COURSE OUTLINE: MH4100

Course Title	**Real Analysis II**		
Course Code	**MH4100**		
Offered	Study Year 4, Semester 2		
Course Coordinator	Tang Wee Kee (Dr)	weekeetang@ntu.edu.sg	6513 8654
Pre-requisites	{MH2100, MH3100}OR{CY1602, MH3100}OR{MH1803, MH3100}		
AU	4		
Contact hours	Lectures: 39, Tutorials: 12		
Approved for delivery from	AY 2019/20 semester 2		
Last revised	25 Sep 2019, 09:01		

Course Aims

This is the second of two courses on Real Analysis. The course aims to present a rigorous treatment of the principal topics of real analysis, such as Lebesgue measure, Lebesgue integrals, differentiation, convexity, and normed linear spaces, and to illustrate the power of the subject through a variety of applications. After learning this course, you will be able to make connections between the abstract settings and the concrete problems studied in various courses in calculus and probability theory.

Intended Learning Outcomes

Upon successfully completing this course, you should be able to:

1. Determine the liminf and limsup of a sequence of extended real numbers;
2. Determine the set of points of continuity of a given function (as a Borel set);
3. Determine if a function is measurable;
4. Compute the Lebesgue Integral of a given funtion;
5. Compute the limit of the integral if a sequence of functions by using various limit theorems like Fatou Lemma, and Lebesgue Domninated Convergence Theorem;
6. Prove major convergence theorems by using Fatou's Lemma;
7. Establish relationship between differentiation and integration, in terms of bounded variations and absolute continuity;
8. Apply Jensen inequality for convex functions;
9. Prove Holder and Minkowski inequalities for Lp spaces;
10. Prove Lp spaces are complete;
11. Prove Riesz Representation Theorem for Lp spaces.

Course Content

Basic topology on the real line and extended real line

Measurable sets and measurable functions

Lebesgue integration

Differentiation, bounded variation, absolute continuity, and convex functions

Classical Banach spaces

Assessment

Component	Course ILOs tested	SPMS-MAS Graduate Attributes tested	Weighting	Team / Individual	Assessment Rubrics
Continuous Assessment					
Lectures					
Assignment	1, 2, 3, 4, 5, 6, 7, 8, 9	**1.** a, b, c **2.** a, b, c, d **3.** a **4.** a **5.** a	20	individual	See Appendix for rubric
Mid-semester Quiz					
Mid Term Test	1, 2, 3, 4, 5, 6, 7, 8, 9	**1.** a, b, c **2.** c **3.** a **5.** a	20	individual	See Appendix for rubric
Examination (2.5 hours)					
Final Exam	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11	**1.** a, b **2.** a, b **3.** a **5.** a	60	individual	See Appendix for rubric
		Total	**100%**		

These are the relevant SPMS-MAS Graduate Attributes.

1. **Competence**
   a. Independently process and interpret mathematical theories and methodologies, and apply them to solve problems
   b. Formulate mathematical statements precisely using rigorous mathematical language
   c. Discover patterns by abstraction from examples
2. **Creativity**
   a. Critically assess the applicability of mathematical tools in the workplace
   b. Build on the connection between subfields of mathematics to tackle new problems
   c. Develop new applications of existing techniques
   d. Critically analyse data from a multitude of sources
3. **Communication**
   a. Present mathematics ideas logically and coherently at the appropriate level for the intended audience
4. **Civic-mindedness**
   a. Develop and communicate mathematical ideas and concepts relevant in everyday life for the benefits of society
5. **Character**
   a. Act in socially responsible and ethical ways in line with the societal expectations of a mathematics professional, particularly in relation to analysis of data, computer security, numerical computations and algorithms

Formative Feedback

Feedback will be given to you through the weekly problem tutorial sets that are covered in tutorial.
Common misunderstanding of major concepts will also be discussed in lecture.
Common mistakes in the midterm tests will be discussed in the provided solution sets.

Learning and Teaching Approach

Lectures (39 hours)	Examples and Explanation ========== Motivations of the concepts given in the learning objectives 1-11 will be provided through examples. The general theory and principles are then explained. This also introduces more abstract mathematical reasonings.
Tutorials (12 hours)	Problem Solving ========== Develops competence in solving a variety of problems in and gaining familiarity with mathematical proofs.

Reading and References

H. L. Royden, Real Analysis, 3rd Edition, MacMillan Publishing Company, ISBN-10: 0024041513

Course Policies and Student Responsibilities

This course aims to develop deep mathematical insights through abstract and rigorous treatment of analysis. The process of establishing the results is more important than the result statements It is therefore essential to attend all lectures.

To gain a better understanding, you need to attempt all tutorial problems instead of waiting for answers to be given. The solutions obtained by you through the necessary struggles will be internalized and this process will help you handle future unfamiliar problems.

Academic Integrity

Good academic work depends on honesty and ethical behaviour. The quality of your work as a student relies on adhering to the principles of academic integrity and to the NTU Honour Code, a set of values shared by the whole university community. Truth, Trust and Justice are at the core of NTU's shared values.

As a student, it is important that you recognize your responsibilities in understanding and applying the principles of academic integrity in all the work you do at NTU. Not knowing what is involved in maintaining academic integrity does not excuse academic dishonesty. You need to actively equip yourself with strategies to avoid all forms of academic dishonesty, including plagiarism, academic fraud, collusion and cheating. If you are uncertain of the definitions of any of these terms, you should go to the Academic Integrity website for more information. Consult your instructor(s) if you need any clarification about the requirements of academic integrity in the course.

Course Instructors

Instructor	Office Location	Phone	Email
Tang Wee Kee (Dr)	SPMS-MAS-05-07	6513 8654	weekeetang@ntu.edu.sg

Planned Weekly Schedule

Week	Topic	Course ILO	Readings/ Activities
1	Basic topology on the real line and extended real line	1, 2, 3	
2	Open Sets Closed Sets	1, 2	
3	Continuous Functions	3, 6	
4	Measurable sets	3, 5	
5	Measurable functions	3, 10	
6	Lebesgue integration	4, 5	
7	Lebesgue integration (continued)	4, 5, 6	
8	Convergence Theorems	4, 5, 6	
9	Differentiation	7, 8	
10	Functions of bounded variation, absolute continuity	7, 8	
11	Convex Functions	7, 8	
12	Classical Banach spaces	9, 10, 11	
13	Classical Banach spaces	9, 10, 11	

Appendix 1: Assessment Rubrics

Rubric for Lectures: Assignment (20%)

a. Description of assessment

- The assignment aims to test your understanding of the subject by giving problems that require long time than in a standard examination setting

b. Assessment Criteria

1. Basic understanding of the course content
2. Ability to apply knowledge learned in the course to solve the given problems
3. Creativity in problem solving

c. What the grader is looking for

1. To pass the assignment, you are expected to exhibit basic understanding by rightly applying relevant theorems to solve problems.
2. To obtain a high score, you must be able to present creative methods to solve difficult problems.
3. To obtain full score, you must show deep understanding of all topics covered in the assignment by being able to modify/generalize/specialize a statement and provide the justification.

Rubric for Mid-semester Quiz: Mid Term Test (20%)

a. Description of assessment

- The test aims to your understanding of the subject by shorter and standard problems covered in the course

b. Assessment Criteria

1. Basic understanding of the course content
2. Ability to apply knowledge learned in the course to solve the given problems
3. Creativity in problem solving

c. What the grader is looking for

1. To pass the test, you are expected to exhibit basic understanding of all topics by being able to state all theorems and prove simple theorems.
2. To obtain a good score, you are expected to show deeper understanding of the course materials. You should be able to apply theorems to solve a given problem.
3. To obtain a full score, you are expected to show deeper understanding of the course materials. You should be able to make sound judgement if a given statement is true or false and provide the necessary justification.

Rubric for Examination: Final Exam (60%)

a. Description of assessment

- The exam is a summative assessment of the course. It aims to measure your understanding of the entire course.

b. Assessment Criteria

1. Basic understanding of the course content.
2. Ability to apply knowledge learned in the course to solve the given problems.
3. Ability to synthesize knowledge learned in the entire course and present analysis thinking in a coherent manner.

c. What the grader is looking for

1. To pass the exam, you are expected to exhibit basic understanding of all topics by being able to state all theorems and prove simple theorems.
2. To obtain a good score, you are expected to show deep understanding of the course materials. You should be able to make sound judgement if a given statement is true or false and provide the necessary justification. You should be able extract the part of the proof of a theorem that is relevant to given claim.
3. To obtain a full score, you must show that he is able to synthesize knowledge acquired and provide a compelling argument to justify a given statement.