# COURSE OUTLINE: MH2220

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|---|---|---|---|
| Course Title | **Algebra I** | | |
| Course Code | **MH2220** | | |
| Offered | Study Year 2, Semester 2 | | |
| Course Coordinators | Ng Keng Meng (Assoc Prof) | kmng@ntu.edu.sg | 6513 8656 |
| | Lim Kay Jin (Dr) | limkj@ntu.edu.sg | 6513 7462 |
| Pre-requisites | (MH1200 or CY1602) and MH1300 | | |
| Mutually exclusive | MH2200 | | |
| AU | 3 | | |
| Contact hours | Lectures: 26, Tutorials: 12 | | |
| Approved for delivery from | AY 2022/23 semester 2 | | |
| Last revised | 13 Oct 2022, 09:46 | | |

## Course Aims

This MAS course aims to introduce group theory that is essential for more advanced algebra courses and applications. The axiomatic concepts serve as a language to study concrete examples in broader sense and helps in developing logical thinking.

## Intended Learning Outcomes

Upon successfully completing this course, you should be able to:

1. State the definitions and fundamental results in group theory.
2. Provide, prove, identify or recognize various examples and non-examples of groups, subgroups, normal subgroups and quotient groups.
3. Manipulate group elements using generators and relations.
4. Present and interpret Cayley tables.
5. Present permutations in different forms and compute their orders and signatures.
6. Read and write simple and logically proofs based on axioms or fundamental results.
7. Analyze groups based on their orders and other properties.
8. Provide, prove and manipulate various examples of group homomorphism or isomorphism.
9. Compute and manipulate cosets of a subgroup in a group.
10. Construct the quotient group G/N given a group G and its normal subgroup N.
11. Provide, prove, identify or recognize various examples and non-examples of group actions.

## Course Content

Axioms of group and examples

Cyclic groups, dihedral groups, symmetric groups, alternating groups, matrix groups, Group of automorphisms, Abelian groups

Group homomorphisms and isomorphisms

Group actions

Subgroups, including centralizers, normalizers, stabilizers, kernels of homomorphisms

Quotient groups

Lagrange's Theorem

Isomorphism Theorems

Direct product, semidirect products

Classification of finite abelian groups

## Assessment

| Component | Course ILOs tested | SPMS-MAS Graduate Attributes tested | Weighting | Team / Individual | Assessment Rubrics |
|---|---|---|---|---|---|
| **Continuous Assessment** | | | | | |
| **Mid-semester Quiz** | | | | | |
| Short Answer Questions 1 | 1, 2, 3, 4, 6, 7 | **1.** a, b, c<br>**2.** b | 25 | individual | See Appendix for rubric |
| Short Answer Questions 2 | 5, 8, 9, 10, 11 | **1.** a, b, c<br>**2.** b | 25 | individual | See Appendix for rubric |
| **Examination (2 hours)** | | | | | |
| Short Answer Questions | 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 | **1.** a, b, c<br>**2.** b<br>**3.** b | 50 | individual | See Appendix for rubric |
| | | **Total** | **100%** | | |

These are the relevant SPMS-MAS Graduate Attributes.

1. **Competence**
   a. Independently process and interpret mathematical theories and methodologies, and apply them to solve problems
   b. Formulate mathematical statements precisely using rigorous mathematical language
   c. Discover patterns by abstraction from examples
2. **Creativity**
   b. Build on the connection between subfields of mathematics to tackle new problems
3. **Communication**
   b. Work in teams on complicated projects that require applications of mathematics, and communicate the results verbally and in written form

## Formative Feedback

You will receive formative feedback for your CA. It is done by commenting on the mistakes and misunderstanding that appeared in your CA. This is possible because this course has a small number of students (typically less than 20). General feedback for your performance as part of the end of course review is done based on the final exam: the most common mistakes, as well as the questions that were best answered, are discussed in the report provided to all students.

## Learning and Teaching Approach

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|---|---|
| **Lectures** (26 hours) | Interactive Lecture:<br>The suggested learning and teaching approach consists of breaking the flow of the lectures by introducing exercises to be solved by the students in small groups during the lectures themselves. A typical pattern would be: the lecturer introduces a new concept, or a new proof, and then asks you to answer a question or solve a small exercise involving the new concept/proof. The lecturer then discusses the answer with you. If the newly introduced concept is understood well enough, the lecturer can then continue to build upon it, otherwise, further explanation is given. This also ensures that you have improved your knowledge in each of the classes that they are attending. This also encourages you to attend the classes, by being more active. |
| **Tutorials** (12 hours) | Problem solving:<br>Develop competence in logical thinking and problem solving especially in abstract algebra. |

## Reading and References

D.S. Dummit, R.M. Foote, Abstract Algebra, third edition, John Wiley & Sons, Inc., Hoboken, NJ, 2004.
ISBN-10 : 0471433349

## Course Policies and Student Responsibilities

You are expected to attend all classes punctually and take all scheduled assignments and tests by due dates. You are expected to take responsibility to follow up with course notes, assignments and course related announcements. You are expected to participate in all discussions and activities.

You are expected to attend the midterms. In case of medical leaves, You should provide a satisfying medical certificate on time.

## Academic Integrity

Good academic work depends on honesty and ethical behaviour. The quality of your work as a student relies on adhering to the principles of academic integrity and to the NTU Honour Code, a set of values shared by the whole university community. Truth, Trust and Justice are at the core of NTU's shared values.

As a student, it is important that you recognize your responsibilities in understanding and applying the principles of academic integrity in all the work you do at NTU. Not knowing what is involved in maintaining academic integrity does not excuse academic dishonesty. You need to actively equip yourself with strategies to avoid all forms of academic dishonesty, including plagiarism, academic fraud, collusion and cheating. If you are uncertain of the definitions of any of these terms, you should go to the Academic Integrity website for more information. Consult your instructor(s) if you need any clarification about the requirements of academic integrity in the course.

## Course Instructors

| Instructor | Office Location | Phone | Email |
|---|---|---|---|
| Ng Keng Meng (Assoc Prof) | MAS-05-09 | 6513 8656 | kmng@ntu.edu.sg |
| Lim Kay Jin (Dr) | SPMS-MAS-05-16 | 6513 7462 | limkj@ntu.edu.sg |

## Planned Weekly Schedule

| Week | Topic | Course ILO | Readings/ Activities |
|---|---|---|---|
| 1 | Axioms of group and examples | 1, 5 | Textbook: 1.1, 1.2, 1.3, 1.4 |
| 2 | Properties of groups | 1, 2, 3, 4, 7 | Textbook: 1.1, 1.2, 1.3, 1.4 |
| 3 | Group homomorphisms and isomorphisms | 1, 4, 8 | Textbook: 1.6 |
| 4 | Subgroups, including centralizers, normalizers, stabilizers, kernels of homomorphisms | 1, 2, 3, 9 | Textbook: 2.1, 2.2, 2.3 |
| 5 | Cosets | 1, 9 | Textbook: 3.1 |
| 6 | Lagrange's Theorem | 1, 6, 9 | Textbook: 3.2 |
| 7 | Normal subgroups and quotient groups | 1, 2, 9, 10 | Textbook: 3.3 |
| 8 | Isomorphism Theorems | 1, 6, 8, 9 | Textbook: 3.3 |
| 9 | Isomorphism Theorems | 1, 6, 8, 9 | Textbook: 3.3 |
| 10 | Direct product, semidirect product | 1, 2, 3, 6 | Textbook: 5.1, 5.5 |
| 11 | Classification of finite abelian groups | 3, 6 | Textbook: 5.2 |
| 12 | Group actions | 3, 6, 11 | Textbook: 1.7, 4.1 |
| 13 | The orbit-stabilizer theorem, Cauchy-Frobenius lemma | 3, 6, 11 | Textbook: 4.3 |

## Appendix 1: Assessment Rubrics

### Rubric for Mid-semester Quiz: Short Answer Questions 1 (25%)
Point-based marking

### Rubric for Mid-semester Quiz: Short Answer Questions 2 (25%)
Point-based marking

### Rubric for Examination: Short Answer Questions (50%)
Point-based marking