COURSE OUTLINE: MH3600

Course Title	**Topology and Manifolds**		
Course Code	**MH3600**		
Offered	Study Year 2, Semester 2		
Course Coordinator	Francois Joachim Marcel Gay-Balmaz (Assoc Prof)	francois.gb@ntu.edu.sg	6513 7187
Pre-requisites	MH2100 OR MH1803		
AU	4		
Contact hours	Lectures: 39, Tutorials: 12		
Approved for delivery from	AY 2023/24 semester 2		
Last revised	2 Nov 2023		

Course Aims

This is a first introduction to topology and calculus on manifolds. The tools introduced in this course are the natural framework for the generalization of the ideas that you learnt in Calculus I, II, and III to infinite-dimensional and non-Euclidean spaces. These methods open the door to other fields in mathematics like algebraic topology, functional analysis, differential/Riemannian/symplectic/Poisson geometry, or Lie theory, to name a few. They also have strong ties with important applications in the physical sciences and engineering like dynamical systems, mechanics, symmetry analysis, or control theory.

The aim of this course is to enable you to formulate and solve mathematical problems using the ideas and the formalism coming from topology and global analysis.

Intended Learning Outcomes

Upon successfully completing this course, you should be able to:

1. Describe the basics of point-set topology and to interpret important notions like convergence and continuity in topological terms.
2. Handle topological notions like connectedness, compactness.
3. Apply notions of topology to approximation and fixed-point theory.
4. Formulate non-Euclidean spaces as manifolds.
5. Handle differentiability problems in globally defined spaces as well as expressing dynamical systems as vector fields.

Course Content

Topological spaces. Basis of a topology. Subspace, product, box, and quotient topologies. Continuous functions. Homeomorphisms, embeddings. Metric topologies. Topologies induced by norms and inner products. Function spaces and uniform convergence. Connectedness and compactness. The Intermediate Value Theorem, the Extreme Value Theorem, Heine-Borel-Lebesgue, the Uniform Continuity Theorem. Countability and separation axioms. The Urysohn lemma and the Metrization Theorem. Approximation. The Stone-Weierstrass Theorems. Fixed Point Theorems and applications. Manifolds. Charts and atlas. Atlas equivalence and manifold topologies. Examples. Submanifolds, products, and mappings. The tangent and the cotangent bundle. Subimmersions, immersions, and transversality. Vector fields, flows, and dynamical systems. The Lie derivative. Vector fields as differential operators.

Assessment

Component	Course ILOs tested	Weighting	Team / Individual	Assessment Rubrics
Continuous Assessment				
Tutorials				
Assignment	1, 2, 3	20	individual	See Appendix for rubric
Presentation	1, 2, 3, 4, 5	10	individual	See Appendix for rubric
Project 1	1, 2, 3, 4, 5	10	individual	See Appendix for rubric
Examination (2 hours)				
Short Answer Questions	1, 2, 3, 4, 5	60	individual	See Appendix for rubric
	Total	**100%**		

Formative Feedback

There is a formal formative feedback designed as follows: 20% of the course assessment is for presenting homework as well as the results of a reading/research project at the end of the course. Students will get informal but individual feedback from the peers and from the course instructor for each of these presentations.

Common mistakes in the assignment will also be discussed in the provided solution set.

Learning and Teaching Approach

Lectures (39 hours)	Present the key ideas behind mathematical concepts. Illustrate the key ideas with examples. Present important steps used to solve different types of problems.
Tutorials (12 hours)	Develop proficiency in problem solving skills. Reinforce concepts already covered in the lectures. Give an opportunity for weaker or more reserved students to clarify doubts. Students will lead the tutorial sessions - presenting their solutions to problems they have selected for discussion by the group.

Reading and References

There are three reference books:

Munkres, James R. Topology, a First Course. Second edition. Pearson, 2014. ISBN: 978-1292023625

Willard. S. General Topology. Dover. 1994. ISBN: 978-0486434797

Abraham, R., Marsden, J. E., and Ratiu, T. S. Manifolds, Tensor Analysis, and Applications. Second Edition. 1988. ISBN: 978-1461269908

Course Policies and Student Responsibilities

Attendance:

Attendance of lectures is not compulsory. However, you are highly encouraged to attend all lectures and keep handwritten notes.

Assessments:

1. A late submission for an assignment is not accepted.
2. No make-up assignments or presentations will be arranged.
3. A student who is late for an assignment or presentation without valid Leave of Absence will be given zero mark.
4. In case of a valid reason for absence, the total course marks would subsequently be rescaled to a base of 100%.

Academic Integrity

Good academic work depends on honesty and ethical behaviour. The quality of your work as a student relies on adhering to the principles of academic integrity and to the NTU Honour Code, a set of values shared by the whole university community. Truth, Trust and Justice are at the core of NTU's shared values.

As a student, it is important that you recognize your responsibilities in understanding and applying the principles of academic integrity in all the work you do at NTU. Not knowing what is involved in maintaining academic integrity does not excuse academic dishonesty. You need to actively equip yourself with strategies to avoid all forms of academic dishonesty, including plagiarism, academic fraud, collusion and cheating. If you are uncertain of the definitions of any of these terms, you should go to the academic integrity website for more information.

On the use of technological tools (such as Generative AI tools), different courses / assignments have different intended learning outcomes. Students should refer to the specific assignment instructions on their use and requirements and/or consult your instructors on how you can use these tools to help your learning.

Consult your instructor(s) if you need any clarification about the requirements of academic integrity in the course.

Course Instructors

Instructor	Office Location	Phone	Email
Francois Joachim Marcel Gay-Balmaz (Assoc Prof)	SPMS-MAS-05-41	6513 7187	francois.gb@ntu.edu.sg

Planned Weekly Schedule

Week	Topic	Course ILO	Readings/ Activities
1	Topological spaces. Basis of a topology. Subspace, product, box, and quotient topologies.	1	Lecture
2	Continuous functions. Homeomorphisms, embeddings.	1	Lecture and tutorial
3	Metric topologies. Topologies induced by norms and inner products. Function spaces and uniform convergence.	1, 2	Lecture and tutorial
4	Connectedness and compactness. The Intermediate Value Theorem, the Extreme Value Theorem, Heine-Borel-Lebesgue, the Uniform Continuity Theorem.	1, 2	Lecture and tutorial
5	Countability and separation axioms. The Urysohn lemma and the Metrization Theorem.	1, 2	Lecture and tutorial
6	The Tietze extension theorem. Paracompactness, partitions of unity, and Euclidean embeddings.	1, 2	Lecture and tutorial
7	Approximation. The Stone-Weierstrass Theorems.	1, 2, 3	Lecture and tutorial
8	Fixed Point Theorems and applications.	1, 2, 3	Lecture and tutorial
9	Manifolds. Charts and atlas. Atlas equivalence and manifold topologies. Examples. Submanifolds, products, and mappings.	4	Lecture and tutorial
10	The tangent and the cotangent bundle. Subimmersions, immersions, and transversality.	4, 5	Lecture and tutorial
11	Vector fields, flows, and dynamical systems.	4, 5	Lecture and tutorial
12	The Lie derivative. Vector fields as differential operators.	4, 5	Lecture and tutorial
13	Project presentations and revision	1, 2, 3, 4, 5	Lecture and tutorial

Appendix 1: Assessment Rubrics

Rubric for Tutorials: Assignment (20%)

Point-based marking (not rubrics based)

Rubric for Tutorials: Presentation (10%)

Each student has to present a certain number of times during the semester (to be determined according to the number of course participants) and is graded individually in the style of an oral examination.

Grading Criteria	Exceptional (9-10)	Effective (7-8)	Acceptable (5-6)	Developing (0-4)
Accuracy	The interpretation is highly accurate, concise and precise.	The interpretation is mostly accurate. Some parts can be better explained or more succinct.	The interpretation is somewhat accurate. However, it contains some inaccuracies, missing points or ideas that are not related to the interpretation.	The interpretation is mostly inaccurate.
Visual and Oral delivery	Slides are informative, good clear voice and constant eye contact	Slides are adequate, oral delivery is satisfactory and frequent eye contact	Slides are bare and somewhat disorganized, voice is sometimes inaudible and little eye contact	Slides are disorganized, voice it too soft, and no eye contact
Presentation	Very clear and organized. It is easy to follow your train of thought	Mostly clear and organized. Some parts can have better transitions.	Somewhat clear. It requires some careful reading to understand what you are writing.	Mostly unclear and messy. It is difficult to understand what you are writing as there is no clear flow of ideas.
Question and Answer (for each individual student)	Very clear and precise answers to all problems. Explain the problems from various different perspectives logically.	Correct answers to most of the problems. Explain the problems in an organized way.	Partially-correct answers to most of the problems. Explain the some of the problems.	Unclear and messy answers. Difficult to understand.

Rubric for Tutorials: Project 1 (10%)

Each student has to give a final presentation in the last week of class. Presentations may be related to research or reading projects. Presentations are graded individually in the style of an oral examination.

Grading Criteria	Exceptional (9-10)	Effective (7-8)	Acceptable (5-6)	Developing (0-4)
Accuracy	The interpretation is highly accurate, concise and precise.	The interpretation is mostly accurate. Some parts can be better explained or more succinct.	The interpretation is somewhat accurate. However, it contains some inaccuracies, missing points or ideas that are not related to the interpretation.	The interpretation is mostly inaccurate.
Visual and Oral delivery	Slides are informative, good clear voice and constant eye contact	Slides are adequate, oral delivery is satisfactory and frequent eye contact	Slides are bare and somewhat disorganized, voice is sometimes inaudible and little eye contact	Slides are disorganized, voice it too soft, and no eye contact
Presentation	Very clear and organized. It is easy to follow your train of thought	Mostly clear and organized. Some parts can have better transitions.	Somewhat clear. It requires some careful reading to understand what you are writing.	Mostly unclear and messy. It is difficult to understand what you are writing as there is no clear flow of ideas.
Question and Answer (for each individual student)	Very clear and precise answers to all problems. Explain the problems from various different perspectives logically.	Correct answers to most of the problems. Explain the problems in an organized way.	Partially-correct answers to most of the problems. Explain the some of the problems .	Unclear and messy answers. Difficult to understand.

Rubric for Examination: Short Answer Questions (60%)

Point-based marking (not rubrics based)