COURSE OUTLINE: MH2814

Course Title	**Probability & Statistics**		
Course Code	**MH2814**		
Offered	Study Year 2, Semester 1		
Course Coordinators	Chan Song Heng (Assoc Prof)	chansh@ntu.edu.sg	6513 7453
Pre-requisites	MH1810 OR MT1001		
Mutually exclusive	MT2001, CV2001, CV2018, HE1005, MH1820, MH2500		
AU	3		
Contact hours	Lectures: 26, Tutorials: 12		
For delivery from	AY 2023/24 semester 1		
Last revised	6 June 2023		

Course Aims

Uncertainties are unavoidable in the design and planning of engineering system. Therefore, engineering analysis should include probability and statistics to evaluate the significance of uncertainty on system performance and design. This course provides the basics of probability and statistical concepts in terms that are more easily understood by engineering students. We present probability and statistical concepts through problems that are meaningful to engineering science. This course should motivate the recognition of the significant roles of the relevance mathematical concepts in engineering.

Intended Learning Outcomes

Upon successfully completing this course, you should be able to:

1. Use the techniques of permutations and combination to calculate the probability of the occurrence of an event
2. Apply Bayes' rule to compute conditional probability and identify independent events
3. State the definition of a random variable (RV), tabulate its probability mass function or density function, and be able to compute its expectation and variance
4. Recognize a joint probability distribution and compute its marginal and conditional distributions
5. Calculate the probability, mean and variance of a RV that has either binomial, geometric or Poisson distribution, and apply the Poisson approximation to binomial distribution
6. Evaluate the probability of a normal RV, its mean and variance, and apply the normal approximation to binomial distribution
7. Distinguish the difference between population and sample, parameter and statistic
8. Apply the Central Limit Theorem (CLT) to the distribution of sample mean
9. Construct confidence interval for sample mean, and determine the sample size with appropriate margin of error
10. State the null and alternative hypotheses of a statistical test, determine the type I and type II errors, and compute its power and p-value
11. Apply a simple linear model to fit a dataset with 2 variables, calculate the values and construct confidence interval for the model coefficients, and perform prediction

Course Content

Topic 1: Basic Probability Theory: • Sample space, Events, Counting Sample Points, Permutations & Combinations. • Probability, Equally Likely Outcomes, Not Equally Likely

outcomes, Useful Rules • Conditional Probability, Independent Events, Partition, Bayes' Rule

Topic 2: Random Variables & Probability Distribution: • Random Variables (RV), Discrete Random variables, Probability Mass Distribution, Cumulative Probability Distribution • Bernoulli Distribution and Binomial Distribution • Continuous Random variables, Probability Density Function, Cumulative Distribution Function

Topic 3: Expectation & Variance : • Expectation of Discrete & Continuous RVs, Expectation of general RV, Useful Properties of Expectation, Variance & Properties

Topic 4: Joint Distribution : • Joint Probability Distribution & Marginal distribution (discrete RVs), Joint Probability Distribution & Marginal distribution (continuous RVs), Expectation E[g(X,Y)] • Conditional Probability Distribution, (Statistically) Independent RV. Variance, Covariance, Correlation coefficients, Useful Results

Topic 5: Special Discrete Probability Distributions : • Binomial, Geometric (return period), Poisson Distribution, Poisson Approximation

Topic 6: Normal Distribution : • Normal Distribution, Standard Normal Distribution (introduction) • Standard Normal Distribution, Mean & Variance (Proofs excluded) • Applications: Problems, Approximation of Binomial Distribution

Topic 7: Statistics: • Population & Parameters, Samples & Sample Statistics

Topic 8: Sampling Distribution and Estimation : • Random Sampling, Sampling Distribution, Distribution of Sample Mean, Central Limit Theorem • Unbiased Estimator of variance, chi-sq-distribution, Samples with Unknown Population Variance, t-distribution • Sampling Distribution of Difference between two Means

Topic 9 Confidence Interval: l • Confidence Intervals (introduction), Known/Unknown Variance, Large/Small Sample • Confidence Intervals (1-sided, 2-sided Confidence interval), Error, Size, Prediction Interval • 2-Population: confidence Interval, Small Size Unknown Variance

Topic 10: Hypothesis Testing: • Introduction, Type-1, Type-2 Error • Hypothesis Testing for Population Mean (1-population), Power, p-value • Hypothesis Testing for Difference between Population Means

Topic 11 Simple Linear Regression: • Linear Regression & Transformation • Estimation of Regression Parameters • Confidence Interval for Parameters in Linear Regression

Assessment

Component	Course ILOs tested	EAB Graduate Attributes tested	Weighting	Team / Individual	Assessment Rubrics
Continuous Assessment					
Mid-semester Quiz					
Short Answer Questions 1	1, 2, 3, 4, 5, 6	a, b, h, i, j, l	18	individual	See Appendix for rubric
Short Answer Questions 2	7, 8, 9, 10	a, b, h, i, j, l	18	individual	See Appendix for rubric
Tutorials					
Tutorial Participation	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11	a, b, h, i, j,l	4	individual	See Appendix for rubric
Examination (2 hours)					
Short Answer Questions	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11	a, b, h, i, j, l	60	individual	See Appendix for rubric
		Total	**100%**		

Mapping of Course Intended Learning Outcomes to Engineering Accreditation Board (EAB) Graduate Attributes

Category	Core				
EAB's 12 Graduate Attributes*					
(a) ●	(b) ●	(c)	(d)	(e)	(f)
(g)	(h) ●	(i) ●	(j) ●	(k)	(l) ●

Overall statement

Uncertainties are unavoidable in the design and planning of engineering system. Therefore, engineering analysis should include probability and statistics to evaluate the significance of uncertainty on system performance and design. This course provides the basics of probability and statistical concepts in terms that are more easily understood by engineering students. We present probability and statistical concepts through problems that are meaningful to engineering science. This course should motivate the recognition of the significant roles of the relevance mathematical concepts in engineering.

Course Student Learning Outcomes		**EAB Graduate Attributes**
1	Use the techniques of permutations and combination to calculate the probability of the occurrence of an event	a, b, h, i, j, l
2	Apply Bayes' rule to compute conditional probability and identify independent events	a, b, h, i, j, l
3	State the definition of a random variable (RV), tabulate its probability mass function or density function, and be able to compute its expectation and variance	a, b, h, i, j, l
4	Recognize a joint probability distribution and compute its marginal and conditional distributions	a, b, h, i, j, l
5	Calculate the probability, mean and variance of a RV that has either binomial, geometric or Poisson distribution, and apply the Poisson approximation to binomial distribution	a, b, h, i, j, l
6	Evaluate the probability of a normal RV, its mean and variance, and apply the normal approximation to binomial distribution	a, b, h, i, j, l
7	Distinguish the difference between population and sample, parameter and statistic	a, b, h, i, j, l
8	Apply the Central Limit Theorem (CLT) to the distribution of sample mean	a, b, h, i, j, l
9	Construct confidence interval for sample mean, and determine the sample size with appropriate margin of error	a, b, h, i, j, l
10	State the null and alternative hypotheses of a statistical test, determine the type I and type II errors, and compute its power and p-value	a, b, h, i, j, l
11	Apply a simple linear model to fit a dataset with 2 variables, calculate the values and construct confidence interval for the model coefficients, and perform prediction	a, b, h, i, j, l

***Legend:**
● Fully consistent (contributes to more than 75% of Student Learning Outcomes)
◐ Partially consistent (contributes to about 50% of Student Learning Outcomes)
○ Weakly consistent (contributes to about 25% of Student Learning Outcomes)
[Blank]Not related to Student Learning Outcomes

The graduate attributes as stipulated by the EAB, are:

a. **Engineering knowledge:** Apply the knowledge of mathematics, natural science, engineering fundamentals, and an engineering specialisation to the solution of complex engineering problems.

b. **Problem Analysis:** Identify, formulate, research literature, and analyse complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.

c. **Design/development of Solutions:** Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for public health and safety, cultural, societal, and environmental considerations.

d. **Investigation:** Conduct investigations of complex problems using research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.

e. **Modern Tool Usage:** Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modelling to complex engineering activities with an understanding of the limitations.

f. **The engineer and Society:** Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal, and cultural issues and the consequent responsibilities relevant to the professional engineering practice.

g. **Environment and Sustainability:** Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for the sustainable development.

h. **Ethics:** Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.

i. **Individual and Team Work:** Function effectively as an individual, and as a member or leader in diverse teams and in multidisciplinary settings.

j. **Communication:** Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.

k. **Project Management and Finance:** Demonstrate knowledge and understanding of the engineering and management principles and economic decision-making, and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.

l. **Life-long Learning:** Recognise the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

Formative Feedback

Lecture: Help you understand the motivation and definitions of the concepts and notions, approaches to solving problems in pursuant to learning outcomes

Tutorial: Develop problem solving skills, reinforce the understanding of the concepts and notions

Learning and Teaching Approach

Lectures (26 hours)	Help you understand the motivation and definitions of the concepts and notions, approaches to solving problems in pursuant to learning outcomes. LO: 1 to 11.
Tutorials (12 hours)	Develop problem solving skills, reinforce the understanding of the concepts and notions. LO:1 to 11

Reading and References

Walpole, Myers, Myers, Ye, Probability and Statistics for Engineers and Scientists, 9th Edition, Pearson. ISBN-13:9780321629111

Course Policies and Student Responsibilities

(1) General

Students are expected to complete all tutorial question, and take the quizzes. Students are expected to take responsibility to follow up with course notes, tutorials and course related announcements if they are absent.

(2) Absenteeism

Absence from quizzes and examination without a valid reason will affect your overall course grade. Valid reasons include falling sick supported by a medical certificate and participation in NTU’s approved activities supported by an excuse letter from the relevant bodies.

(3) Absence Due to Medical or Other Reasons

If you are sick and not able to attend the quizzes, you have to submit the Medical Certificate (or another relevant document) to the administration to obtain official leave. In this case, the missed assessment component will not be counted towards the final grade. There are no make-up quizzes.

Academic Integrity

Good academic work depends on honesty and ethical behaviour. The quality of your work as a student relies on adhering to the principles of academic integrity and to the NTU Honour Code, a set of values shared by the whole university community. Truth, Trust and Justice are at the core of NTU's shared values.

As a student, it is important that you recognize your responsibilities in understanding and applying the principles of academic integrity in all the work you do at NTU. Not knowing what is involved in maintaining academic integrity does not excuse academic dishonesty. You need to actively equip yourself with strategies to avoid all forms of academic dishonesty, including plagiarism, academic fraud, collusion and cheating. If you are uncertain of the definitions of any of these terms, you should go to the Academic Integrity website for more information. Consult your instructor(s) if you need any clarification about the requirements of academic integrity in the course.

Course Instructors

Instructor	Office Location	Phone	Email
Chan Song Heng (Assoc Prof)	SPMS-MAS-04-13	6513 7453	chansh@ntu.edu.sg

Planned Weekly Schedule

Week	Topic	Course ILO	Readings/ Activities
1	Topic 1: Basic Probability Theory: • Sample space, Events, Counting Sample Points, Permutations & Combinations. • Probability, Equally Likely Outcomes, Not Equally Likely outcomes, Useful Rules • Conditional Probability, Independent Events, Partition, Bayes' Rule	1, 2	Lecture notes
2	Topic 2: Random Variables & Probability Distribution: • Random Variables (RV), Discrete Random variables, Probability Mass Distribution, Cumulative Probability Distribution • Bernoulli Distribution and Binomial Distribution • Continuous Random variables, Probability Density Function, Cumulative Distribution Function	3	Lecture notes / Tutorial
3	Topic 3: Expectation & Variance : • Expectation of Discrete & Continuous RVs, Expectation of general RV, Useful Properties of Expectation, Variance & Properties	3, 5	Lecture notes / Tutorial
4	Topic 4: Joint Distribution : • Joint Probability Distribution & Marginal distribution (discrete RVs), Joint Probability Distribution & Marginal distribution (continuous RVs), Expectation E[g(X,Y)] • Conditional Probability Distribution, (Statistically) Independent RV. Variance, Covariance, Correlation coefficients, Useful Results	4, 5	Lecture notes / Tutorial
5	Topic 5: Special Discrete Probability Distributions : • Binomial, Geometric (return period), Poisson Distribution, Poisson Approximation	5	Lecture notes / Tutorial
6	Topic 6: Normal Distribution : • Normal Distribution, Standard Normal Distribution (introduction) • Standard Normal Distribution, Mean & Variance (Proofs excluded) • Applications: Problems, Approximation of Binomial Distribution	6	Lecture notes / Tutorial
7	Topic 7: Statistics: • Population & Parameters, Samples & Sample Statistics	7	Lecture notes / Tutorial
8	Topic 8: Sampling Distribution and Estimation : • Random Sampling, Sampling Distribution, Distribution of Sample Mean, Central Limit Theorem • Unbiased Estimator of variance, chi-sq-distribution, Samples with Unknown Population Variance, t-distribution • Sampling Distribution of Difference between two Means	8, 9	Lecture notes / Tutorial
9	Topic 9 Confidence Interval: • Confidence Intervals (introduction), Known/Unknown Variance, Large/Small Sample • Confidence Intervals (1-sided, 2-sided Confidence interval), Error, Size, Prediction Interval • 2-Population: confidence Interval, Small Size Unknown Variance	9	Lecture notes / Tutorial
10	Topic 9 Confidence Interval: • 2-Population: confidence Interval, Small Size Unknown Variance	8, 9	Lecture notes / Tutorial
11	Topic 10: Hypothesis Testing: • Introduction, Type-1, Type-2 Error • Hypothesis Testing for Population Mean (1-population), Power, p-value • Hypothesis Testing for Difference between Population Means	10	Lecture notes / Tutorial
12	Topic 11 Simple Linear Regression: • Linear Regression & Transformation • Estimation of Regression Parameters • Confidence Interval for Parameters in Linear Regression	11	Lecture notes / Tutorial
13	Revision	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11	Lecture notes

Appendix 1: Assessment Rubrics

Rubric for Mid-semester Quiz: Short Answer Questions 1 (18%)

Criteria	**Standards**		
	Fail standard	**Pass standard**	**High standard**
Methods of approach	· Using methods that are irrelevant or do not apply to the given problem. · Invoking theorems whose conditions are not satisfied.	· Using relevant methods that help solve the problem. · Invoking theorems whose conditions are satisfied.	Finding methods and utilizing theorems that are both relevant and effective
Validity of reasoning	Reasoning is logically invalid.	Reasoning is logically valid.	Reasoning is logically valid and effective.
Clarity of argument	Reasoning is poorly explained or not explained at all.	Reasoning is clear but may contain some gaps.	Reasoning is clear, precise with no or insignificant gaps.

Rubric for Mid-semester Quiz: Short Answer Questions 2 (18%)

Criteria	**Standards**		
	Fail standard	**Pass standard**	**High standard**
Methods of approach	· Using methods that are irrelevant or do not apply to the given problem. · Invoking theorems whose conditions are not satisfied.	· Using relevant methods that help solve the problem. · Invoking theorems whose conditions are satisfied.	Finding methods and utilizing theorems that are both relevant and effective
Validity of reasoning	Reasoning is logically invalid.	Reasoning is logically valid.	Reasoning is logically valid and effective.
Clarity of argument	Reasoning is poorly explained or not explained at all.	Reasoning is clear but may contain some gaps.	Reasoning is clear, precise with no or insignificant gaps.

Rubric for Tutorial Participation (4%)

Criteria	**Standards**		
	Fail standard	**Pass standard**	**High standard**
Presentation of solution	Did not volunteer to present tutorial solution and decline to present when called.	Made one tutorial presentation during the 12 tutorial sessions.	Made two to three tutorial presentations during the 12 tutorial sessions.
Other forms of participation*	No engagement in the tutorial activities.	Engaged during tutorial to some extent, limited involvement.	Actively engaged during tutorial, such as consistently contributed to discussions, asking and answering questions.

*(Student will not be assessed on the quality of presentation and solution.)

Rubric for Examination: Short Answer Questions (60%)

Criteria	**Standards**		
	Fail standard	**Pass standard**	**High standard**
Methods of approach	· Using methods that are irrelevant or do not apply to the given problem. · Invoking theorems whose conditions are not satisfied.	· Using relevant methods that help solve the problem. · Invoking theorems whose conditions are satisfied.	Finding methods and utilizing theorems that are both relevant and effective

Validity of reasoning	Reasoning is logically invalid.	Reasoning is logically valid.	Reasoning is logically valid and effective.
Clarity of argument	Reasoning is poorly explained or not explained at all.	Reasoning is clear but may contain some gaps.	Reasoning is clear, precise with no or insignificant gaps.