

Annexe A: New/Revised Course Content in OBTL+ Format

Course Overview

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|------------------------------------------------------------|----------------------|
| Expected Implementation in Academic Year | AY2023-2024 |
| Semester/Trimester/Others (specify approx. Start/End date) | Semester 1 |
| Course Author * Faculty proposing/revising the course | Ng Keng Meng |
| Course Author Email | kmng@ntu.edu.sg |
| Course Title | SET THEORY AND LOGIC |
| Course Code | MH4301 |
| Academic Units | 4 |
| Contact Hours | 51 |
| Research Experience Components | Not Applicable |

Course Requisites (if applicable)

| | |
|-----------------------|--------------------------------------------|
| Pre-requisites | (MH1300 and MH3100) OR (MH1300 and MH2220) |
| Co-requisites | |
| Pre-requisite to | |
| Mutually exclusive to | |
| Replacement course to | |
| Remarks (if any) | |

Course Aims

This course aims to provide you with a basic understanding of formal mathematical logic and axiomatic set theory. Mathematical logic provides a foundational basis for the practice of mathematics, and axiomatic set theory provides a rigorous justification for the existence of mathematical objects and structures. This course will equip you with the awareness of foundational issues in mathematics. It will allow you to understand how proofs in mathematics are rigorously defined, and how the concepts of “truth” and “provability” interact. It will also enable you to judge which objects and processes in mathematics are well-defined, and which ones require further axiomatic justifications. It is aimed at increasing awareness among mathematics students for why the common practices in mathematics are rigorously grounded, and which ones are ill-defined and should be avoided.

This course is aimed at 3rd and 4th year students interested in learning about the foundations upon which all of mathematics is built upon, particularly mathematics and computer science students who are interested in the theoretical aspects of mathematics. This course is also crucial for students intending to pursue further studies in theoretical mathematics.

Course's Intended Learning Outcomes (ILOs)

Upon the successful completion of this course, you (student) would be able to:

| | |
|--------|-------------------------------------------------------------------------------------------------------------------------|
| ILO 1 | Identify the language of propositional logic. |
| ILO 2 | Explain how truth and proofs work in propositional logic. |
| ILO 3 | Explain the soundness and completeness theorems in propositional logic. |
| ILO 4 | Apply the compactness theorem in propositional logic. |
| ILO 5 | Explain language of first-order logic. |
| ILO 6 | Derive the relationship between truth and proofs in first-order logic. |
| ILO 7 | Relate the concept of a first-order structure to the commonly studied objects in mathematics. |
| ILO 8 | Explain the workings the compactness theorem in first-order logic, particularly to non-standard structures. |
| ILO 9 | Discuss the issues of definability and judge how this relates to the definition of objects commonly used in mathematics |
| ILO 10 | Describe the formal axioms of Zermelo-Fraenkel set theory. |
| ILO 11 | Explain Godel's Incompleteness Theorems and the impact it has on mathematics. |
| ILO 12 | State the use of ordinals and cardinals, as well as well-orderings |
| ILO 13 | Apply transfinite induction and recursion to mathematical problems. |
| ILO 14 | Apply the Axiom of Choice to different areas of mathematics |
| ILO 15 | Justify the existence of the natural numbers, the real numbers, and the other commonly studied objects in mathematics. |

Course Content

The language of propositional logic. Truth assignments and Boolean functions. Induction and recursion. The proof system used in propositional logic. The Soundness Theorem, Completeness Theorem and compactness Theorem for propositional logic. The language of first order logic. Truths and models The proof system used in first order logic. The Soundness Theorem, Completeness Theorem and compactness Theorem for first order logic. The axioms of set theory. Relations, functions and orderings. Natural numbers. Equipotent sets and bijections. Ordinals and well-orderings. The axiom of choice.

Reading and References (if applicable)

H. B. Enderton - A Mathematical Introduction to Logic (3rd Edition). Academic Press. ISBN-13:978-0123869777

K. Hrbacek and T. Jech - Introduction to Set Theory (3rd Edition). Marcel Dekker. ISBN-13: 978-0824779153

Planned Schedule

| Week or Session | Topics or Themes | ILO | Readings | Delivery Mode | Activities |
|-----------------------|-----------------------|---------------------------------------|-------------------------------------|------------------|------------|
| 1 | Mathematical Logic | 1, 2, 3, 4, 5, 6, 7, 8, 9 | H.B. Enderton - Sections 1.0 to 2.5 | In-person | |
| 2 | Mathematical Logic | 1, 2, 3, 4, 5, 6, 7, 8, 9 | H.B. Enderton - Sections 1.0 to 2.5 | In-person | |
| 3 | Mathematical Logic | 1, 2, 3, 4, 5, 6, 7, 8, 9 | H.B. Enderton - Sections 1.0 to 2.5 | In-person | |
| 4 | Mathematical Logic | 1, 2, 3, 4, 5, 6, 7, 8, 9 | H.B. Enderton - Sections 1.0 to 2.5 | In-person | |
| 5 | Mathematical Logic | 1, 2, 3, 4, 5, 6, 7, 8, 9 | H.B. Enderton - Sections 1.0 to 2.5 | In-person | |
| 6 | Mathematical Logic | 1, 2, 3, 4, 5, 6, 7, 8, 9 | H.B. Enderton - Sections 1.0 to 2.5 | In-person | |
| 7 | Mathematical Logic | 1, 2, 3, 4, 5, 6, 7, 8, 9 | H.B. Enderton - Sections 1.0 to 2.5 | In-person | |

| Week or Session | Topics or Themes | ILO | Readings | Delivery Mode | Activities |
|-----------------------|------------------|---------------------------------------|-----------------------------------|------------------|------------|
| 8 | Set Theory | 10, 11, 12, 13, 14, 15 | Hrbacek and Jech - Chapters 1-6,8 | In-person | |
| 9 | Set Theory | 10, 11, 12, 13, 14, 15 | Hrbacek and Jech - Chapters 1-6,8 | In-person | |
| 10 | Set Theory | 10, 11, 12, 13, 14, 15 | Hrbacek and Jech - Chapters 1-6,8 | In-person | |
| 11 | Set Theory | 10, 11, 12, 13, 14, 15 | Hrbacek and Jech - Chapters 1-6,8 | In-person | |
| 12 | Set Theory | 10, 11, 12, 13, 14, 15 | Hrbacek and Jech - Chapters 1-6,8 | In-person | |
| 13 | Set Theory | 10, 11, 12, 13, 14, 15 | Hrbacek and Jech - Chapters 1-6,8 | In-person | |

Learning and Teaching Approach

| Approach | How does this approach support you in achieving the learning outcomes? |
|-----------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Lectures | Present the key ideas behind mathematical concepts. Present important steps used to solve different types of problems. |
| Tutorials | Develop proficiency in problem solving skills. Reinforce concepts already covered in the lectures. Gives an opportunity for weaker or more reserved students to clarify doubts |

Assessment Structure

Assessment Components (includes both continuous and summative assessment)

| No. | Component | ILO | Related PLO or Accreditation | Weightage | Description of Assessment Component | Team/Individual | Rubrics | Level of Understanding |
|-----|---------------------------------------------------------------|--------------------------------------------------|------------------------------|-----------|-------------------------------------|-----------------|----------|------------------------|
| 1 | Continuous Assessment (CA): Others(Homework) | 1, 2, 3, 4, 5, 6, 7, 8, 9,10, 11, 12, 13, 14, 15 | 1.a,b,c, 2.a,b,c, 3.a, 5.a | 15 | | Individual | Analytic | Multistructural |
| 2 | Continuous Assessment (CA): Test/Quiz(Mid-term Examination 1) | 1, 2, 3, 4, 5, 6, 7, 8, 9 | 1. a,b,c, 2.a, 5.a | 25 | | Individual | Analytic | Extended Abstract |
| 3 | Summative Assessment (EXAM): Final exam() | 1, 2, 3, 4, 5, 6, 7, 8, 9,10, 11, 12, 13, 14, 15 | 1.a,b,c, 2.a, 5.a | 60 | | Individual | Analytic | Extended Abstract |

Description of Assessment Components (if applicable)

Formative Feedback

For the midterm and final exams, feedback on the common mistakes are given on NTULearn after the grades are announced. This includes the examiner's report which will be released on NTU Learn after the results are announced. Common mistakes are often repeated and addressing this will be important for achieving the learning outcomes 1-15. For the homework assignment, the solutions will be discussed during the next tutorial session and feedback on common mistakes will be given then

NTU Graduate Attributes/Competency Mapping

This course intends to develop the following graduate attributes and competencies (maximum 5 most relevant)

| Attributes/Competency | Level |
|-----------------------|--------------|
| Creative Thinking | Advanced |
| Curiosity | Intermediate |
| Problem Solving | Intermediate |
| Sense Making | Advanced |

Course Policy

Policy (Academic Integrity)

Good academic work depends on honesty and ethical behaviour. The quality of your work as a student relies on adhering to the principles of academic integrity and to the NTU Honour Code, a set of values shared by the whole university community. Truth, Trust and Justice are at the core of NTU's shared values. As a student, it is important that you recognize your responsibilities in understanding and applying the principles of academic integrity in all the work you do at NTU. Not knowing what is involved in maintaining academic integrity does not excuse academic dishonesty. You need to actively equip yourself with strategies to avoid all forms of academic dishonesty, including plagiarism, academic fraud, collusion and cheating. If you are uncertain of the definitions of any of these terms, you should go to the academic integrity website for more information. On the use of technological tools (such as Generative AI tools), different courses / assignments have different intended learning outcomes. Students should refer to the specific assignment instructions on their use and requirements and/or consult your instructors on how you can use these tools to help your learning. Consult your instructor(s) if you need any clarification about the requirements of academic integrity in the course.

Policy (General)

You are expected to complete all assigned pre-class readings and activities, attend all tutorial classes punctually and take all scheduled assignments and tests by due dates. You are expected to participate in all tutorial discussions and activities.

Policy (Absenteeism)

Absence from the midterm without a valid reason will affect your overall course grade. Valid reasons include falling sick supported by a medical certificate and participation in NTU's approved activities supported by an excuse letter from the relevant bodies. There will be no make-up opportunities for CA components.

All project assignments must be submitted on time. Failure to do so will affect your score.

Policy (Others, if applicable)

Diversity and inclusion policy

Integrating a diverse set of experiences is important for a more comprehensive understanding of science.

It is our goal to create an inclusive and collaborative learning environment that supports a diversity of perspectives and learning experiences, and that honours your identities; including ethnicity, gender, socioeconomic status, sexual orientation, religion or ability.

To help accomplish this:

If you are neuroatypical or neurodiverse, have dyslexia or ADHD (for example), or have a social anxiety disorder or social phobia;

If you feel like your performance in the class is being impacted by your experiences outside of class;

If something was said in class (by anyone, including the instructor) that made you feel uncomfortable;

Please speak to your teaching team, a peer or senior (either in-person or via email) about how we can help

facilitate your learning experience.

As a participant in course discussions, you should also strive to honour the diversity of your classmates. You can do this by: using preferred pronouns and names; being respectful of others opinions and actively making sure all voices are being heard; and refraining from the use of derogatory or demeaning speech or actions.

All members of the class are expected to adhere to the NTU anti-harassment policy. if you witness something that goes against this or have any other concerns, please speak to your instructors or a faculty member.