Academic Year	2022/23	**Semester**	2
Course Coordinator	Assoc. Prof. Cheong Siew Ann		
Course Code	MH2801		
Course Title	Complex Methods for the Sciences		
Pre-requisites	(MH1801 & MH2800) or (MH1101 & MH1200) or (MH1802 & MH1803 & MH1200) or (MH1802 & MH1803 & MH2802) or (CY1601 & CY1602)		
Mutually Exclusive	MH3101		
No of AUs	3 AU		
Contact Hours	Lecture: 26 hours; Tutorial: 12 hours (2 hr – lecture; 1 hr – tutorial)		
Proposal Date	15 January 2023		

Course Aims

This course introduces the use of mathematical techniques based on complex numbers and their applications in physics and the other sciences. The techniques include contour integration, Fourier transforms, and Green's functions, and the applications include the solution of definite integrals and differential equations, and the modelling and analysis of oscillators and waves.

Intended Learning Outcomes (ILO)

Upon the successful completion of this course, you (as a student) would be **able to**:

Complex Algebra:

1. Manipulate algebraic expressions involving complex numbers to calculate the real part, imaginary part, magnitude, argument, and related quantities.
2. Use Euler's formula to translate between complex exponentials, trigonometric functions, and hyperbolic functions, and to solve integrals containing combinations of such functions.
3. Use the complex plane to visualise complex numbers and simple operations acting on them.
4. Formulate the branch functions for a multi-valued operation based on roots and/or the complex logarithm, including identifying branch points and selecting branch cuts.
5. Plot the contour, or complex plane trajectory, of a complex function of a real variable.
6. Use complex numbers to solve harmonic oscillator and wave problems.

Complex Calculus:

7. Identify the domains of analyticity for common complex functions such as reciprocals.
8. Use the Cauchy-Riemann equations to determine if a given complex function is analytic, or to reconstruct an analytic function from its real or imaginary part, and related tasks.
9. Determine the simple poles and residues of a complex function.
10. Evaluate contour integrals via parameterisation and via Cauchy's integral theorem.
11. Solve definite integrals via contour integration, the calculus of residues, and Jordan's lemma.

Fourier Transforms and Green's Functions:

12. Calculate the Fourier series coefficients of a periodic function of a real variable.
13. Calculate the Fourier transform of a square-integrable function of a real variable.
14. Deduce the features of a function from its Fourier spectrum, and vice versa.
15. Use Fourier transforms to solve linear differential equations.
16. Derive the Green's function for a driven oscillator or wave problem, and use it to obtain the solution for an arbitrary driving source.

Course Contents

Review of Real Functions

- Exponential, logarithm, trigonometric, and hyperbolic functions
- Power operations
- Definition of real derivatives and integrals
- Basic integration techniques (integration by parts and change of variables)
- The Gaussian integral

Complex Algebra

- Definition of complex numbers; basic complex algebraic manipulations; magnitude and argument of a complex number
- Complex exponential and trigonometric functions
- Euler's formula and its uses (e.g. for solving definite integrals)
- The complex plane and trajectories in the complex plane (contours)

Complex Oscillators and Waves

- Formulation and solution of the complex damped harmonic oscillator equation
- Interpretation of complex frequencies
- Under-damped, critically-damped, and over-damped motion

Complex Derivatives

- Definition of complex differentiability; the domain of analyticity of a complex function
- Derivation of the Cauchy-Riemann Equations and their applications

Branch Cuts and Branch Points

- Root and logarithms as multiple-valued operations
- Branch points of root and logarithm operations
- Branch cuts and the formulation of branch functions

Contour Integrals

- Evaluation of contour integral via parameterisation
- Cauchy's Integral Theorem
- The calculus of residues and Jordan's lemma
- Cauchy's principal value integrals

Fourier Transforms

- The Fourier series and its properties
- The Fourier Transform and Inverse Fourier Transform
- Interpretation of Fourier spectra

Green's Functions

- Formulation and application of the Green's function for a damped driven harmonic oscillator
- Formulation and application of the Green's function for waves in empty space
- Causality and the Green's function in space and time

Assessment (includes both continuous and summative assessment)

Component	Course LO Tested	Related Programme LO or Graduate Attributes	Weighting	Team / Individual	Assessment Rubrics
1. Final Examination	All	Competence (Written)	60%	Individual	Point-based marking (not rubric-based)
2. CA1: Homeworks	All	Competence (Written)	20%	Individual	Point-based marking (not rubric-based)
3. CA2: Midterm Test	1-11	Competence (Written)	20%	Individual	Point-based marking (not rubric-based)
Total			100%		

Formative feedback

You will receive formative feedback through discussion within tutorial lessons and via written feedback on graded assignments and the midterm. Solutions will be provided for assignments and the midterm.

Learning and Teaching approach

Approach	How does this approach support students in achieving the learning outcomes?
Lectures	The lectures build up the central concepts of the course, and provide concrete examples for calculations of the type and difficulty students are expected to be able to do.
Homework	The homework comprises textbook-style practice questions, and are discussed during the tutorials.

Reading and References

1. Y. D. Chong, *Complex Methods for the Sciences* (online notes)

Course Policies and Student Responsibilities

Absence Due to Medical or Other Reasons

If you are sick and unable to attend your class / Mid-terms, you have to:

1. Send an email to the instructor regarding the absence and request for a replacement class and make-up mid-terms.
2. Submit the original Medical Certificate* or official letter of excuse to administrator.
3. Attend the assigned replacement class (*subject to availability*) and make-up mid-terms.

* The medical certificate mentioned above should be issued in Singapore by a medical practitioner registered with the Singapore Medical Association.

Academic Integrity

Good academic work depends on honesty and ethical behaviour. The quality of your work as a student relies on adhering to the principles of academic integrity and to the NTU Honour Code, a set of values shared by the whole university community. Truth, Trust and Justice are at the core of NTU's shared values.

As a student, it is important that you recognize your responsibilities in understanding and applying the principles of academic integrity in all the work you do at NTU. Not knowing what is involved in maintaining academic integrity does not excuse academic dishonesty. You need to actively equip yourself with strategies to avoid all forms of academic dishonesty, including plagiarism, academic fraud, collusion and cheating. If you are uncertain of the definitions of any of these terms, you should go to the academic integrity website for more information. Consult your instructor(s) if you need any clarification about the requirements of academic integrity in the course.

Course Instructors

Instructor	Office Location	Phone	Email
Assoc. Prof. Cheong Siew Ann	SPMS-PAP-04-03	+65 6513 8084	cheongsa@ntu.edu.sg

Planned Weekly Schedule

Week	Topic	Course LO	Readings/ Activities
1	Review of complex numbers; Cartesian and polar representations, Euler formula; trigonometric, hyper-trigonometric functions, and their inverses	1-3	Homework 1
2	Visualization of complex numbers on Argand diagram; visualizing functions of a single complex variable; multi-valuedness, Riemann surfaces	4	Homework 2
3	Branch points and branch cuts; review differentiating function of a single real variable; differentiating function of a single complex variable; Cauchy-Riemann relations	4-8	Homework 2, Homework 3
4	Contour integrals, open and closed; Cauchy integral theorem; Cauchy integral formula	5-11	Homework 3, Homework 4
5	Laurent series; classification of singularities; residue theorem; Cauchy principal value	9-10	Homework 4
6	Applications of contour integration to evaluate real integrals; Jordan lemma	11	Homework 4
7	Periodic functions; real Fourier series; complex Fourier series	12	Homework 5
8	Aperiodic functions; Fourier transforms; inverse Fourier transforms; evaluating inverse Fourier transforms using contour integration	13	Midterm Test, Homework 6
9	Dirac delta function; convolution	14	Homework 6
10	Applications of Fourier series to solve	15	Homework 7

	differential equations		
11	Applications of Fourier transforms to solve differential equations	15	Homework 7
12	Green functions	16	Homework 8
13	Application of Green functions to solve differential equations	16	Homework 8

Graduate Attributes

What we want our graduates from Physics and Applied Physics to be able to do:

Upon the successful completion of the PHY, APHY, PHME, PHMP and PHMS programs, graduates should be able to:

Competency	1	demonstrate a rigorous understanding of the core theories and principles of physics involving (but not limited to) areas such as classical mechanics, electromagnetism, thermal physics and quantum mechanics [PHMS only] demonstrate a rigorous understanding of the core theories and principles of mathematical sciences involving (but not limited to) areas such as analysis, algebra and statistical analysis
	2	read and understand undergraduate level physics content independently;
	3	make educated guesses / estimations of physical quantities in general;
	4	apply fundamental physics knowledge, logical reasoning, mathematical and computational skills to analyse, model and solve problems;
	5	develop theoretical descriptions of physical phenomena with an understanding of the underlying assumptions and limitations;
	6	critically evaluate and distinguish sources of scientific/non-scientific information and to recommend appropriate decisions and choices when needed;
	7	demonstrate the ability to design and conduct experiments in a Physics laboratory, to make measurements, analyse and interpret data to draw valid conclusions.

Creativity	1	propose valid approaches to tackle open-ended problems in unexplored domains;
	2	offer valid alternative perspectives/approaches to a given situation or problem.

Communication	1	describe physical phenomena with scientifically sound principles;
	2	communicate (in writing and speaking) scientific and non-scientific ideas effectively to professional scientists and to the general public;
	3	communicate effectively with team members when working in a group.

Character	1	uphold absolute integrity when conducting scientific experiments, reporting and using the scientific results;
	2	readily pick up new skills, particularly technology related ones, to tackle new problems;
	3	contribute as a valued team member when working in a group.

Civic Mindedness	1	put together the skills and knowledge into their work in an effective, responsible and ethical manner for the benefits of society.