Annexe A: New/Revised Course Content in OBTL+ Format

Course Overview

Expected Implementation in Academic Year	AY2025-2026
Semester/Trimester/Others (specify approx. Start/End date)	Semester 2
Course Author * Faculty proposing/revising the course	François Joachim Marcel Gay-Balmaz
Course Author Email	francois.gb@ntu.edu.sg
Course Title	Topology and Manifolds
Course Code	MH3600
Academic Units	4
Contact Hours	51
Research Experience Components	Not Applicable

Course Requisites (if applicable)

Pre-requisites	MH2100 OR MH1803
Co-requisites	
Pre-requisite to	
Mutually exclusive to	
Replacement course to	
Remarks (if any)	

Course Aims

This is a first introduction to topology and calculus on manifolds. The tools introduced in this course are the natural framework for the generalization of the ideas that you learnt in Calculus I, II, and III to infinite-dimensional and non-Euclidean spaces. These methods open the door to other fields in mathematics like algebraic topology, functional analysis, differential/Riemannian/symplectic/Poisson geometry, or Lie theory, to name a few. They also play a central role in contemporary applications across the physical sciences, engineering, and data-driven disciplines. Examples include dynamical systems and mechanics, symmetry and conservation laws, control theory, general relativity, gauge theories, and geometric methods in fluid dynamics. In recent years, manifold-based techniques have also become fundamental in machine learning, shape analysis, geometric data science, robotics and motion planning, and topological data analysis.

The aim of this course is to enable you to formulate and solve mathematical problems using the concepts and the formalism of topology and global analysis, and to appreciate how these ideas unify many modern developments in pure and applied mathematics.

Course's Intended Learning Outcomes (ILOs)

Upon the successful completion of this course, you (student) would be able to:

ILO 1	Describe the fundamental concepts of point-set topology and interpret key notions such as convergence and continuity in topological terms.
ILO 2	Explain and apply core topological properties such as connectedness, compactness, and completeness.
ILO 3	Use topological concepts to analyze and solve problems in fixed-point theory.
ILO 4	Formulate and describe non-Euclidean spaces in terms of manifolds.
ILO 5	Define and apply the notion of differentiability on manifolds.

Course Content

Topological spaces. Basis of a topology. Subspace, product, box, and quotient topologies. Continuous functions. Homeomorphisms, embeddings. Metric topologies. Topologies induced by norms and inner products. Function spaces and uniform convergence. Connectedness, compactness, and completeness. The Intermediate Value Theorem, the Extreme Value Theorem, Heine-Borel-Lebesgue, the Uniform Continuity Theorem. Countability and separation axioms. The Urysohn Lemma and the Metrization Theorem. Fixed Point Theorems and applications. Homotopy and the fundamental group. Topological and differential manifolds. Charts and atlas. Atlas equivalence and manifold topologies. Examples of manifolds. Submanifolds, products, and mappings. The tangent bundle.

Reading and References (if applicable)

Reference books:

Topology:

Lopez, R. Point-Set Topology, A Working Textbook, Springer Undergraduate Mathematics Series, 2024. ISBN: 978-3-031-58512-8

Munkres, James R. Topology, a First Course. Second edition. Pearson. 2014. ISBN: 978- 1292023625

Simmons, G. Introduction to Topology and Modern Analysis, McGraw-Hill Inc., US. 1963. ISBN: 978-0070573895

Willard. S. General Topology. Dover. 1994. ISBN: 978-0486434797

Manifolds:

Lee, J.M. Introduction to Topological Manifolds, Graduate Texts in Mathematics, 2011.

Abraham, R., Marsden, J. E., and Ratiu, T. S. Manifolds, Tensor Analysis, and Applications. Second Edition. 1988. ISBN: 978-1461269908

Gross, G. and Meinrenken, G. Manifolds, Vector Fields, and Differential Forms, Springer Undergraduate Mathematics Series, 2023. ISBN: 978-3-031-25408-6.

Planned Schedule

Week or Session	Topics or Themes	ILO	Readings	Delivery Mode	Activities
1	Preliminaries: sets, functions, and relations.	1	Chap 1, Munkres.	In-person	Lecture
2	Topological spaces, bases, subspace topology, product and box topologies, closed sets, limit points.	1	Chapter 2, Munkres. Chapter 2, Willard.	In-person	Lecture and tutorial
3	Continuous functions, homeomorphisms, embeddings.	1, 2	Chapter 2, Munkres. Chapter 3, Willard.	In-person	Lecture and tutorial
4	Metric spaces, topologies induced by norms and inner products, function spaces and uniform convergence.	1, 2	Chapter 2, Munkres. Chapter 1, Willard.	In-person	Lecture and tutorial
5	Completeness and fixed point theorems.	1, 2	Chapter 7, Munkres. Appendix one, Simmons. Chapter 6, Willard.	In-person	Lecture and tutorial
6	Connectedness and path connectedness, connected components.	1, 2	Chapter 3, Munkres.	In-person	Lecture and tutorial
7	Topological manifolds and Examples.	1, 2, 3	Chapter 2, Lee.	In-person	Lecture and tutorial

Week or Session	Topics or Themes	ILO	Readings	Delivery Mode	Activities
8	Homotopy, path-homotopy, and the fundamental group. Mid-term exam.	1, 2, 3	Chapter 9, Munkres. Chapter 7, Lee.	In-person	Lecture and tutorial
9	Compactness, local compactness.	4	Chapter 3, Munkres. Chapter 6, Willard.	In-person	Lecture and tutorial
10	The Intermediate Value Theorem, the Extreme Value Theorem, Heine-Borel-Lebesgue, the Uniform Continuity Theorem.	4, 5	Chapter 3, Munkres..	In-person	Lecture and tutorial
11	Differentiable manifolds, charts and atlases, examples.	4, 5	Chapter 3, Abraham, Marsden, Ratiu.	In-person	Lecture and tutorial
12	Submanifolds, products, and mappings.	4, 5	Chapter 3, Abraham, Marsden, Ratiu.	In-person	Lecture and tutorial
13	Revision	1, 2, 3, 4, 5		In-person	Lecture and tutorial

Learning and Teaching Approach

Approach	How does this approach support you in achieving the learning outcomes?
Lectures	Present the key ideas behind mathematical concepts. Illustrate the key ideas with examples. Present important steps used to solve different types of problems.
Tutorials	Develop proficiency in problem solving skills. Reinforce concepts already covered in the lectures. Give an opportunity for weaker or more reserved students to clarify doubts. If they desire, students can presenting their solutions to problems they have selected for discussion by the group.

Assessment Structure

Assessment Components (includes both continuous and summative assessment)

No.	Component	ILO	Related PLO or Accreditation	Weightage	Description of Assessment Component	Team/Individual	Rubrics	Level of Understanding
1	Continuous Assessment (CA): Assignment(Assignment)	1, 2, 3		20	Home assignment	Individual	Analytic	Extended Abstract
2	Continuous Assessment (CA): Test/Quiz(Short Answer Questions)	1, 2, 3, 4, 5		25	Mid-Term exam	Individual	Analytic	Extended Abstract
3	Summative Assessment (EXAM): Final exam(Short Answer Questions)	1, 2, 3, 4, 5		50	Exam	Individual	Analytic	Extended Abstract
4	Continuous Assessment (CA): Class Participation(In-class participation: Wooclap)	1. 2. 3. 4. 5.		5	In-class quizz with Wooclap regarding current topics.	Individual	Analytic	Extended Abstract

Description of Assessment Components (if applicable)

Formative Feedback

Common mistakes in the Home Assignment (20%) and the Mid-Term (25%) will be discussed in the provided solution set.

NTU Graduate Attributes/Competency Mapping

This course intends to develop the following graduate attributes and competencies (maximum 5 most relevant)

Attributes/Competency	Level
Communication	Advanced
Creative Thinking	Advanced
Curiosity	Intermediate
Learning Agility	Advanced
Problem Solving	Advanced

Course Policy

Policy (Academic Integrity)

Good academic work depends on honesty and ethical behaviour. The quality of your work as a student relies on adhering to the principles of academic integrity and to the NTU Honour Code, a set of values shared by the whole university community. Truth, Trust and Justice are at the core of NTU's shared values. As a student, it is important that you recognize your responsibilities in understanding and applying the principles of academic integrity in all the work you do at NTU. Not knowing what is involved in maintaining academic integrity does not excuse academic dishonesty. You need to actively equip yourself with strategies to avoid all forms of academic dishonesty, including plagiarism, academic fraud, collusion and cheating. If you are uncertain of the definitions of any of these terms, you should go to the academic integrity website for more information. On the use of technological tools (such as Generative AI tools), different courses / assignments have different intended learning outcomes. Students should refer to the specific assignment instructions on their use and requirements and/or consult your instructors on how you can use these tools to help your learning. Consult your instructor(s) if you need any clarification about the requirements of academic integrity in the course.

Policy (General)

Assessments:
1. A late submission for the Home Assignment is not accepted
2. No make-up Home Assignment and Mid-Term test will be arranged
3. A student who is absent from a Mid-Term test without valid Leave of Absence will be given zero mark.
4. If you are sick and not able to attend the Mid-Term test, you must submit the original Medical Certificate to an administrator. In this case, the total course marks would subsequently be rescaled to a base of 100%.

Policy (Absenteeism)

Attendance:
Attendance of lectures is not compulsory. However, you are highly encouraged to attend all lectures and keep handwritten notes.

Policy (Others, if applicable)

Diversity and inclusion policy

Integrating a diverse set of experiences is important for a more comprehensive understanding of science.

It is our goal to create an inclusive and collaborative learning environment that supports a diversity of perspectives and learning experiences, and that honours your identities; including ethnicity, gender, socioeconomic status, sexual orientation, religion or ability.

To help accomplish this:
- If you are neuroatypical or neurodiverse, have dyslexia or ADHD (for example), or have a social anxiety disorder or social phobia;
- If you feel like your performance in the class is being impacted by your experiences outside of class;
- If something was said in class (by anyone, including the instructor) that made you feel uncomfortable;

Please speak to your teaching team, our school pastoral officer or a peer or senior (either in-person or via email)

about how we can help facilitate your learning experience.

As a participant in course discussions, you should also strive to honour the diversity of your classmates. You can do this by: using preferred pronouns and names; being respectful of others opinions and actively making sure all voices are being heard; and refraining from the use of derogatory or demeaning speech or actions.

All members of the class are expected to adhere to the NTU anti-harassment policy. if you witness something that goes against this or have any other concerns, please speak to your instructors or a faculty member.

Appendix 1: Assessment Rubrics

Rubric for Home Assignment (20%)
Point-based marking (not rubrics based)

Rubric for Mid-term (25%)
Point-based marking (not rubrics based)

Rubric for Examination (50%)
Point-based marking (not rubrics based)

Rubric for In-Class Activity (5%)
Point-based marking (not rubrics based)
Complete 80% of Wooclap in-class activities to receive full marks