Academic Year	1819	**Semester**	2
Course Coordinator	TONG Ping		
Course Code	MH4110		
Course Title	Partial Differential Equations		
Pre-requisites	{MH3100 Real Analysis I and MH3110 Ordinary Differential Equations} OR {MH1803 Calculus for Physics and MH3100 Real Analysis I}		
No of AUs	4		
Contact Hours	3 Hours Lecture + 1 hour tutorial		
Proposal Date	18-Jun-2018		

Course Aims

This course aims to introduce you to some basic concepts and properties of first-order partial differential equations, wave equation, heat equation, and Laplace's equation as well as the numerical finite-difference methods for solving partial differential equations. This course will also develop your skills in solving some important partial differential equations with various auxiliary conditions. What you learned in this course will have a wide application in real life applications as well as for numerous further graduate courses.

Intended Learning Outcomes (ILO)

By the end of the course, you should be able to:

1. Solve the first-order partial differential equations with constant and variable coefficients
2. Solve the heat equation and wave equation problems on the whole real axis, the half line, and a finite interval.
3. Expand functions in Fourier cosine series, Fourier sine series, and full Fourier series.
4. Prove the uniqueness of solution using the energy method or the maximum principle.
5. Design a numerical scheme and implement the algorithm on computer to solve simple partial differential equation problems.

Course Content

- *Definition of partial differential equation (PDE); linearity, order, solution, heterogeneity of a partial differential equation; Uniqueness of the solution to a partial differential equation; Types of partial differential equation.*
- *Geometric method and Cartesian method for solving the first-order partial differential equation with constant or variable coefficients; Operator factorization method for solving the second-order constant coefficient PDEs.*
- *Derivations of the D'Alembert solution formula for the wave equation and the solution formula for the heat equation on the whole axis; Introduction to the reflection method for the solution formulas of the wave equations and heat equations on the half line with Dirichlet boundary condition and Neumann boundary condition;*
- *Introduction to the method of separation of variables and solve wave equations, heat equations, and Laplace equations on finite intervals.*
- *Fourier series expansion including Fourier cosine series, Fourier sine series, and full Fourier series.*
- *Finite-difference method for solving PDEs.*

Assessment (includes both continuous and summative assessment)

Component	*Course ILO Tested*	*Related Programme LO or Graduate Attributes*	*Weighting*	*Team/Individual*	*Assessment rubrics*

1. Assignment 1	*1*	*A1, A2*	*10%*	*Individual/Team*	*Appendix 1*
2. Midterm Test	*1, 2, 4*	*A1, A2*	*20%*	*Individual*	
3. Assignment 2	*2, 3, 4, 5*	*A1, A2, A4*	*10%*	*Individual/Team*	*Appendix 2*
4. Final Exam	*1, 2, 3, 4*	*A1, A2*	*60%*	*Individual*	
Total			*100%*		

Graduates of MAS programmes should be able to:

Competence	
A1: {Understanding}	*independently process and interpret mathematical theories and methodologies, and apply them to solve problems*
A2: {Rigour}	*formulate mathematical statements precisely using rigorous mathematical language*
A3: {Intuition}	*discover patterns by abstraction from examples*
A4: {Modern Tool Usage}	*use computer technology to solve problems, and to communicate mathematical ideas*
Creativity	
B1: {Critical Thinking}	*critically assess the applicability of mathematical tools in the workplace*
B2: {Analysis}	*critically analyse data from a multitude of sources*
B3: {Interdisciplinarity}	*build on the connection between subfields of mathematics to tackle new problems*
B4: {Creativity}	*develop new applications of existing techniques*
Communication	
C1: {Communication}	*present mathematics ideas logically and coherently at the appropriate level for the intended audience*
C2: {Teamwork}	*work in teams on complicated projects that require applications of mathematics, and communicate the results verbally and in written form*
Civic-Mindedness	
D: {Professionalism}	*develop and communicate mathematical ideas and concepts relevant in everyday life for the benefits of society*
Character	
E: {Ethics}	*act in socially responsible and ethical ways in line with the societal expectations of a mathematics professional, particularly in relation to analysis of data, computer security, numerical computations and algorithms*

Formative feedback

Timely feedback is central to this course. You will receive formative feedback through written responses to your assignments and midterm test, and verbal feedback through in-class discussion. You will receive summative group feedback on the exam following the conclusion of the module.

Learning and Teaching approach

Approach	**How does this approach support students in achieving the learning outcomes?**
Lecture, Self-directed learning, Tutorial	*The course focuses on introducing basic concepts about Partial Differential Equations (PDEs), the common tools for solving classic PDEs problems, modern computer technology to solve real problems closely associated with PDEs. We aim at inculcating a culture of proactive individual learning. Lecture: There are regular weekly lectures during the semester. The*

	roles of the instructors are to guide you to understand fundamental concepts and theories and to facilitate discussion. *Self-directed learning: You are expected to adopt, adapt and synthesize the acquired concepts and theories into practice. Since the regular lectures cannot cover every aspect, you are expected to read and explore the references as much as you can.* *Tutorial: Weekly tutorial questions will be made available on NTULearn. The tutorial questions not only help you have a better understanding of the fundamental knowledge, but also help to develop your individual learning abilities and attitudes toward active learning. You need to attempt the tutorial questions within one week after they are posted.*

Reading and References

1. *Partial Differential Equations: An Introduction, Walter Strauss, 2nd edition, 978-0470054567*
2. *Partial Differential Equations: Second Edition, Lawrence C. Evans: University of California, Berkeley, Berkeley, CA, 978-0-8218-4974-3*

Course Policies and Student Responsibilities

(1) General

Students are expected to complete all assigned pre-class readings and activities, attend all classes and take all scheduled assignments and tests by due dates. Students are expected to take responsibility to follow up with course notes, assignments and course related announcements for seminar sessions they have missed. Students are expected to participate in all discussions and activities.

(2) Absenteeism

Absence from class without a valid reason will affect your overall course grade. Valid reasons include falling sick supported by a medical certificate and participation in NTU's approved activities supported by an excuse letter from the relevant bodies.

If you miss a lecture, you must inform the course instructor via email prior to the start of the class.

(3) Absence Due to Medical or Other Reasons

If you are sick and not able to attend the midterm, you have to submit the original Medical Certificate (or another relevant document) to the administration to obtain official leave. In this case, the weight of the missed assessment component will be shifted towards the final exam. There is no make-up midterm.

Academic Integrity

Good academic work depends on honesty and ethical behaviour. The quality of your work as a student relies on adhering to the principles of academic integrity and to the NTU Honour Code, a set of values shared by the whole university community. Truth, Trust and Justice are at the core of NTU's shared values.

As a student, it is important that you recognize your responsibilities in understanding and applying the principles of academic integrity in all the work you do at NTU. Not knowing what is involved in maintaining academic integrity does not excuse academic dishonesty. You need to actively equip yourself with strategies to avoid all forms of academic dishonesty, including plagiarism, academic

fraud, collusion and cheating. If you are uncertain of the definitions of any of these terms, you should go to the academic integrity website for more information. Consult your instructor(s) if you need any clarification about the requirements of academic integrity in the course.

Collaboration is encouraged for your tutorial and assignment problems because peer-to-peer learning helps you understand the subject better and working in a team trains you to better communicate with others. As part of academic integrity, crediting others for their contribution to your work promotes ethical practice.

You **must write up your solutions by yourself and understand anything that you hand in.** If you do collaborate, **you must write on your solution sheet the names of the students you worked with. If you did not collaborate with anyone, please explicitly write, "No collaborators."**

Course Instructors

Instructor	Office Location	Phone	Email
TONG Ping	SPMS-MAS-04-17	65137457	tongping@ntu.edu.sg

Planned Weekly Schedule

Week	Topic	Course ILO	Readings/ Activities
1	Course Overview, basics about PDEs, geometric method and Cartesian methods for solving first-order PDE.	1	
2	Transport equation, wave equation	1, 2	
3	Initial and boundary conditions, types of second-order PDEs	1	

4	Solution to wave equation on the whole line, energy method	2, 4	
5	Solution to heat equation on the whole line, Maximum principle	2, 4	
6	Reflection methods for solving wave equation and heat equation on the half line with different boundary conditions	2, 4	Assignment 1 (Returned by the end of Week 7)
7	Reflection methods for solving wave equation and heat equation on the half line with different boundary conditions	2, 4	
8	Reflection methods for solving wave and heat equations on the half line with sources	2, 4	Midterm test
9	Separation of variables for solving problems on finite intervals	2, 4	
10	Separation of variables for solving boundary problems	2, 4	
11	Fourier series	3	Assignment 2 (Returned by the end of Week 14)
12	Laplace's equation	2, 4	
13	Numerical methods for solving PDEs	5	

Appendix 1: Assessment Criteria for Assignment 1

This is the grading criteria for assignment 1 which are related to your ability to:

1. Solve the first-order partial differential equations with constant and variable coefficients

Marks	**Criteria**
> 90%	Correctly attempt all the assignment questions individually or through team work; all the necessary details are included in the solutions.
75% to 89%	Attempt all the questions; all the necessary details are included in the solutions. More than 90% but not all the questions are correctly attempted.
65% to 74%	More than 80% questions are attempted. The solutions of the attempted questions have the necessary step-by-step details.
50% to 64%	More than 50% questions are attempted. The solutions are present in a logic way.
< 50%	Less than 50% questions are attempted.

Appendix 2: Assessment Criteria for Assignment 2

This is the grading criteria for assignment 2 which are related to your ability to:

2. Solve the heat equation and wave equation problems on the whole real axis, the half line, and a finite interval.
3. Expand functions in Fourier cosine series, Fourier sine series, and full Fourier series.
4. Prove the uniqueness of solution using the energy method or the maximum principle.
5. Design a numerical scheme and implement the algorithm on computer to solve simple partial differential equation problems.

Marks	**Criteria**
> 90%	Solutions, proofs and design are detailed and correct for all given questions.
75% to 89%	Solutions, proofs and design are detailed and correct for most questions. There are some minor errors in some questions.
65% to 74%	Solutions, proofs and design are detailed and correct for most questions. There are several minor errors or missing details across the various questions.
50% to 64%	Solutions, proofs and design are correct for most questions. There are several errors across the various questions that suggest conceptual misunderstanding. Details are lacking too.
< 50%	Did not attempt most of the questions; OR The solutions, proofs and design lack details and are mostly incorrect.