Course Requisites (if applicable)

Pre-requisites	For graduate students: No pre-requisites For undergraduates: PH3502/MH3320 Dynamical System Theory with Chaos and Fractals or equivalent
Co-requisites	
Pre-requisite to	
Mutually exclusive to	
Replacement course to	
Remarks (if any)	

Course Aims

This course aims to equip you with the basic concepts of determinism and randomness in the physical world. You will develop a basic understanding of dynamical system theory which is an essential component in physics, engineering, chemistry, biology, and the social sciences. You will also gain basic computational and analytical skills to solve and understand real-world problems involving chaotic and non-linear systems.

Course's Intended Learning Outcomes (ILOs)

Upon the successful completion of this course, you (student) would be able to:

ILO 1	Analyse and solve problems in linear system and linear dynamical system in N-dimensions mathematically and computationally.
ILO 2	Analyse and solve problems in nonlinear system and nonlinear dynamical system in N-dimensions mathematically and computationally.
ILO 3	Formulate first-order differential equations to model the evolution of diverse continuous-time dynamical phenomena and solve the equations mathematically and computationally.
ILO 4	Formulate recurrence equations to model the evolution of diverse discrete-time dynamical phenomena and solve the equations mathematically and computationally.
ILO 5	Perform geometric analysis on the phase portrait of linear and nonlinear dynamical systems.
ILO 6	Perform mathematical definition and analysis of self-similar and fractal sets.
ILO 7	Determine the fixed points, limit cycles/periodic orbits, strange and non-strange attractors of the dynamical system under-study analytically and numerically.
ILO 8	Determine the stability properties of the fixed points and limit cycles/periodic orbits of the dynamical system under-study analytically and numerically.
ILO 9	Explain the concept of stable manifold and unstable manifold as geometric structures in phase space that guide the flow of the dynamical trajectories.
ILO 10	Demonstrate mastery in the use of the phase portrait (as a geometric picture of phase space) that contains the set of fixed points, limit cycles, strange attractors, stable and unstable manifolds, as the solution to dynamical system problems.
ILO 11	Analyse dynamical systems that are chaotic, compute its trajectories, and yield the level of chaos by evaluating its Lyapunov exponents numerically or analytically.
ILO 12	Account for the different type of bifurcations that occur in nonlinear dynamical systems.
ILO 13	Explain the concepts of countable and uncountable sets
ILO 14	Identify sets that have fractional dimension.
ILO 15	Construct and analyse fractal sets that are self-similar and non self-similar.
ILO 16	Determine diverse fractal dimensions of fractal sets that are self-similar and non self-similar.

Course Content

Introduction

- A dynamical view of the world
- What is nonlinear dynamics?

Linear Dynamical System

- Examples
- General formulation

Ingredients of a Dynamical System

- Phase space
- Evolution equations
- Initial conditions

Stability Properties of Linear Dynamical System

- Two-dimensional linear dynamical system
- N-dimensional linear dynamical system

Phase Portraits from the Stability Properties of Manifolds of Fixed Points in Continuous-Time Nonlinear Dynamical System

- Phase portraits
- Existence and uniqueness, no-intersection theorem
- Stability properties of fixed points in nonlinear dynamical systems
- Stable and unstable manifolds

Bifurcations

- Saddle-node bifurcation
- Transcritical bifurcation
- Pitchfork bifurcation
- Hopf bifurcation

- Global bifurcation of cycles

Lorenz Equations

- Linear stability of Poincaŕe orbits via Poincaŕe map
- Homoclinic and Heteroclinic orbits
- Bifurcations, chaos, and strange attractors

Nonlinear Mapping and their Dynamical Properties

- Fixed points and cobwebs
- Periodic points
- Logistic map
- The fully chaotic logistic map at A = 4
- Symbolic dynamics and the Bernoulli shift map

Fractals

- Countable and uncountable sets
- Cantor set
- Dimension of self-similar fractals
- Box dimension
- Hausdorff dimension

Reading and References (if applicable)

Nonlinear dynamics and Chaos, 2nd Edition, Steven Strogatz, (2014). ISBN-13: 978-0738204536; ISBN-10: 0738204536

Planned Schedule

Week or Session	Topics or Themes	ILO	Readings	Delivery Mode	Activities
1	Introduction	1			Lecture
2	Linear Dynamical System and Ingredients of Dynamical System	1			Lecture, Tutorial
3	Stabilities of Linear Dynamical System	1, 3, 5, 7, 8			Lecture, Tutorial
4	Phase Portraits	2, 3, 5, 7, 8			Lecture, Tutorial
5	Bifurcations	12			Lecture, Tutorial
6	Poincaŕe map, Homoclinic and Heteroclinic orbits	9			Lecture, Tutorial
7	Lorenz equations	2, 3, 5, 9, 10, 11			Lecture, Tutorial
8	Lorenz equations	2, 3, 5, 9, 10, 11			Lecture, Tutorial
9	Nonlinear Mappings	2, 4, 5, 7, 8, 10, 11			Lecture, Tutorial

Week or Session	Topics or Themes	ILO	Readings	Delivery Mode	Activities
10	Nonlinear Mappings	2, 4, 5, 7, 8, 10, 11			Lecture, Tutorial
11	Fractals	6, 13, 14			Lecture, Tutorial
12	Fractals, Revision	6, 15, 16			Lecture, Tutorial
13	Project Presentation	All			Project

Learning and Teaching Approach

Approach	How does this approach support you in achieving the learning outcomes?
Problem solving (tutorial and lecture)	This is to develop your competence and perseverance in solving physics problems
Problem sets (home work)	This enables you to apply the theory and mathematical formulation learnt in class to solve problems in nonlinear dynamics in order to develop the understanding, competence, and intuition on the topic, as well as to develop both analytical and computational skills.
Projects (home work, lecture and tutorial)	This will sharpen your knowledge in nonlinear dynamics through creatively working on a project in a team of two/three persons. It will also enhance your analytical and computational skills as you work to deliver the requirement of the project. Furthermore, your presentation and communication skills will be developed through project presentation and answering critical questions from your peers and seniors during the Question and Answer session.
Technology-Enhanced Learning (TEL)	This will test your understanding and address any misconception you have by providing instant feedback with correct solution. It will be conducted via online quizzes using Wooclap.

Assessment Structure

Assessment Components (includes both continuous and summative assessment)

No.	Component	ILO	Related PLO or Accreditation	Weightage	Team/Individual	Rubrics	Level of Understanding
1	Continuous Assessment (CA): Assignment(CA1: Problem Sets)	All		20	Individual	Analytic	Multistructural
2	Continuous Assessment (CA): Project(CA2: Project)	All		40	Individual	Holistic	Relational
3	Continuous Assessment (CA): Project(CA3: Project)	All		40	Individual	Holistic	Extended Abstract

Description of Assessment Components (if applicable)

CA1 consists of problem sets for students to practice the application of theory and mathematical formulation learnt in class by solving problems in dynamical systems. The purpose is to develop understanding, competence, and intuition on the topic, as well as attaining both analytical and computational skills. CA2 is known as a Theme project where each student works on a project of the same theme as defined by the course instructor. CA3 is known as a Term project where each student defines his/her own project related to nonlinear science and the project is encouraged to be relevant to his/her PhD research topic.

Formative Feedback

You will receive formative feedback which will be given through discussion within tutorial lessons.

Feedback will also be provided for each marked problem set, where any particularly problematic areas will be identified in the marked scripts.

Finally, feedback will be given constantly during lectures and tutorials on the common mistakes and level of difficulty of the course materials and applied examples/problems. Past exam questions and examiner's report are also made available for you. They will be discussed near the end of the course.

NTU Graduate Attributes/Competency Mapping

This course intends to develop the following graduate attributes and competencies (maximum 5 most relevant)

Attributes/Competency	Level
Communication	Intermediate
Creative Thinking	Intermediate
Curiosity	Intermediate
Problem Solving	Advanced

Course Policy

Policy (Academic Integrity)

Good academic work depends on honesty and ethical behaviour. The quality of your work as a student relies on adhering to the principles of academic integrity and to the NTU Honour Code, a set of values shared by the whole university community. Truth, Trust and Justice are at the core of NTU's shared values. As a student, it is important that you recognize your responsibilities in understanding and applying the principles of academic integrity in all the work you do at NTU. Not knowing what is involved in maintaining academic integrity does not excuse academic dishonesty. You need to actively equip yourself with strategies to avoid all forms of academic dishonesty, including plagiarism, academic fraud, collusion and cheating. If you are uncertain of the definitions of any of these terms, you should go to the academic integrity website for more information. On the use of technological tools (such as Generative AI tools), different courses / assignments have different intended learning outcomes. Students should refer to the specific assignment instructions on their use and requirements and/or consult your instructors on how you can use these tools to help your learning. Consult your instructor(s) if you need any clarification about the requirements of academic integrity in the course.

Policy (General)

You are expected to complete all assigned pre-class readings and activities, attend all seminar classes punctually and take all scheduled assignments and tests by due dates. You are expected to take responsibility to follow up with course notes, assignments and course related announcements for seminar sessions they have missed. You are expected to participate in all seminar discussions and activities.

Policy (Absenteeism)

Absence Due to Medical or Other Reasons

If you are sick and unable to attend your class, you must:

1. Send an email to the instructor regarding the absence and request for a replacement class
2. Submit the original Medical Certificate* or official letter of excuse to administrator.
3. Attend the assigned replacement class (subject to availability)

*The medical certificate mentioned above should be issued in Singapore by a medical practitioner registered with the Singapore Medical Association.

Academic Integrity

Policy (Others, if applicable)

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