COURSE OUTLINE: MH1201

Course Title	**Linear Algebra II**		
Course Code	**MH1201**		
Offered	Study Year 1, Semester 2		
Course Coordinator	Kiah Han Mao (Asst Prof)	hmkiah@ntu.edu.sg	6513 7185
Pre-requisites	MH1200		
Mutually exclusive	MH2800, MH2802, CY1602		
AU	4		
Contact hours	Lectures: 39, Tutorials: 12		
Approved for delivery from	AY 2021/22 semester 2		
Last revised	19 Nov 2021, 09:14		

Course Aims

This first year course is the second of two courses on linear algebra. It is a core module for MATH students. The course aims to develop your understanding of fundamental topics of linear algebra with particular emphasis on abstract vector spaces and linear maps.

Intended Learning Outcomes

Upon successfully completing this course, you should be able to:

1. Identify when a set of objects form a vector space or vector subspace.
2. Determine the span and linear independence of a set of vectors.
3. Determine a basis and the dimension of a vector space.
4. Describe and determine the null space and range of a linear map.
5. Apply the Rank-Nullity theorem to determine properties of a certain linear map.
6. Determine the eigenvector, eigenvalue and eigenspace of a matrix.
7. Determine whether a given matrix is diagonalizable.
8. Identify when a set of objects form an inner product space
9. Apply Gram-Schmidt process to obtain an orthonormal basis.

Course Content

Abstract Vector Spaces: Real and Complex Vector Space: Definition and Examples; Subspaces.

Basis and Dimension: Linear Combination and Span; Linear Independence; Basis and Coordinate Vectors; Finite Dimensional Space and its Dimension, Basis Construction.

Linear Transformations: Linear Transformation and Examples; Null Space, Range and Rank-Nullity Theorem; Injection, Surjection, Bijection and Isomorphisms.

Matrix Representation of Linear Transformations between Finite Dimensional Vectors Spaces: Examples of Linear Transformations and their Matrix Representations, Change of Basis and Applications.

Eigenvectors and Eigenvalues: Eigenvalues and Eigenvectors of Matrices and Linear Transformations, Characteristic Polynomials; Eigenspace, Diagonalization & Applications.

Inner Product Spaces: Inner product and Norm, Inner product space, Some inequalities: Cauchy Schwarz Inequality; Orthogonality, Orthonormal sets, Gram-Schmidt process;

Orthogonality, Orthogonal projection, Orthogonal complement. Applications.

Assessment

Component	Course ILOs tested	SPMS-MAS Graduate Attributes tested	Weighting	Team / Individual	Assessment Rubrics
Continuous Assessment					
Lectures					
Midterm Test	1, 2, 3, 4, 5	**1.** a, b, c **2.** b	30	individual	See Appendix for rubric
Participation	1, 2, 3, 4, 5, 6, 7, 8, 9	**1.** a, b, c **2.** a, b **3.** a	20	individual	See Appendix for rubric
Examination (2 hours)					
Final Examination	1, 2, 3, 4, 5, 6, 7, 8, 9	**1.** a, b, c **2.** a, b	50	individual	See Appendix for rubric
		Total	**100%**		

These are the relevant SPMS-MAS Graduate Attributes.

1. Competence

a. Independently process and interpret mathematical theories and methodologies, and apply them to solve problems

b. Formulate mathematical statements precisely using rigorous mathematical language

c. Discover patterns by abstraction from examples

2. Creativity

a. Critically assess the applicability of mathematical tools in the workplace

b. Build on the connection between subfields of mathematics to tackle new problems

3. Communication

a. Present mathematics ideas logically and coherently at the appropriate level for the intended audience

Formative Feedback

Formative feedback will be provided in the following manner.

1. During each problem-solving session (in-class activity), we will work (with the lecturer) on a specific problem. After each step, you will communicate your solution and the lecturer will provide immediate feedback.

2. Feedback is given after the midterm on the common mistakes.

Learning and Teaching Approach

Lectures (39 hours)	Derivation and demonstration: -Explains the motivation behind certain concepts. -Presents systematic ways to solve problems related to the concepts developed. -Derives methods to finding bases, eigenvalues and orthonormal bases. Problem solving session (in-class activities): -Develops competence in solving a variety of problems related to linear algebra.
Tutorials (12 hours)	Derivation and demonstration: -Explains the motivation behind certain concepts. -Presents systematic ways to solve problems related to the concepts developed. -Derives methods to finding bases, eigenvalues and orthonormal bases. Problem solving: -Develops competence in solving a variety of problems related to linear algebra. Peer Instruction: -Develops communication and presentation skills and deepen understanding. You will have the opportunity to work with peers and present your solution to the class.

Reading and References

1. Sheldon Axler, Linear Algebra Done Right, 3rd Edition, Springer, 2015.
ISBN: 978-3-319-11079-0
Book site: https://linear.axler.net/
Ebook can be downloaded using NTU's library account
https://link.springer.com/book/10.1007/978-3-319-11080-6

2. Gilbert Strang, Introduction to Linear Algebra, 5th Edition, Wellesley-Cambridge Press, 2016
ISBN: 978-0980232776

Course Policies and Student Responsibilities

(1) General

You are expected to diligently attend all lectures, whether in-person or online, and attempt in-class activities. While you are not expected to complete all tutorial questions prior to class, please be aware of the topic that will be covered and participate wholeheartedly in discussions with your group mates. A general observation: students who struggle together do well together.

(2) Absenteeism

Missing in-class activity will mean an automatic zero for that class activity. Nevertheless, you will be able to makeup the score through the later in-class activities. Past data demonstrates a strong correlation between in-class participation and the final course grade.

Academic Integrity

Good academic work depends on honesty and ethical behaviour. The quality of your work as a student relies on adhering to the principles of academic integrity and to the NTU Honour Code, a set of values shared by the whole university community. Truth, Trust and Justice are at the core of NTU’s shared values.

As a student, it is important that you recognize your responsibilities in understanding and applying the principles of academic integrity in all the work you do at NTU. Not knowing what is involved in maintaining academic integrity does not excuse academic dishonesty. You need to actively equip yourself with strategies to avoid all forms of academic dishonesty, including plagiarism, academic fraud, collusion and cheating. If you are uncertain of the definitions of any of these terms, you should go to the Academic Integrity website for more information. Consult your instructor(s) if you need any clarification about the requirements of academic integrity in the course.

Course Instructors

Instructor	Office Location	Phone	Email
Kiah Han Mao (Asst Prof)	SPMS-MAS-05-39	6513 7185	hmkiah@ntu.edu.sg

Planned Weekly Schedule

Week	Topic	Course ILO	Readings/ Activities
1	Vector Spaces - Definition of Vector Space	1	Chapter 1A, 1B
2	Vector Spaces - Subspaces	1	Chapter 1C
3	Finite-Dimensional Vector Spaces - Span and Linear Independence	2, 3	Chapter 2A
4	Finite-Dimensional Vector Spaces -Bases -Dimension	2, 3	Chapter 2B, 2C
5	Linear Maps - Vector Space of Linear Maps	4, 5	Chapter 3A
6	Linear Maps - Null Spaces and Ranges -- Null Space and Injectivity -- Range and Surjectivity -- Rank-Nullity Theorem	4, 5	Chapter 3B
7	Linear Maps - Matrices -- Repesenting a Linear Map by a matrix -- Matrix multiplication	4, 5	Chapter 3C
8	Linear Maps -- Invertible Linear Maps -- Isomorphic Vector Spaces -- Linear Maps is Matrix Multiplication	4, 5	Chapter 3D
9	Eigenvalues and Eigenvectors - Finding Eigenvectors and Eigenvalues - Characteristic polynomials - Diagonalization	6, 7	Chapter 5B, 5C
10	Eigenvalues and Eigenvectors - Characteristic and Minimal Polynomials - Jordan Form	6, 7	Chapter 8B, 8C, 8D
11	Inner Product Spaces - Inner Products and Norms	8, 9	Chapter 6A
12	Chapter 11 (Continued) - Orthonormal basis. - Gram-Schmidt Process	8, 9	Chapter 6B
13	Revision	1, 2, 3, 4, 5, 6, 7, 8, 9	

Appendix 1: Assessment Rubrics

Rubric for Lectures: Midterm Test (30%)

Point-based marking.

Rubric for Lectures: Participation (20%)

Point-based marking.

Class participation scores will be determined based on quizzes conducted during lectures. The quizzes will in turn be scored using point-based marking.

Rubric for Examination: Final Examination (50%)

Point-based Marking.