COURSE OUTLINE: MH1811

Course Title	**Mathematics 2**		
Course Code	**MH1811**		
Offered	Study Year 1, Sem 1 | Study Year , Sem 2		
Course Coordinator	Fedor Duzhin (Dr)	fduzhin@ntu.edu.sg	6513 7469
Pre-requisites	MH1810		
Co-requisites	MH1810		
Mutually exclusive	MS2900,FE1007,MH1100,MH1101,MH1801		
AU	3		
Contact hours	Lectures: 26, Tutorials: 12		
Approved for delivery from	AY 2021/22 semester 2		
Last revised	17 Jan 2022, 11:34		

Course Aims

This course extends the basic concepts of differentiation and integration learned in Mathematics 1 to the operations on functions of multiple variables. Advanced applications of differential and integral calculus are included. In addition, the course covers topics on sequences, series and ordinary differential equations.

This course aims to equip you with
• mathematical knowledge and analytical skills so that you are able to apply techniques of advanced calculus (along with their existing mathematical skills) to solve engineering or scientific problems whenever applicable;
• mathematical reading skills so that you can read and understand related mathematical content in the basic and popular scientific and engineering literature; and
• mathematical communication skills so that you can effectively and rigorously present your mathematical ideas to mathematicians, scientists and engineers.

Intended Learning Outcomes

Upon successfully completing this course, you should be able to:

1. Express the general term of a sequence, determine and evaluate the limit of a convergent sequence, apply squeeze theorem for sequences, explain why a sequence is convergent or divergent.
2. Manipulate basic series such as geometric series, telescoping series and determine its convergence and sum. Identify properties of convergent and divergent series.
3. Determine whether a series is divergent by Divergence Test. Apply alternating test to determine an alternating series is convergent. Apply integral test, comparison test and limit comparison test, ratio test, and root test appropriately to series to determine convergence of series with positive terms. Determine absolute, conditional or divergence of a general series,
4. Manipulate some power series related to geometric series; determine the radius and the interval of convergence for power series. Apply term-by-term differentiation and integration of power series. Apply power series to approximation.
5. Evaluate Taylor's series (Maclaurin series) of functions, including exponential and sine functions. Determine Taylor series of some functions using known Taylor series. Use the Taylor's Remainder Theorem and Estimation theorem to determine the upper bound of the estimation error of a series.
6. Determine the domain of simple 2-variable function and sketch the domain and level curve.
7. Evaluate the limits of 2-variable and 3-variable functions; explain why the limit does not exist.
8. Interpret the concept and meaning of partial derivative and evaluate partial derivatives. Apply

Chain rule for partial differentiation.

9. Evaluate the gradient vector and use its orthogonality to determine the equation of tangent plane to a level surface and the graph z=f(x,y). Determine the equation of normal line. Apply the equation of tangent plane to z=f(x,y) to linear approximation. Use total differential to approximate the errors or changes.
10. Evaluate directional derivative, and use it to determine rate of change.
11. Find stationary points and classify them as local maximum, minimum or saddle points.
12. Use Lagrange Multiplier method to determine global maximum or minimum of a function subject to an equality constraint. Apply the method to solve optimization problems.
13. Sketch the region of integration for 2-variable function and evaluate double integral, and apply it to some problems. Generalize the concept of double integral to triple integrals.
14. Solve separable first order ordinary differential equations (ODEs), homogeneous first order ODEs, first order linear ODEs by integrating factors, Bernoulli's equations and exact ODEs.
15. Solve linear homogeneous second order ODEs with constant real-number coefficients. Use undetermined coefficient method and method of variation of parameters to find a particular solution to non-homogeneous second order ODEs with constant real-number coefficients. Obtain the general solution for a non-homogeneous second order ODEs with constant real-number coefficients.
16. Apply ODEs to model and solve simple practical problems.

Course Content

Sequences -- convergence and Limit Evaluation

Series -- Geometric series, telescoping series, harmonic series and applications

Series (Convergence Test): Comparison Tests, Ratio and Root Test, Integral test, Alternating Series Test, Absolute and conditional convergence.

Power Series & Applications: Radiu and Interval of Convergence, Differentiation and Integration of Power Series

Taylor Series: Evaluation and Applications

Functions of 2 or 3 variables, Domain & Level Sets, Limits and Continuity

Partial Derivatives, Gradient Vectors, Applications

Directional Derivatives & Applications

Classify stationary points. Lagrange Multiplier Method

Double Integration

First order Ordinary Differential Equations -- Separable, Linear, Bernouli, Homogeneous, Exact

Second Order Homogeneous Ordinary Differential Equations: with constant coefficients.

Second Order Non-Homogeneous Ordinary Differential Equations: Undetermined Coefficients, Variation of Parameters

Assessment

Component	Course ILOs tested	EAB Graduate Attributes tested	Weighting	Team / Individual	Assessment Rubrics
Continuous Assessment					
Tutorials					
Take-home online assignment	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16	a, b	25	individual	See Appendix for rubric
Mid-semester Quiz					
Midterm Test	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16	a, b	25	individual	See Appendix for rubric
Examination (2 hours)					
Written Examination	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16	a, b	50	individual	See Appendix for rubric
		Total	**100%**		

Mapping of Course Intended Learning Outcomes to Engineering Accreditation

Board (EAB) Graduate Attributes

Category	Core					
EAB's 12 Graduate Attributes*						
(a) ●	(b) ●	(c)	(d)	(e)	(f)	
(g)	(h)	(i)	(j)	(k)	(l)	

Overall statement

This course extends the basic concepts of differentiation and integration learned in Mathematics 1 to the operations on functions of multiple variables. Advanced applications of differential and integral calculus are included. In addition, the course covers topics on sequences, series and ordinary differential equations. This course aims to equip you with • mathematical knowledge and analytical skills so that you are able to apply techniques of advanced calculus (along with their existing mathematical skills) to solve engineering or scientific problems whenever applicable; • mathematical reading skills so that you can read and understand related mathematical content in the basic and popular scientific and engineering literature; and • mathematical communication skills so that you can effectively and rigorously present your mathematical ideas to mathematicians, scientists and engineers.

Course Student Learning Outcomes		**EAB Graduate Attributes**
1	Express the general term of a sequence, determine and evaluate the limit of a convergent sequence, apply squeeze theorem for sequences, explain why a sequence is convergent or divergent.	a, b
2	Manipulate basic series such as geometric series, telescoping series and determine its convergence and sum. Identify properties of convergent and divergent series.	a, b
3	Determine whether a series is divergent by Divergence Test. Apply alternating test to determine an alternating series is convergent. Apply integral test, comparison test and limit comparison test, ratio test, and root test appropriately to series to determine convergence of series with positive terms. Determine absolute, conditional or divergence of a general series,	a, b
4	Manipulate some power series related to geometric series; determine the radius and the interval of convergence for power series. Apply term-by-term differentiation and integration of power series. Apply power series to approximation.	a, b
5	Evaluate Taylor's series (Maclaurin series) of functions, including exponential and sine functions. Determine Taylor series of some functions using known Taylor series. Use the Taylor's Remainder Theorem and Estimation theorem to determine the upper bound of the estimation error of a series.	a, b
6	Determine the domain of simple 2-variable function and sketch the domain and level curve.	a, b
7	Evaluate the limits of 2-variable and 3-variable functions; explain why the limit does not exist.	a, b
8	Interpret the concept and meaning of partial derivative and evaluate partial derivatives. Apply Chain rule for partial differentiation.	a, b
9	Evaluate the gradient vector and use its orthogonality to determine the equation of tangent plane to a level surface and the graph z=f(x,y). Determine the equation of normal line. Apply the equation of tangent plane to z=f(x,y) to linear approximation. Use total differential to approximate the errors or changes.	a, b
10	Evaluate directional derivative, and use it to determine rate of change.	a, b
11	Find stationary points and classify them as local maximum, minimum or saddle points.	a, b
12	Use Lagrange Multiplier method to determine global maximum or minimum of a function subject to an equality constraint. Apply the method to solve optimization problems.	a, b
13	Sketch the region of integration for 2-variable function and evaluate double integral, and apply it to some problems. Generalize the concept of double integral to triple integrals.	a, b

14	Solve separable first order ordinary differential equations (ODEs), homogeneous first order ODEs, first order linear ODEs by integrating factors, Bernoulli's equations and exact ODEs.	a, b
15	Solve linear homogeneous second order ODEs with constant real-number coefficients. Use undetermined coefficient method and method of variation of parameters to find a particular solution to non-homogeneous second order ODEs with constant real-number coefficients. Obtain the general solution for a non-homogeneous second order ODEs with constant real-number coefficients.	a, b
16	Apply ODEs to model and solve simple practical problems.	a, b

***Legend:**
● Fully consistent (contributes to more than 75% of Student Learning Outcomes)
◐ Partially consistent(contributes to about 50% of Student Learning Outcomes)
○ Weakly consistent (contributes to about 25% of Student Learning Outcomes)
[Blank]Not related to Student Learning Outcomes

The graduate attributes as stipulated by the EAB, are:

a. **Engineering knowledge:** Apply the knowledge of mathematics, natural science, engineering fundamentals, and an engineering specialisation to the solution of complex engineering problems.
b. **Problem Analysis:** Identify, formulate, research literature, and analyse complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
c. **Design/development of Solutions:** Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for public health and safety , cultural, societal, and environmental considerations.
d. **Investigation:** Conduct investigations of complex problems using research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
e. **Modern Tool Usage:** Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modelling to complex engineering activities with an understanding of the limitations.
f. **The engineer and Society:** Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal, and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
g. **Environment and Sustainability:** Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for the sustainable development.
h. **Ethics:** Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
i. **Individual and Team Work:** Function effectively as an individual, and as a member or leader in diverse teams and in multidisciplinary settings.
j. **Communication:** Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
k. **Project Management and Finance:** Demonstrate knowledge and understanding of the engineering and management principles and economic decision-making, and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary envir onments.
l. **Life-long Learning:** Recognise the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

Formative Feedback

Instructors and tutors will discuss tutorial solution and common mistakes in student work during tutorial sessions or lectures or via online materials.

Learning and Teaching Approach

Lectures (26 hours)	Derivation of formulas and demonstrating problem solving: Train you to be independent learners who are able to derive ideas/concepts from first principles and take ownership of your own learning. Help you understand the motivation behind mathematical theorems, definitions and formulas. Develop the train of thought in problem solving and presentation skills in presenting mathematical solutions. Problem solving: Develop competence in solving related problems. Peer Instruction: Develop communication skills and competence in mathematics. You also have an opportunity to work with your peers during lectures and tutorial sessions.
Tutorials (12 hours)	Derivation of formulas and demonstrating problem solving: Train you to be independent learners who are able to derive ideas/concepts from first principles and take ownership of your own learning. Help you understand the motivation behind mathematical theorems, definitions and formulas. Develop the train of thought in problem solving and presentation skills in presenting mathematical solutions. Problem solving: Develop competence in solving related problems. Peer Instruction: Develop communication skills and competence in mathematics. You also have an opportunity to work with your peers during lectures and tutorial sessions.

Reading and References

TEXT
Thomas' Calculus, 13th edt, Thomas, GB Jr., Weir MD and Hass J, Pearson-Addison-Wesley, 978-1292089799, 2016

REFERENCE
Calculus (International Student Edition), 6th edt, James Stewart, Thomson, 9780495482826 Brooks/Cole

Course Policies and Student Responsibilities

Absence Due to Medical or Other Reasons
If you are unable to attend a common test with a valid reason, you have to:
1. Send an email to the instructor regarding the absence.
2. Submit the original Medical Certificate* or documents (if it is not due to medical reason) to administrator (in School Office).

* The medical certificate mentioned above should be issued in Singapore by a medical practitioner registered with the Singapore Medical Association.

Academic Integrity

Good academic work depends on honesty and ethical behaviour. The quality of your work as a student relies on adhering to the principles of academic integrity and to the NTU Honour Code, a set of values shared by the whole university community. Truth, Trust and Justice are at the core of NTU's shared values.

As a student, it is important that you recognize your responsibilities in understanding and applying the principles of academic integrity in all the work you do at NTU. Not knowing what is involved in maintaining academic integrity does not excuse academic dishonesty. You need to actively equip yourself with strategies to avoid all forms of academic dishonesty, including plagiarism, academic fraud, collusion and cheating. If you are uncertain of the definitions of any of these terms, you should go to the Academic Integrity website for more information. Consult your instructor(s) if you need any clarification about the requirements of academic integrity in the course.

Course Instructors

Instructor	Office Location	Phone	Email
Fedor Duzhin (Dr)	SPMS-MAS-05-23	6513 7469	fduzhin@ntu.edu.sg

Planned Weekly Schedule

Week	Topic	Course ILO	Readings/ Activities
1	Sequences (Order of topics may change according to instructors.)	1	Lecture
2	Sequences & Series	1, 2	Tutorial 1
3	Series (Convergence Test)	2, 3	Tutorial 2
4	Power Series	4	Tutorial 3
5	Taylor Series	5	Tutorial 4
6	Functions of 2 or 3 variables, Limits	6, 7	Tutorial 5
7	Partial Derivatives, Gradient Vectors	8, 9	Tutorial 6
8	Applications Directional Derivatives & Applications	9, 10	Tutorial 7
9	Classify stationary points. Lagrange Multiplier Method	11, 12	Tutorial 8
10	Double Integration	13	Tutorial 9
11	Double Integration/ODE	13, 14, 15	Tutorial 10
12	ODE	14, 15, 16	Tutorial 11
13	ODE	14, 15, 16	Tutorial 12

Appendix 1: Assessment Rubrics

Rubric for Tutorials: Take-home online assignment (25%)
Multiple-choice questions, no rubric is needed.

Rubric for Mid-semester Quiz: Midterm Test (25%)
Point-based marking (not rubrics based)

Rubric for Examination: Written Examination (50%)
Point-based marking (not rubrics based)