

## COURSE OUTLINE: MH4514

Course Title	<b>Financial Mathematics</b>		
Course Code	<b>MH4514</b>		
Offered	Study Year 4, Semester 2		
Course Coordinator	Nicolas Privault (Prof)	nprivault@ntu.edu.sg	6513 7176
Pre-requisites	MH2500 and MH3512		
Mutually exclusive	MH3513		
AU	4		
Contact hours	Lectures: 39, Tutorials: 12		
Approved for delivery from	AY 2020/21 semester 1		
Last revised	12 May 2020, 12:13		

### Course Aims

This course aims at developing quantitative skills for the pricing and hedging of financial derivatives, using stochastic calculus and partial differential equations. It will enable you to design both discrete and continuous-time financial pricing models by combining the power of analytical and probabilistic methods. This is a level 4 course and no finance prerequisite is required.

### Intended Learning Outcomes

Upon successfully completing this course, you should be able to:

1. Price and hedge vanilla options in discrete-time models.
2. Apply the Itô change of variable formula to derive pricing partial differential equations in continuous time.
3. Price and hedge call and put options in the continuous-time Black-Scholes framework.
4. Solve basic pricing differential equations by analytical and probabilistic methods.
5. Perform model calibration and volatility estimation based on market data.

### Course Content

Introduction and motivations

Discrete-Time Martingales

Assets, Portfolios, and Arbitrage

Discrete-Time Model

Pricing in Discrete Time

Hedging in Discrete Time

Brownian Motion

Stochastic Calculus

The Black-Scholes PDE

Martingale Approach to Pricing and Hedging

Estimation of Volatility

Basic Numerical Methods

Revisions and additional tutorial exercises

## Assessment

Component	Course ILOs tested	SPMS-MAS Graduate Attributes tested	Weighting	Team / Individual	Assessment Rubrics
<b>Continuous Assessment</b>					
<b>Tutorials</b>					
Assignment 1	1, 2, 3, 4, 5	1. a, b, c, d 2. a, b, c 3. a, b 4. a	10	individual	See Appendix for rubric
Assignment 2	1, 2, 3, 4, 5	1. a, b, c, d 2. a, b, c 3. a, b 4. a	10	individual	See Appendix for rubric
<b>Mid-semester Quiz</b>					
Short Answer Questions	1, 2, 3, 4	1. a, b, c 2. a, b 5. a	20	individual	See Appendix for rubric
<b>Examination (2 hours)</b>					
Short Answer Questions	1, 2, 3, 4	1. a, b, c 2. a, b 5. a	60	individual	See Appendix for rubric
<b>Total</b>			<b>100%</b>		

These are the relevant SPMS-MAS Graduate Attributes.

### 1. Competence

- a. Independently process and interpret mathematical theories and methodologies, and apply them to solve problems
- b. Formulate mathematical statements precisely using rigorous mathematical language
- c. Discover patterns by abstraction from examples
- d. Use computer technology to solve problems, and to communicate mathematical ideas

### 2. Creativity

- a. Critically assess the applicability of mathematical tools in the workplace
- b. Build on the connection between subfields of mathematics to tackle new problems
- c. Develop new applications of existing techniques

### 3. Communication

- a. Present mathematics ideas logically and coherently at the appropriate level for the intended audience
- b. Work in teams on complicated projects that require applications of mathematics, and communicate the results verbally and in written form

### 4. Civic-mindedness

- a. Develop and communicate mathematical ideas and concepts relevant in everyday life for the benefits of society

### 5. Character

- a. Act in socially responsible and ethical ways in line with the societal expectations of a mathematics professional, particularly in relation to analysis of data, computer security, numerical computations and algorithms

## Formative Feedback

Midterm test: Written and verbal feedback will be given from the lecturer and graders on your midterm test results, including through in-class discussion.

Assignments 1 and 2: Written and verbal feedback will be given from the lecturer and graders on your written assignments.

Students will also receive formative feedback for all learning outcomes during weekly tutorial classes from Week 2-Week 13.

Final examination: summative group feedback on the exam will be given following the conclusion of the module.

## Learning and Teaching Approach

<b>Lectures</b> (39 hours)	Derivations and proofs: Helps students improve their problem solving skills.
<b>Tutorials</b> (12 hours)	Derivations and proofs: Helps students improve their problem solving skills.  Peer instruction: Develops communication and presentation skills and deepens understanding.  Self-practice: Self-practice through solved exercises helps to build fundamental techniques and to develop your individual learning abilities.

## Reading and References

T. Mikosch, Elementary Stochastic Calculus with Finance in View, Advanced Series on Statistical Science & Applied Probability: Volume 6, World Scientific 1998, 978-9810235437.

N. Privault. Stochastic Finance - An Introduction with Market Examples (Chapters 1-7 and 16) Chapman & Hall/CRC Financial Mathematics Series, 2013, ISBN 978-1466594029.

## Course Policies and Student Responsibilities

### (1) General

Students are expected to complete all assigned class readings and activities, attend classes punctually and take all scheduled assignments and tests by due dates. Students are expected to take responsibility to follow up with course notes and assignments, and to participate in tutorial discussions and activities.

### (2) Absenteeism

Absence from class without a valid reason will affect your overall course grade. Valid reasons include falling sick supported by a medical certificate and participation in NTU's approved activities supported by an excuse letter from the relevant bodies.

If you miss a lecture, you must inform the course instructor via email prior to the start of the class.

### (3) Absence Due to Medical or Other Reasons

If you are sick and not able to attend a quiz or midterm, you have to submit the original Medical Certificate (or another relevant document) to the administration to obtain official leave. In this case, the missed assessment component will not be counted towards the final grade.

## Academic Integrity

Good academic work depends on honesty and ethical behaviour. The quality of your work as a student relies on adhering to the principles of academic integrity and to the NTU Honour Code, a set of values shared by the whole university community. Truth, Trust and Justice are at the core of NTU's shared values.

As a student, it is important that you recognize your responsibilities in understanding and applying the principles of academic integrity in all the work you do at NTU. Not knowing what is involved in maintaining academic integrity does not excuse academic dishonesty. You need to actively equip yourself with strategies to avoid all forms of academic dishonesty, including plagiarism, academic fraud, collusion and cheating. If you are uncertain of the definitions of any of these terms, you should go to the [Academic Integrity website](#) for more information. Consult your instructor(s) if you need any clarification about the requirements of academic integrity in the course.

## Course Instructors

Instructor	Office Location	Phone	Email
Nicolas Privault (Prof)	SPMS-MAS-05-43	6513 7176	nprivault@ntu.edu.sg

## Planned Weekly Schedule

Week	Topic	Course ILO	Readings/ Activities
1	Introduction and motivations	1, 2, 3, 4, 5	Lecture notes
2	Discrete-Time Martingales	1	Lecture notes and assignment
3	Assets, Portfolios, and Arbitrage	1	Lecture notes and assignment
4	Discrete-Time Model	1	Lecture notes and assignment
5	Pricing in Discrete Time	1	Lecture notes, assignments and simulations
6	Hedging in Discrete Time	1	Lecture notes, assignments and simulations
7	Brownian Motion	2	Lecture notes, assignments and simulations
8	Stochastic Calculus	2	Lecture notes and assignments
9	The Black-Scholes PDE	2, 3, 4	Lecture notes, assignments and simulations
10	Martingale Approach to Pricing and Hedging	2, 3, 4	Lecture notes and assignments
11	Estimation of Volatility	5	Lecture notes, assignments and simulations
12	Basic Numerical Methods	3, 4, 5	Lecture notes and simulations
13	Revisions and additional tutorial exercises	1, 2, 3, 4, 5	Lecture notes

## Appendix 1: Assessment Rubrics

### Rubric for Tutorials: Assignment 1 (10%)

This is the grading criteria for assignment 1 which are related to your ability to:

1. Price and hedge vanilla options in discrete-time models.
2. Apply the Itô change of variable formula to derive pricing partial differential equations in continuous time.
3. Price and hedge call and put options in the continuous-time Black-Scholes framework.

4. Solve basic pricing differential equations by analytical and probabilistic methods.
5. Perform model calibration and volatility estimation based on market data.

Marks	Criteria
> 90%	Solutions to the given problems and questions are correct and logically proven in all cases.
75% to 89%	Solutions to the given problems and questions are correct and logically proven in most cases. Some minor calculation or logic error exist.
65% to 74%	Solutions to the given problems and questions are correct and logically proven in most cases. Several minor calculation or logic error exist throughout the assignment.
50% to 64%	Solutions to the given problems and questions are correct and logically proven in more often than not. However, there appears to be major misconceptions for some topics.
< 50%	Did not attempt most of the questions; OR Solutions to the given problems and questions are incorrect and illogical in most cases. There appears to be major misconceptions for most topics.

### Rubric for Tutorials: Assignment 2 (10%)

*This is the grading criteria for assignment 2 which are related to your ability to:*

1. Price and hedge vanilla options in discrete-time models.
2. Apply the Itô change of variable formula to derive pricing partial differential equations in continuous time.
3. Price and hedge call and put options in the continuous-time Black-Scholes framework.
4. Solve basic pricing differential equations by analytical and probabilistic methods.
5. Perform model calibration and volatility estimation based on market data.

Marks	Criteria
> 90%	Solutions to the given problems and questions are correct and logically proven in all cases.
75% to 89%	Solutions to the given problems and questions are correct and logically proven in most cases. Some minor calculation or logic error exist.
65% to 74%	Solutions to the given problems and questions are correct and logically proven in most cases. Several minor calculation or logic error exist throughout the assignment.
50% to 64%	Solutions to the given problems and questions are correct and logically proven in more often than not. However, there appears to be major misconceptions for some topics.
< 50%	Did not attempt most of the questions; OR Solutions to the given problems and questions are incorrect and illogical in most cases. There appears to be major misconceptions for most topics.

### Rubric for Mid-semester Quiz: Short Answer Questions (20%)

*Point-based marking (not rubrics based)*

### Rubric for Examination: Short Answer Questions (60%)

*Point-based marking (not rubrics based)*