MH1810 Mathematics I

[Lecture: 26 hrs; Tutorial: 12 hrs; Pre-requisite: Nil; Academic Unit: 3.0]

Learning Objective

In this course, the basic concepts of limits, differentiation and integration are introduced. Applications of differential and integral calculus are included. In addition, the course also covers topics on complex numbers, vectors and matrices to prepare the students for other courses in Year 1.

Content

1. Complex numbers
2. vectors and matrices
3. Limits and continuity of functions
4. Derivatives
5. Applications of derivatives
6. Integration
7. Integration methods
8. Applications of integration

Course Outline

1. **COMPLEX NUMBERS, VECTORS AND MATRICES**
   Complex plane, polar coordinates and polar form of complex numbers, powers and roots, fundamental theorem of algebra. Vector algebra in 2-D and 3-D space, dot product and cross product, lines and planes in space, applications. Matrices, matrix addition and multiplication, determinants, Cramer's rule.

2. **LIMITS AND CONTINUITY OF FUNCTIONS**
   Limits of functions, types of limits, the Sandwich Theorem, evaluation of limits, continuity of functions, properties of continuous functions.

3. **DERIVATIVES**
   Derivatives, differentiability, rules and properties, differentiation of transcendental functions, higher order derivatives, implicit differentiation, formulae, linearization, estimate changes, Newton's Method, hyperbolic functions.

4. **APPLICATIONS OF DERIVATIVES**
   Extreme values, points of inflection and curve sketching, Rolle's Theorem, Mean Value Theorem, optimization, indeterminate forms, L'Hopital's Rule.

5. **INTEGRATION**
   Anti-derivatives, Riemann Sum, indefinite and definite integration, Mean Value Theorem for definite integral, Fundamental Theorem of Calculus. Trapezoidal Rule, Simpson's Rule.

6. **INTEGRATION METHODS**
   Basic integration formulae for algebraic and transcendental functions. Integration by special devices: integration by parts, rationalizing substitution or trigonometric substitution, partial fractions, reduction formulas, improper integrals, convergence tests.

7. **APPLICATIONS OF INTEGRATION**
   Area between curves, volume of a general solid by slicing and cylindrical shell methods, volume of a solid of revolution, length of a plane curve, area of a surface of revolution.

Learning Outcome

After completing this course, the student will be ready to take the next level of mathematics subjects where the basic concepts will be further developed and applied. At the end of this course, the student will be able to:

- Evaluate product, quotient, power and roots of complex numbers.
- Use vector operators (dot product and cross product) to solve simple mechanics and geometry problems (e.g. find work done, moment, equations for planes, distance form a point to a plane etc.).
- Evaluate matrix determinants and use Cramer's rule to solve simultaneous equations.
- Evaluate limits by the substitution method, sandwich theorem or L'Hopital's rule.
- Use limit to determine if a function is continuous.
- Evaluate the derivatives of simple functions from the definition.
- Evaluate the derivatives of more complicated functions by using rules of differentiation (e.g. product rule and chain rule).
- Use the derivatives to establish linear approximations, estimate changes and solve nonlinear equations (by Newton's method).
- Use the derivatives to assist in curve sketching and solving optimization problems.
- Evaluate integrals by using integration formulae, rules of algebra and substitution techniques.
- Evaluate more complicated integrals by using "integration by parts", "trigonometric substitutions", "partial fractions", "reduction formulae" and special techniques for integrands involving quadratic or trigonometric expressions.
- Evaluate definite integrals by using trapezium & Simpson's rules and perform error analysis.
- Evaluate improper integrals of the first, second & third kinds, and test for convergence.
- Interpret the meaning of integration by using the concept of Riemann sum.
- Use integration to find areas between curves (by dividing into vertical or horizontal strips) and volumes of solids (slicing & cylindrical shells methods).
- Use integration to find volumes of solids of revolution (slicing & cylindrical shells methods).
- Use integration to find lengths of plane curves and areas of surfaces of revolution.
- Use mean value theorem, fundamental theorem of calculus and Leibniz's rule to evaluate expressions involving differentiation and integration.

Textbooks/References

Thomas, GB Jr., Weir MD, Hass J and Giordano FR, **Thomas' Calculus**, 11th edition, Pearson-Addison-Wesley, 2005

References

1. Smith RT and Minton RB, **Calculus: Concepts and Connections**, McGraw Hill, 2006.
2. Kreysgiz E, **Advanced Engineering Mathematics**, 9th edition, John Wiley & Sons, 2006.
3. Anton H, **Calculus with Analytic Geometry**, 5th edition, John Wiley & Sons, 1995.
4. Ayres F Jr and Mendelson E, **Schaum's Outline of Theory and Problems of Calculus**, 4th edition, McGraw Hill, 2000.