“The Force and Gravity of Events”

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Robert Delbourgo, University of Tasmania
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The framework for a local event is described by

- WHERE and WHEN - location (x, y, z) and time (t);

- WHAT - change of property (quantum number, momentum, etc.)

How to incorporate property mathematically? Try using anticommuting complex numbers ζ, of which there are only a FINITE number.

But WHY anticommuting?

- When an entity possesses a property or attribute ζ it cannot doubly have it.

- An entity can be a melange of several properties (necessarily *finite*);

- Its complex conjugate $\bar{\zeta}$ connotes opposite attributes;

- Generations can be built up with *finite* numbers of $\bar{\zeta}\zeta$.

This is similar to N-extended SUSY, except that there is no spin proliferation.

1 How many properties?

Minimal number of ζ needed seems to be 5 in order to encompass the three known lepton and quark generations. This ties in nicely with popular grand unified groups SU(5) and SO(10). *Four ζ are insufficient* so Sp(8), SO(8) and SU(4) are ruled out.

But quite a few more states are engendered, including ones which are completely neutral and therefore sterile.

In the supercoordinate $X^M = (x^m, \zeta^\mu, \zeta^{\bar{\mu}})$ framework, superBose fields $\Phi(X)$ are even functions of ζ, SuperFermi fields $\Psi(X)$ are odd functions thereof. The action is obtained by integrating the field theory over spacetime and property.

Reserve the labels and quantum numbers (tied to gauge fields)

- $m = 0, 1, 2, 3$ for spacetime
- $\mu = 1, 2, 3$ for 'chromicity' property or colour (of D-type quarks)
- $\mu = 0, 4$ for 'neutrinicity' and 'electricity' respectively
- Charge $Q(\zeta^0, \zeta^{\bar{1}}, \zeta^{\bar{2}}, \zeta^{\bar{3}}, \zeta^4) = (0, 1/3, 1/3, 1/3, -1)$
- Fermion Number $F(\zeta^0, \zeta^{\bar{1}}, \zeta^{\bar{2}}, \zeta^{\bar{3}}, \zeta^4) = (1, -1/3, -1/3, -1/3, 1)$

2 Where are Lepton and Quark Generations?

Leptons are easily identified as $(\zeta^{0}, \zeta^{4})\times$ powers of $Z \equiv \zeta^{\bar{\mu}}\zeta^{\mu}$. Although there are many 5's and 10's of SU(5) in expansions of $\Phi(X), \Psi(X)$, the quarks generations arise more subtly in the combos:

$$\begin{pmatrix} U^{[\bar{\mu}\bar{\nu}]} \sim \zeta^{\bar{\mu}}\zeta^{\bar{\nu}}\zeta^{0} \\ D^{[\bar{\mu}\bar{\nu}]} \sim \zeta^{\bar{\mu}}\zeta^{\bar{\nu}}\zeta^{4} \end{pmatrix}, \begin{pmatrix} U'^{[\bar{\mu}\bar{\nu}]} \sim \zeta^{\bar{\mu}}\zeta^{\bar{\nu}}\zeta^{0}\zeta^{\bar{4}}\zeta^{4} \\ D'^{[\bar{\mu}\bar{\nu}]} \sim \zeta^{\bar{\mu}}\zeta^{\bar{\nu}}\zeta^{4}\zeta^{\bar{0}}\zeta^{0} \end{pmatrix}, \begin{pmatrix} U''^{\lambda} \sim \zeta^{\lambda}\zeta^{\bar{4}}\zeta^{0} \\ D''^{\lambda} \sim \zeta^{\lambda}(\zeta^{\bar{0}}\zeta^{0}, \zeta^{\bar{4}}\zeta^{4}) \\ X''^{\lambda} \sim \zeta^{\lambda}\zeta^{\bar{0}}\zeta^{4} \end{pmatrix}$$

isodoublet/singlet *isodoublet/singlet* *isotriplet/doublet/singlet*.

Thus prediction of (a) a brand new quark X'' of charge -4/3 in the third generation (found via $e\bar{e}$ annihilation?), (b) nonunitarity of CKM, (c) smaller V_{tb} coupling to W because isotriplet couples more strongly than isodoublet, (d) more D-type quarks, unaccompanied by U-type, (e) more leptons.

3 Where are the force fields?

These arise in the extended metric. (Separation in location & property)^2 is

$$ds^2 = dX^M dX^N G_{NM}; \quad G_{NM} = \mathcal{E}_N{}^B \mathcal{E}_M{}^A \eta_{AB}(-1)^{[B][M]}$$

where $\mathcal{E}$ are the frame vectors which 'curve' X from flat space:

$$ds^2 = dx^a dx^b \eta_{ba} + \ell^2 (d\zeta^{\bar{\alpha}} d\zeta^{\beta} \eta_{\beta\bar{\alpha}} + d\zeta^{\alpha} d\zeta^{\bar{\beta}} \eta_{\bar{\beta}\alpha})/2.$$

The tensorial transformations rules for the metric:

$$G'_{SR}(X') = \left(\frac{\partial X^M}{\partial X'^R}\right)\left(\frac{\partial X^N}{\partial X'^S}\right) G_{NM}(X)(-1)^{[S]([R]+[M])},$$

oblige us to introduce gauge fields V via $\mathcal{E}_m{}^\alpha = -iV_m{}^{\alpha\bar{\nu}}\zeta^\nu$ if we want the metric to behaves properly under local unitary property rotations: $\zeta^\mu \rightarrow \zeta'^\mu = [\exp(i\Theta(x))]^{\mu\bar{\nu}}\zeta^\nu$, whereupon, $iV'_m(x') = \exp[i\Theta(x)](iV_m(x) + \partial_m)\exp[-i\Theta(x)]$.

Comments:

- A length scale ℓ must be introduced for dimensional reasons to tie property separation $d\zeta$ to spacetime separation dx.

- There is very little room for manoeuvre in placing the Gauge Field within the spacetime-property element: it has to be overall fermionic and involve the product of V and ζ. Its placement in $G_{m\nu}$ makes good physical sense, since gauge fields communicate property from one location to another location. (This is similar to placement in standard KK theory.)

- The only freedom left, without destroying gauge symmetry of the group (or a subgroup perhaps), is the inclusion of invariants $Z \equiv \zeta^{\bar{\mu}}\zeta^{\mu}$ and powers thereof.

4 The extended metric

The only metric fully consistent with local SU(N) gauge transformations is

$$\begin{pmatrix} G_{mn} & G_{m\nu} & G_{m\bar\nu} \\ G_{\mu n} & G_{\mu\nu} & G_{\mu\bar\nu} \\ G_{\bar\mu n} & G_{\bar\mu\nu} & G_{\bar\mu\bar\nu} \end{pmatrix} =$$

$$\begin{pmatrix} g_{mn}C+\ell^2\bar\zeta\{V_m,V_n\}\zeta C'/2 & -i\ell^2(\bar\zeta V_m)^{\bar\nu}C'/2 & i\ell^2(V_m\zeta)^{\nu}C'/2 \\ -i\ell^2(\bar\zeta V_n)^{\bar\mu}C'/2 & 0 & \ell^2\delta_\mu{}^\nu C'/2 \\ i\ell^2(V_n\zeta)^{\mu}C'/2 & -\ell^2\delta_\nu{}^\mu C'/2 & 0 \end{pmatrix}.$$

where $C(Z)=\sum_{n=1}^N c_nZ^n, C'(Z)=1+\sum_{n=1}^N c'_nZ^n$ are independent polynomials of degree N in Z, which are in principle allowed; the c_n are interpreted as property curvature coefficients.

5 Lagrangian of Gauge Fields

The procedure is straightforward from hereon, but requires technical gymnastics: Work out the superRicci scalar (in Palatini form),

$$\mathcal{R} = G^{MK}\mathcal{R}_{KM} = (-1)^{[L]}G^{MK}[(-1)^{[L][M]}\Gamma_{KL}{}^{N}\Gamma_{NM}{}^{L} - \Gamma_{KM}{}^{N}\Gamma_{NL}{}^{L}],$$

from the generalized Christoffel symbols

$$2\Gamma_{MN}{}^{K} = \Big[(-1)^{[M][N]}G_{ML,N} + G_{NL,M} - (-1)^{[L]([M]+[N])}G_{MN,L}\Big](-1)^{[L]}G^{LK}$$

and integrate it over x and ζ. We get

$$\left(\frac{\ell^2}{2}\right)^{2(N-1)}\int d^N\zeta d^N\bar{\zeta}\,\sqrt{G_{..}}\,\mathcal{R} = \frac{\mathcal{A}R^{[g]}}{\ell^2} + \mathcal{B}\mathrm{Tr}(F.F) + \frac{\mathcal{C}}{\ell^4},$$

where $F_{mn} = V_{n,m} - V_{m,n} + i[V_m, V_n]$ and we have not as yet included coupling constants which are *embedded* in V.

We have finally found a way of calculating N-dependent coefficients $\mathcal{A}, \mathcal{B}, \mathcal{C}$ without using Mathematica. The coefficients for gravity, the force fields and the cosmological constant are listed in the references in the conference proceedings. It is possible to get any value of $\mathcal{C}$ for $N \geq 2$ by adjusting the property curvature coefficients. One may even set $C = C'$ for one universal property polynomial. If $\mathcal{C}$ is tiny then fine-tuning of c_n is indicated.

We find, as a bonus, that the stress tensor T_{mn} for the force field is automatically incorporated in the components of the superRicci tensor $\mathcal{R}_{mn}$ when we extract the 'equations of motion', i.e. T_{mn} appears on the left hand side of the Einstein equation; what is more, with the correct sign!

6 Matter Field Lagrangian

These arise in the usual way

$$
\begin{aligned}
\mathcal{L}_\phi &= \int d^N\zeta d^N\bar{\zeta}\,\sqrt{-G_{..}}\,G^{MN}\partial_N\Phi\partial_M\Phi; \\
\mathcal{L}_\psi &= \int d^N\zeta d^N\bar{\zeta}\,\sqrt{-G_{..}}\,\bar{\Psi}\,i\Gamma^A E_A{}^M\partial_M\Psi.
\end{aligned}
$$

once we have made the superBose Φ and SuperFermi Ψ fields (anti)selfdual.

The gauge and gravitational interactions of the component fields ϕ, ψ then just fall out, but may require wave function renormalizations due to influence of ζ curvature coefficients c_n, as we will see. The key point is that $E_A{}^M\partial_M \supset E_a{}^M\partial_M \equiv D_a = e_a{}^m[\partial_m + i(V_m\zeta)^\mu\,\partial_\mu - i(\bar{\zeta}V_m)^{\bar{\mu}}\,\partial_{\bar{\mu}}]$, which is our version of covariant differentiation, with $V_m \equiv gW_m$.

Three important comments about fermions:

- The adjoint $\overline{\mathbf{\Psi}}$ must be carefully defined to produce a series of $\overline{\psi}\psi$ terms after property integration

- In the full expansion of $\mathbf{\Psi}(\zeta, \bar{\zeta})$, $\bar{\zeta}\psi$ and $\psi^c\zeta$ both appear. The latter just doubles up the eventual answers so we can simplify calculation by 'halving' the expansion to $\mathbf{\Psi} \supset \bar{\zeta}\psi$ terms

- Super Γ matrices must obey $(\Gamma^A P_A)^2 = \eta^{AB} P_B P_A$. Spacetime $\Gamma^a =$ usual γ^a matrices with $\{\gamma^a, \gamma^b\} = 2\eta^{ab}$; but for property

$$[\Gamma^\alpha, \Gamma^\beta] = [\Gamma^{\bar{\alpha}}, \Gamma^{\bar{\beta}}] = 0, \text{ and } [\Gamma^\alpha, \Gamma^{\bar{\beta}}] = 2\eta^{\alpha\bar{\beta}} = 2\delta_\beta{}^\alpha.$$

One way solve these last conditions is mimic Dirac and extend the space with auxiliary coordinates θ^1 to θ^N and set

$$\Gamma^\alpha \equiv \sigma_+\theta^\alpha, \quad \Gamma^{\bar{\alpha}} \equiv \sigma_-\partial/\partial\theta^\alpha, \quad \Psi \to \Psi \times (1+\sigma_3)\theta^1\theta^2\cdots\theta^N/2$$

and integrate over the auxiliary coordinates.

There is probably a less extravagant way of satisfying the constraints on Γ^α.

(Any suggestions welcome!)

7 Chromic and Electric Relativity

Confine ourselves to the four coordinates ζ^1 to ζ^4 and the subgroup SU(3)×U(1). Those forces seem to be parity invariant and we need not distinguish left from right. We have two separate property invariants, $\zeta^{\bar{\kappa}}\zeta^{\kappa}$ and $\zeta^{\bar{4}}\zeta^4$ which can appear in the property curvature polynomial.

For simplicity, assume this is universal to spacetime and property with

$$C = C' = 1 + \cdots c_e(\zeta^{\bar{4}}\zeta^4)(\zeta^{\bar{\kappa}}\zeta^{\kappa})^2 + c_f(\zeta^{\bar{\kappa}}\zeta^{\kappa})^3.$$

The gauge fields (gluon B and em A with their respective couplings f and e) enter through the frame vectors, $\mathcal{E}_m{}^{\kappa} = -i(fB_m - eA_m/3)^{\kappa\bar{\iota}}\zeta^{\iota}, \quad \mathcal{E}_m{}^4 = ieA_m$, leading to elements $G_{m4} = i\ell^2\zeta^{\bar{4}}eA_mC'/2, \quad G_{m\iota} = i\ell^2[\zeta^{\bar{\iota}}eA_m/3 - \zeta^{\bar{\kappa}}fB_m{}^{\kappa\bar{\iota}}]C'/2.$

A careful calculation, using $\sqrt{-G..} = (2/\ell^2)^4\sqrt{-g..}C^{-2}$ produces the gauge field terms

$$\mathcal{R}\sqrt{-G..} \supset \quad [1 - 3c_e(\zeta^{\bar{4}}\zeta^4)(\zeta^{\bar{\kappa}}\zeta^\kappa)^2 - 3c_f(\zeta^{\bar{\kappa}}\zeta^\kappa)^3 + \cdots] \\ \sqrt{-g..}g^{km}g^{ln}[4e^2\zeta^{\bar{4}}F_{kl}F_{mn}\zeta^4/3 + f^2\zeta^{\bar{\kappa}}(E_{kl}E_{mn})^{\kappa\bar{\iota}}\zeta^\iota],$$

where $F_{mn} = A_{n,m} - A_{m,n}$ and $E_{mn} = B_{n,m} - B_{m,n} + if[B_n, B_m]$ are the usual 'curls' of the gauge potentials. Hence

$$\int d^4\zeta d^4\bar{\zeta}\,\sqrt{-G..}\mathcal{R} \supset (-12\sqrt{-g..}/\ell^2)[4c_f e^2 F.F + c_e f^2 \mathrm{Tr}(E.E)].$$

To ensure gravitational universality we have to set $c_e f^2 = 4c_f e^2$, which can be arranged without requiring equality of strong and em couplings.

The colour and em interactions of the matter fields emerge as expected..

8 Electroweak relativity

Apply this scheme to the original electroweak model. Because weak isospin and hypercharge assignments change with chirality invoke *distinct* properties ζ_L and ζ_R for left- and right-handed leptons. so we have a quartet of properties with

$$Y(\zeta_L^0, \zeta_L^4, \zeta_R^0, \zeta_R^4) = (-1, -1, 0, -2).$$

The full SU(4) gauge field is not needed; just the reduced SU$(2)_L \times$U(1) rotations, so

$$(V_m\zeta) = (L_m\zeta_L) + (R_m\zeta_R) = (g\mathbf{W}_m.\tau - g'B_m)\zeta_L/2 + g'B_m(\tau_3 - 1)\zeta_R/2$$

R is associated with right property derivative $\partial/\partial\zeta_R$... It is sufficiently general to take property curvature as direct product of left and right polynomials:

$$C = C_R C_L = [1 + c_R Z_R + c_{RR} Z_R^2][1 + c_L Z_L + c_{LL} Z_L^2];\ Z_R \equiv \bar{\zeta}_R\zeta_R,\ Z_L \equiv \bar{\zeta}_L\zeta_L.$$

Using the metric components,

$$
\begin{aligned}
G_{m\zeta_L} &= -i\ell^2\bar{\zeta}_L L_m C/2; \qquad G_{m\zeta_R} = -i\ell^2\bar{\zeta}_R R_m C/2, \\
G_{\zeta_L\bar{\zeta}_L} &= G_{\zeta_R\bar{\zeta}_R} = \ell^2 C/2, \quad G_{\zeta_L\zeta_R} = G_{\bar{\zeta}_L\bar{\zeta}_R} = G_{\zeta_L\bar{\zeta}_R} = G_{\zeta_R\bar{\zeta}_L} = 0.
\end{aligned}
$$

one eventually obtains

$$
\begin{aligned}
\int d^2\zeta_R..d^2\bar{\zeta}_L\sqrt{G..}\mathcal{R}\supset \frac{3}{2}\sqrt{g..}\left(\frac{2}{\ell^2}\right)^3\Big[c_L(2c_{RR}-3c_R^2)(g^2\mathbf{W}_{mn}.\mathbf{W}^{mn}+g'^2B_{mn}B^{mn}) \\
+g'^2 2c_R(2c_{LL}-3c_L^2)B_{mn}B^{mn}\Big],
\end{aligned}
$$

where $W_{mn}\equiv W_{n,m}-W_{m,n}+ig[W_n,W_m]$ and $B_{mn}\equiv B_{n,m}-B_{m,n}$.

Universality of gravity (and correct normalization of gauge fields) requires

$$
c_L(3c_R^2-2c_{RR})(g^2-g'^2)=2c_R(3c_L^2-2c_{LL})g'^2.
$$

Assuming gravity is parity blind, $c_R=c_L=c, c_{RR}=c_{LL}=c_2$, so $g^2=3g'^2$.

Thus universality of G_N predicts a weak mixing angle of 30^o in this scheme.

So far as (selfdual) matter fields are concerned, there are predicted to be *two* generations ψ, ψ' of leptons:

$$\begin{aligned} 2\Psi &= \bar{\zeta}_L[\psi_L(1+Z_R^2/2) + \psi'_L Z_R](1+Z_L) + (L \leftrightarrow R), \\ \text{adjoint } 2\overline{\Psi} &= [\overline{\psi_L}(1+Z_R^2/2) + \overline{\psi'_L} Z_R]\zeta_L(1+Z_L) + (L \leftrightarrow R). \end{aligned}$$

An even parity superBose field $\mathbf{\Phi}$ contains three singlets $(\varphi, \varphi', \chi)$ and a quartet $\phi^{\mu\bar{\nu}} \equiv (\phi_0 I + \phi.\tau)^{\mu\bar{\nu}}/\sqrt{2}$ with quantum numbers:

$$\begin{aligned} Y(\phi^{0\bar{0}}, \phi^{0\bar{4}}, \phi^{4\bar{0}}, \phi^{4\bar{4}}) &= (1, 1, -1, -1); \\ 2I_{3L}(\phi^{0\bar{0}}, \phi^{0\bar{4}}, \phi^{4\bar{0}}, \phi^{4\bar{4}}) &= (-1, 1, -1, 1); \\ Q(\phi^{0\bar{0}}, \phi^{0\bar{4}}, \phi^{4\bar{0}}, \phi^{4\bar{4}}) &= (0, 1, -1, 0). \end{aligned}$$

The Higgs field is associated with $\phi_0 + \phi_3$, as we will see.

To introduce gauge interactions, note that the fermion kinetic energy can be written as $\overline{\Psi} i \Gamma^A D_A \Psi$ where (temporarily ignoring property curvature through C):

$$D_A = E_A{}^M \partial_M = E_A{}^m \partial_m + E_A{}^\mu \partial_\mu + E_A{}^{\bar\mu} \partial_{\bar\mu}$$

$$\begin{pmatrix} E_a{}^m & E_a{}^\mu & E_a{}^{\bar\mu} \\ E_\alpha{}^m & E_\alpha{}^\mu & E_\alpha{}^{\bar\mu} \\ E_{\bar\alpha}{}^m & E_{\bar\alpha}{}^\mu & E_{\bar\alpha}{}^{\bar\mu} \end{pmatrix} = \frac{1}{\sqrt{C}} \begin{pmatrix} e_a{}^m & i[(L_a\zeta_L)+(R_a\zeta_R)]^\mu & -i[(\bar\zeta_L L_a)+(\bar\zeta_R R_a)]^{\bar\mu} \\ 0 & \delta_\alpha{}^\mu & 0 \\ 0 & 0 & \delta_{\bar\alpha}{}^{\bar\mu} \end{pmatrix}.$$

Interpreting $(\psi^0, \psi^4) = (\nu, l)$ one ends up with the standard Lagrangian:

$$\begin{aligned} \mathcal{L}_\psi &= \bar l \gamma.(i\partial - eA)l + \bar\nu i\gamma.\partial\nu + \frac{e}{\sqrt{2}\sin\theta}[\overline{\nu_L}\gamma.W^+ l_L + \overline{l_L}\gamma.W^-\nu_L] \\ &\quad + \frac{e}{\sin 2\theta}(\overline{\nu_L}\gamma.Z\nu_L) + e\tan\theta(\overline{l_R}\gamma.Zl_R) - e\cot 2\theta(\overline{l_L}\gamma.Zl_L) \\ &\quad + (l,\nu) \to (l',\nu'), \qquad \textcolor{red}{PERFECT!} \end{aligned}$$

where $\cos\theta = g/\sqrt{g^2+g'^2}$, $\sin\theta = g'/\sqrt{g^2+g'^2}$, $e = gg'/\sqrt{g^2+g'^2}$.

9 Induced masses

To identify the Higgs field and its expectation value, we need to consider the kinetic energy term of the superBose field,

$$2D\Phi.D\Phi = (1{+}2Z_L)(1{+}2Z_R)[\bar{\zeta}_R\{\partial\phi{+}i(\phi L{-}R\phi)\}\zeta_L\bar{\zeta}_L\{\partial\phi{+}i(\phi R{-}L\phi)\zeta_R\}]$$

and in particular the terms associated with the uncharged fields ϕ_0, ϕ_3 which can develop expectation values:

$$2\mathrm{Tr}[(\phi R - L\phi)(\phi L - R\phi)] \;\to\; \frac{1}{2}g^2W^+W^-(\phi_+^2 + \phi_-^2) + \frac{1}{4}\phi_+^2(gW_3 - g'B)^2$$
$$+\frac{1}{4}\phi_-^2(gW_3 + g'B)^2 - g'^2\phi_-^2;\;\; \phi_\pm \equiv \phi_0 \pm \phi_3.$$

To recover the standard vector meson masses, we must take

$$\langle\phi_-\rangle = 0 \text{ or } \langle\phi_0\rangle = \langle\phi_3\rangle, \text{ and } \langle\phi_+\rangle = v,$$

whereupon

$$\begin{aligned}\langle 2\mathrm{Tr}[(\phi R - L\phi)(\phi L - R\phi)]\rangle &\rightarrow \frac{1}{2}v^2 g^2 W^+ W^- + \frac{1}{4}v^2(g^2 + g'^2)Z^2 \\ &= \frac{e^2 v^2}{2\sin^2\theta}W^+W^- + \frac{e^2v^2}{\sin^2 2\theta}Z^2.\end{aligned}$$

All is as it should be with A massless, and if we take $\theta = 30^o$ on the assumption of parity-invariant universal gravitation, Z couples axially to the two charged leptons.

The masses of the fermions are induced through the Yukawa interaction. To include νs, recall that $i\tau_2 H^*$ can serve as a doublet like the Higgs field H. In our context it means we can replace ϕ by the linear combination $\hat{\phi} = c_l \tau_2 \phi^* \tau_2 + s_l \phi$, whence

$$-16\int d^2\zeta_R..d^2\bar{\zeta}_L\,\overline{\Psi}\Phi\Psi = (2\bar{\psi} + \bar{\psi}')\hat{\phi}(2\psi + \psi') \equiv 5\overline{\hat{\psi}}\hat{\phi}\hat{\psi}.$$

Taking expectation values of $\hat{\phi}$ of the Yukawa term produces

$$5\mathfrak{g}(\overline{\nu_l}\,,\,\bar{l})\begin{pmatrix} c_l\langle\phi_+\rangle & 0 \\ 0 & s_l\langle\phi_+\rangle \end{pmatrix}\begin{pmatrix} \nu_l \\ l \end{pmatrix} = 5v\mathfrak{g}(c_l\overline{\nu_l}\nu_l + s_l\bar{l}l).$$

The other generation $(\psi - 2\psi')/\sqrt{5}$ does not acquire a mass.

The effect of spacetime curvature (through ${e_m}^a$ or g_{mn}) and property curvature $C(Z)$ is to make the interactions generally covariant; the influence of C is to mix and component fields further via the superdeterminant

$$\sqrt{G_{..}} = \sqrt{g_{..}}\left(\frac{2}{\ell^2\sqrt{C}}\right)^4 \propto (1 - 2c_R - 2c_{RR}Z_{RR} + 3c_R^2 Z_R^2)(1 - 2c_L - 2c_{LL}Z_{LL} + 3c_L^2 Z_L^2).$$

but affects the derivative and gauge fields equally so does not otherwise compromise the interactions found earlier. For details and earlier refs. see R.D & P.D.Stack. Mod. Phys. Lett. 31A, 1650019 (2016)