

## COURSE OUTLINE: MH2500

|                            |  |                   |           |
|----------------------------|--|-------------------|-----------|
| Course Title               | <b>Probability and Introduction to Statistics</b>  |                   |           |
| Course Code                | <b>MH2500</b>  |                   |           |
| Offered                    | Study Year 2, Semester 1   |                   |           |
| Course Coordinator         | Wu Guohua (Assoc Prof)   | guohua@ntu.edu.sg | 6513 7186 |
| Pre-requisites             | MH1100 AND MH1101<br>OR MH1800 AND MH1801<br>OR MH1101 AND MH110S<br>OR MH1100 AND MH111S<br>OR MH1802<br>OR CY1601<br>OR MH1805 |                   |           |
| AU                         | 4  |                   |           |
| Contact hours              | Lectures: 39, Tutorials: 12  |                   |           |
| Approved for delivery from | AY 2021/22 semester 1  |                   |           |
| Last revised               | 24 May 2021, 17:08   |                   |           |

### Course Aims

This is a core mathematical course, aiming to develop your understanding of fundamental concepts in probability and statistics such as random variables, independence, basic probability distributions, and confidence intervals. The course also prepares you for further statistics courses such as MH3500 in the Statistics Track.

### Intended Learning Outcomes

Upon successfully completing this course, you should be able to:

1. Calculate probabilities of events concerning discrete distributions by counting
2. Calculate conditional probabilities with Bayes Theorem
3. Describe a probability distribution using cdf/pdf, including joint and marginal distributions
4. Identify the scenarios which certain probability distributions are appropriate to model
5. Calculate the expectation/variance/mgf/quantiles of a probability distribution/random variable, including marginal distributions
6. Calculate the distribution of functions of random variables (especially the sum and quotient of two random variables) and the covariance of two random variables
7. Prove or disprove that two random variables are independent
8. Explain central limit theorem and its significance, and apply central limit theorem to approximate the probability of event concerning a sum of random variables
9. Conduct simple hypothesis tests

### Course Content

Events, probabilities, law of total probability, Bayes theorem

Independence events, discrete distributions

Discrete distributions, continuous distributions

Continuous distributions

Continuous distributions; joint distribution, marginal and conditional distribution for discrete variables

Joint distribution, marginal and conditional distribution for conditional variables

Functions of two or more random variables, order statistics

Expectation, variance, Chebyshev inequality

Covariance

Conditional expectation and moment generating functions

Law of large numbers, central limit theorem

Random sampling, estimation of parameters

Hypothesis testing

## Assessment

| Component                    | Course ILOs tested        | SPMS-MAS Graduate Attributes tested | Weighting   | Team / Individual | Assessment Rubrics      |
|------------------------------|---------------------------|-------------------------------------|-------------|-------------------|-------------------------|
| <b>Continuous Assessment</b> |                           |                                     |             |                   |                         |
| <b>Tutorials</b>             |                           |                                     |             |                   |                         |
| Participation                | 1, 2, 3, 4, 5, 6, 7, 8, 9 | 1. a, b, c<br>2. a, b<br>3. a       | 10          | individual        | See Appendix for rubric |
| Test 1                       | 1, 2, 3, 5                | 1. a, b<br>2. a, b                  | 20          | individual        | See Appendix for rubric |
| Test 2                       | 5, 6, 7                   | 1. a, b<br>2. a, b                  | 20          | individual        | See Appendix for rubric |
| <b>Examination (2 hours)</b> |                           |                                     |             |                   |                         |
| Short Answer Questions       | 1, 2, 3, 4, 5, 6, 7, 8, 9 | 1. a, b<br>2. a, b                  | 50          | individual        | See Appendix for rubric |
| <b>Total</b>                 |                           |                                     | <b>100%</b> |                   |                         |

These are the relevant SPMS-MAS Graduate Attributes.

### 1. Competence

- a. Independently process and interpret mathematical theories and methodologies, and apply them to solve problems
- b. Formulate mathematical statements precisely using rigorous mathematical language
- c. Discover patterns by abstraction from examples

### 2. Creativity

- a. Critically assess the applicability of mathematical tools in the workplace
- b. Build on the connection between subfields of mathematics to tackle new problems

### 3. Communication

- a. Present mathematics ideas logically and coherently at the appropriate level for the intended audience

## Formative Feedback

Feedback will be given after each midterm on common mistakes and level of difficulty of the problems. For the final exam, comments on answers and common errors will also be given to students after the exams are marked.

Feedback can also be given through discussion within tutorial lessons.

## Learning and Teaching Approach

|                                |  |
|--------------------------------|--|
| <b>Lectures</b><br>(39 hours)  | Help the students understand the motivation and definitions of the concepts and notions, approaches to solving the problems in pursuant to learning outcomes |
| <b>Tutorials</b><br>(12 hours) | Develop communication and presentation skills, help the students understand better the concepts and notions better and the techniques in problem solving     |

## Reading and References

Textbook:

[R1] Sheldon Ross. A First Course in Probability. Pearson. (Any of 8th-10th edition. ISBN: 978-0136033134, 978-0321794772 or 978-0134753119)

[R2] Sheldon Ross. Introductory Statistics. Academic Press/Elsevier. 4th edition. ISBN: 978-0128043172.

## Course Policies and Student Responsibilities

Absence due to medical or other reasons

If you are sick and unable to attend a test you must:

1. Send an email to the instructor regarding the absence.
2. Submit the original Medical Certificate\* to an administrator.

\*The Medical Certificate mentioned above should be issued in Singapore by a medical practitioner registered with the Singapore Medical Association.

## Academic Integrity

Good academic work depends on honesty and ethical behaviour. The quality of your work as a student relies on adhering to the principles of academic integrity and to the NTU Honour Code, a set of values shared by the whole university community. Truth, Trust and Justice are at the core of NTU's shared values.

As a student, it is important that you recognize your responsibilities in understanding and applying the principles of academic integrity in all the work you do at NTU. Not knowing what is involved in maintaining academic integrity does not excuse academic dishonesty. You need to actively equip yourself with strategies to avoid all forms of academic dishonesty, including plagiarism, academic fraud, collusion and cheating. If you are uncertain of the definitions of any of these terms, you should go to the [Academic Integrity website](#) for more information. Consult your instructor(s) if you need any clarification about the requirements of academic integrity in the course.

## Course Instructors

| Instructor             | Office Location | Phone     | Email             |
|------------------------|-----------------|-----------|-------------------|
| Wu Guohua (Assoc Prof) | SPMS-MAS-05-40  | 6513 7186 | guohua@ntu.edu.sg |

## Planned Weekly Schedule

| Week | Topic  | Course ILO | Readings/ Activities         |
|------|--|------------|------------------------------|
| 1    | Events, probabilities, law of total probability, Bayes theorem | 1, 2, 6, 7 | [R1] §2.1-2.5, §3.1-3.2, 3.5 |
| 2    | Independence events, discrete distributions                    | 3, 4       | [R1] §3.3-3.4                |
| 3    | discrete random variables                                      | 3, 4, 5    | [R1] §4.1-4.2                |
| 4    | discrete random variables                                      | 3, 4, 5    | [R1] §4.3-4.7                |
| 5    | discrete random variables                                      | 3, 4, 5    | [R1] §4.7-4.10               |
| 6    | continuous random variables                                    | 3, 6, 7    | [R1] §5.1-5.4                |
| 7    | continuous random variables                                    | 3, 6, 7    | [R1] §5.1-5.4                |
| 8    | jointly distributed random variables                           | 3, 6, 7    | [R1] §6.1-6.3                |
| 9    | jointly distributed random variables                           | 6          | [R1] §6.4-6.6                |
| 10   | expectations involving multiple random variables               | 5, 6       | [R1] §7.1-7.4                |
| 11   | expectations involving multiple random variables               | 5, 6       | [R1] §7.5-7.8                |
| 12   | Law of large numbers, central limit theorem                    | 8          | [R1] §8.1-8.4                |
| 13   | Hypothesis testing   | 9          | [R2] §9.1-9.3                |

## Appendix 1: Assessment Rubrics

### Rubric for Tutorials: Participation (10%)

| Criteria      | Standards                                   |   |  |
|---------------|---|---|--|
|               | Fail standard                               | Pass standard   | High standard  |
| Participation | Lack participation                          | Participation takes place                                   | Active participation, including volunteering on questions and helping other presenters |
| Presentation  | Poor explanation or incoherent presentation | Explanation can be followed but may have a few vague points | Explanation is clear   |

### Rubric for Tutorials: Test 1 (20%)

| Criteria              | Standards   |   |   |
|-----------------------|---|---|---|
|                       | Fail standard   | Pass standard   | High standard   |
| Methods of approach   | Using methods that are irrelevant or do not apply to the given problem. Invoking theorems whose conditions are not satisfied. | Using relevant methods that help solve the problem. Invoking theorems whose conditions are satisfied. | Finding methods and utilizing theorems that are both relevant and effective |
| Validity of reasoning | Reasoning is logically invalid.   | Reasoning is logically valid.   | Reasoning is logically valid and effective.                                 |
| Clarity of argument   | Reasoning is poorly explained or not explained at all.  | Reasoning is clear but may contain some gaps.   | Reasoning is clear, precise with no or insignificant gaps.                  |

**Rubric for Tutorials: Test 2 (20%)**

| Criteria              | Standards  |  |   |
|-----------------------|--|--|---|
|                       | Fail standard  | Pass standard  | High standard   |
| Methods of approach   | Using methods that are irrelevant or do not apply to the given problem.<br>Invoking theorems whose conditions are not satisfied. | Using relevant methods that help solve the problem.<br>Invoking theorems whose conditions are satisfied. | Finding methods and utilizing theorems that are both relevant and effective |
| Validity of reasoning | Reasoning is logically invalid.  | Reasoning is logically valid.  | Reasoning is logically valid and effective.                                 |
| Clarity of argument   | Reasoning is poorly explained or not explained at all.   | Reasoning is clear but may contain some gaps.  | Reasoning is clear, precise with no or insignificant gaps.                  |

**Rubric for Examination: Short Answer Questions (50%)**

| Criteria              | Standards  |  |   |
|-----------------------|--|--|---|
|                       | Fail standard  | Pass standard  | High standard   |
| Methods of approach   | Using methods that are irrelevant or do not apply to the given problem.<br>Invoking theorems whose conditions are not satisfied. | Using relevant methods that help solve the problem.<br>Invoking theorems whose conditions are satisfied. | Finding methods and utilizing theorems that are both relevant and effective |
| Validity of reasoning | Reasoning is logically invalid.  | Reasoning is logically valid.  | Reasoning is logically valid and effective.                                 |
| Clarity of argument   | Reasoning is poorly explained or not explained at all.   | Reasoning is clear but may contain some gaps.  | Reasoning is clear, precise with no or insignificant gaps.                  |