-MAS Graduate Attributes tested	Weighting	Team / Individual	Assessment Rubrics
Continuous Assessment					
Tutorials					
Assignment	1, 2, 3, 4, 5	**1.** a, b, c **2.** a, b, c **3.** a **4.** a **5.** a	20	individual	See Appendix for rubric
Presentation	1, 2, 3, 4, 5	**1.** a, b, c **2.** a, b, c **3.** a **4.** a **5.** a	10	individual	See Appendix for rubric
Project	1, 2, 3, 4, 5, 6	**1.** a, b, c **2.** a, b, c **3.** a **4.** a **5.** a	10	individual	See Appendix for rubric
Examination (2 hours)					
Final Examination	1, 2, 3, 4, 5	**1.** a, b, c **2.** a, b, c **3.** a **4.** a **5.** a	60	individual	See Appendix for rubric
		Total	**100%**		

These are the relevant SPMS-MAS Graduate Attributes.

1. Competence

a. Independently process and interpret mathematical theories and methodologies, and apply them to solve problems

b. Formulate mathematical statements precisely using rigorous mathematical language

c. Discover patterns by abstraction from examples

2. Creativity

a. Critically assess the applicability of mathematical tools in the workplace

b. Build on the connection between subfields of mathematics to tackle new problems

c. Develop new applications of existing techniques

3. Communication

a. Present mathematics ideas logically and coherently at the appropriate level for the intended audience

4. Civic-mindedness

a. Develop and communicate mathematical ideas and concepts relevant in everyday life for the benefits of society

5. Character

a. Act in socially responsible and ethical ways in line with the societal expectations of a mathematics professional, particularly in relation to analysis of data, computer security, numerical computations and algorithms

Formative Feedback

Feedback on common mistakes and the level of difficulty of the homework problems is given.

Feedbacks on performance in the group project will also be given to each student.

Learning and Teaching Approach

Lectures (39 hours)	Present the key ideas behind mathematical concepts. Present important steps used to solve different types of problems.
Tutorials (12 hours)	Develop proficiency in problem solving skills. Reinforce concepts already covered in the lectures. Give an opportunity for weaker or more reserved students to clarify doubts.

Reading and References

Kosniowski, C., A first course in algebraic topology, Cambridge University Press. ISBN 978-0-521-29864-3

Munkres J.R., Elements of algebraic topology, Perseus Publishing. ISBN: 0-201-62728-0

Hatcher A., Algebraic Topology, Cambridge University Press. ISBN 978-0-521-79540-1

Course Policies and Student Responsibilities

Absence Due to Medical or Other Reasons

If you are sick and not able to attend a quiz or midterm, you have to submit the original Medical Certificate (or another relevant document) to the administration to obtain official leave. In this case, the missed assessment component will not be counted towards the final grade. There are no make-up quiz or make-up midterm.

Academic Integrity

Good academic work depends on honesty and ethical behaviour. The quality of your work as a student relies on adhering to the principles of academic integrity and to the NTU Honour Code, a set of values shared by the whole university community. Truth, Trust and Justice are at the core of NTU's shared values.

As a student, it is important that you recognize your responsibilities in understanding and applying the principles of academic integrity in all the work you do at NTU. Not knowing what is involved in maintaining academic integrity does not excuse academic dishonesty. You need to actively equip yourself with strategies to avoid all forms of academic dishonesty, including plagiarism, academic fraud, collusion and cheating. If you are uncertain of the definitions of any

of these terms, you should go to the Academic Integrity website for more information. Consult your instructor(s) if you need any clarification about the requirements of academic integrity in the course.

Course Instructors

Instructor	Office Location	Phone	Email
Andrew James Kricker (Assoc Prof)	SPMS-MAS-04-18	6513 7458	ajkricker@ntu.edu.sg

Planned Weekly Schedule

Week	Topic	Course ILO	Readings/ Activities
1	Introduction to the purpose of the theory of algebraic topology. The language of category theory. Some basic examples.	5, 6	Study lecture notes. Solve problems.
2	Theory of simplices. Definition of simplicial complexes. Examples of simplicial complexes.	1	Study lecture notes. Solve problems. Presentations of solutions and theory.
3	Definition of chain groups of a complex. The boundary map. The chain complex of a simplicial complex. Definitions of cycles, boundaries, and homology groups.	1, 2	Study lecture notes. Solve problems. Presentations of solutions and theory.
4	Definition of chain groups of a complex. The boundary map. The chain complex of a simplicial complex. Definitions of cycles, boundaries, and homology groups.	1, 2	Study lecture notes. Solve problems. Presentations of solutions and theory.
5	Algorithms to compute homology groups. Examples of computations.	1, 2	Study lecture notes. Solve problems. Presentations of solutions and theory.
6	Definition of relative homology. Simplicial maps. Chain maps. Induced homomorphisms of homology groups.	1, 2	Study lecture notes. Solve problems. Presentations of solutions and theory.
7	Brief survey of singular homology groups. Statement of homotopy invariance property. Axioms of a homology theory.	1, 2	Study lecture notes. Solve problems. Presentations of solutions and theory.
8	Fundamental group. Definition as a functor. Homotopy invariance.	3, 5	Study lecture notes. Solve problems. Presentations of solutions and theory.
9	Introduction to the theory of covering spaces by the computation of the fundamental group of the circle. Definition of a covering map. Fundamental development of the theory including lemmas about the existence of lifts of paths and homotopies.	4	Study lecture notes. Solve problems. Presentations of solutions and theory.
10	Introduction to the theory of covering spaces by the computation of the fundamental group of the circle. Definition of a covering map. Fundamental development of the theory including lemmas about the existence of lifts of paths and homotopies.	4	Study lecture notes. Solve problems. Presentations of solutions and theory.

11	The theory that classifies covering spaces of a fixed space in terms of subgroups of the fundamental group of the space. Key points include: characterization of covering spaces in terms of subgroups. Lifting theorem for general maps into the base space. Isometries of the covers. Quotient spaces by group actions. Construction of the universal covering space.	4	Study lecture notes. Solve problems. Presentations of solutions and theory.
12	The theory that classifies covering spaces of a fixed space in terms of subgroups of the fundamental group of the space. Key points include: characterization of covering spaces in terms of subgroups. Lifting theorem for general maps into the base space. Isometries of the covers. Quotient spaces by group actions. Construction of the universal covering space.	4	Study lecture notes. Solve problems. Presentations of solutions and theory.
13	Student presentations of their projects.	1, 2, 3, 4, 5, 6	Project presentations.

Appendix 1: Assessment Rubrics

Rubric for Tutorials: Assignment (20%)

Point-based marking (not rubrics based)

Rubric for Tutorials: Presentation (10%)

Each student has to present 5 times during the semester, and is graded individually in the style of an oral examination.

Grading Criteria	Exceptional (36-40)	Effective (30-35)	Acceptable (24-29)	Developing (0-23)
Accuracy	The interpretation is highly accurate, concise and precise.	The interpretation is mostly accurate. Some parts can be better explained or more succinct.	The interpretation is somewhat accurate. However, it contains some inaccuracies, missing points or ideas that are not related to the interpretation.	The interpretation are mostly inaccurate.
Visual and Oral delivery	Slides are informative, good clear voice and constant eye contact	Slides are adequate, oral delivery is satisfactory and frequent eye contact	Slides are bare and somewhat disorganized, voice is sometimes inaudible and little eye contact	Slides are disorganized, voice it too soft, and no eye contact
Presentation	Very clear and organized. It is easy to follow your train of thought	Mostly clear and organized. Some parts can have better transitions.	Somewhat clear. It requires some careful reading to understand what you are writing.	Mostly unclear and messy. It is difficult to understand what you are writing as there is no clear flow of ideas.
Question and Answer (for each individual student)	Very clear and precise answers to all problems. Explain the problems from various different perspectives logically.	Correct answers to most of the problems. Explain the problems in an organized way.	Partially-correct answers to most of the problems. Explain the some of the problems .	Unclear and messy answers. Difficult to understand.

Rubric for Tutorials: Project (10%)

Students are graded individually and will have to give a final presentation in the last

week of class.

Grading Criteria	Exceptional (17-20)	Effective (14-16)	Acceptable (10-13)	Developing (0-9)
Accuracy	The interpretation is highly accurate, concise and precise.	The interpretation is mostly accurate. Some parts can be better explained or more succinct.	The interpretation is somewhat accurate. However, it contains some inaccuracies, missing points or ideas that are not related to the interpretation.	The interpretation are mostly inaccurate.
Thoroughness	The literature review was comprehensive and rigorous. It includes several different perspectives, including a good spread of the first and latest ideas on the topic.	The literature review was mostly comprehensive and rigorous. It can improve in terms of the selection of the works relating to the topic.	The literature review was adequate. It covers some of the major works relating to the topic. References to primary source is largely missing.	The literature review was not thorough. It is based on a single source of information and/or inaccurate or unreliable secondary sources.
Presentation	Very clear and organized. It is easy to follow your train of thought	Mostly clear and organized. Some parts can have better transitions.	Somewhat clear. It requires some careful reading to understand what you are writing.	Mostly unclear and messy. It is difficult to understand what you are writing as there is no clear flow of ideas.
Originality	Evidence of extensive synthesis of ideas from different perspectives such that there is a very convincing original interpretation and that goes beyond what is already discussed in literature.	Evidence of some synthesis of ideas which lead to an original interpretation. The interpretation is good original summary of what is discussed in literature.	Evidence of an attempt to synthesise ideas. However, the attempt contains some misunderstandings.	No synthesis of ideas or originality. It is a repetition of what people have said or a laundry list of ideas with little interpretation.

Rubric for Examination: Final Examination (60%)

Point-based marking (not rubrics based)