A new look at Newton-Cartan gravity

Eric Bergshoeff

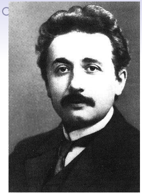
Groningen University

Memorial Meeting for

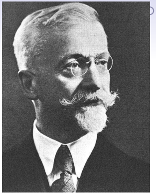
Nobel Laureate Professor Abdus Salam's 90th Birthday

NTU, Singapore, January 27 2016

rijksuniversiteit groningen



Einstein (1905/1915)

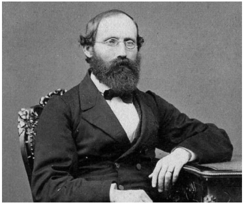


Élie Cartan (1923)

Einstein achieved two things in 1915:

- He made gravity consistent with special relativity
- He used an arbitrary coordinate frame formulation

Geometry



Riemann (1867)

Einstein used Riemannian geometry $\Rightarrow$ General relativity

Cartan used NC geometry $\Rightarrow$ NC gravity

Newton-Cartan (NC) gravity is Newtonian gravity in arbitrary frame why non-relativistic gravity ?

Motivation

- gauge-gravity duality

  Liu, Schalm, Sun, Zaanen, Holographic Duality in Condensed Matter Physics (2015)

  Christensen, Hartong, Kiritsis Obers and Rollier (2013-2015)

- condensed matter physics

  Son (2013), Can, Laskin, Wiegmann (2014), Gromov, Abanov (2015)

- Hořava-Lifshitz gravity, flat-space holography, etc.

  Hořava (2009); Hartong, Obers (2015); Duval, Gibbons, Horvathy, Zhang (2014)

- non-relativistic strings/branes

  Gomis, Ooguri (2000); Gomis, Kamimura, Townsend (2004)

How do we construct (Non-)relativistic Gravity ?

(1) gauging a (non-)relativistic algebra

(2) taking a non-relativistic limit

(3) using a nonrelativistic version of the conformal tensor calculus

Outline

NC Gravity from gauging Bargmann

Outline

Outline

Outline

Outline

Einstein Gravity

In the relativistic case free-falling frames are connected by the Poincare symmetries:

- space-time translations: $\delta x^\mu = \xi^\mu$
- Lorentz transformations: $\delta x^\mu = \lambda^\mu{}_\nu \, x^\nu$

In free-falling frames there is no gravitational force

in arbitrary frames the gravitational force is described by an invertable Vierbein field $e_\mu{}^A(x)$

$\mu = 0, 1, 2, 3$; A=0,1,2,3

Non-relativistic Gravity

In the non-relativistic case free-falling frames are connected by the Galilean symmetries:

- time translations : $\delta t = \xi^0$
- space translations : $\delta x^i = \xi^i \qquad i = 1, 2, 3$
- spatial rotations : $\delta x^i = \lambda^i{}_j \, x^j$
- Galilean boosts : $\delta x^i = \lambda^i t$

In free-falling frames there is no gravitational force

Newtonian gravity versus Newton-Cartan gravity

- in frames with constant acceleration $(\delta x^i = \frac{1}{2}a^i t^2)$ the gravitational force is described by the Newton potential $\Phi(\vec{x}) \rightarrow$

Newtonian gravity

- in arbitrary frames the gravitational force is described by a temporal Vierbein $\tau_\mu(x)$, spatial Vierbein $e_\mu{}^a(x)$ plus a vector $m_\mu(x) \rightarrow$

$\mu = 0, 1, 2, 3$; a=1,2,3

Newton-Cartan (NC) gravity

The Galilei Algebra versus the Bargmann algebra

- Einstein gravity follows from gauging the Poincare algebra
- The Galilei algebra is the contraction of the Poincare algebra
- does NC gravity follow from gauging the Galilei algebra?
- Can NC gravity be obtained by taking the non-relativistic limit of Einstein gravity?

No!

one needs Bargmann instead of Galilei and Poincare $\otimes$ U(1) !

Gauging the Bargmann algebra

cp. to Chamseddine and West (1977)

$$[J_{ab}, P_c] = -2\delta_{c[a}P_{b]}\,, \qquad [J_{ab}, G_c] = -2\delta_{c[a}G_{b]}\,,$$

$$[G_a, H] = -P_a\,, \qquad [G_a, P_b] = -\delta_{ab}Z\,, \qquad a = 1, 2, \ldots, d$$

symmetry	generators	gauge field	parameters	curvatures
time translations	H	τ_μ	$\zeta(x^\nu)$	$\mathcal{R}_{\mu\nu}(H)$
space translations	P^a	${e_\mu}^a$	$\zeta^a(x^\nu)$	${\mathcal{R}_{\mu\nu}}^a(P)$
Galilean boosts	G^a	${\omega_\mu}^a$	$\lambda^a(x^\nu)$	${\mathcal{R}_{\mu\nu}}^a(G)$
spatial rotations	J^{ab}	${\omega_\mu}^{ab}$	$\lambda^{ab}(x^\nu)$	${\mathcal{R}_{\mu\nu}}^{ab}(J)$
central charge transf.	Z	m_μ	$\sigma(x^\nu)$	$\mathcal{R}_{\mu\nu}(Z)$

Imposing Constraints

$\mathcal{R}_{\mu\nu}{}^{a}(P) = 0\,, \qquad \mathcal{R}_{\mu\nu}(Z) = 0$: solve for spin-connection fields

$\mathcal{R}_{\mu\nu}(H) = \partial_{[\mu}\tau_{\nu]} = 0 \;\rightarrow\; \tau_\mu = \partial_\mu \tau$: foliation of Newtonian spacetime ('zero torsion')

$\mathcal{R}_{\mu\nu}{}^{ab}(J) \neq 0$: restriction on-shell

$\mathcal{R}_{0(a,b)}(G) \neq 0$: Poisson equation on-shell

The Final Result

The independent NC fields $\{\tau_\mu, e_\mu{}^a, m_\mu\}$ transform as follows:

$$\delta\tau_\mu = 0\,,$$
$$\delta e_\mu{}^a = \lambda^a{}_b\, e_\mu{}^b + \lambda^a \tau_\mu\,,$$
$$\delta m_\mu = \partial_\mu \sigma + \lambda_a\, e_\mu{}^a$$

The spin-connection fields $\omega_\mu{}^{ab}$ and $\omega_\mu{}^a$ are functions of e, τ and m

There are two Galilean-invariant metrics:

$$\tau_{\mu\nu} = \tau_\mu \tau_\nu\,, \qquad h^{\mu\nu} = e^\mu{}_a e^\nu{}_b\, \delta^{ab}$$

The NC Equations of Motion

Taking the non-relativistic limit of the Einstein equations $\Rightarrow$

Rosseel, Zojer + E.B. (2015)

$$\tau^{\mu} e^{\nu}{}_{a} \mathcal{R}_{\mu\nu}{}^{a}(G) = 0$$

$$e^{\nu}{}_{a} \mathcal{R}_{\mu\nu}{}^{ab}(J) = 0$$

- after gauge-fixing and assuming flat space the first NC e.o.m. becomes $\triangle\Phi = 0$
- note: there is no action that gives rise to these equations of motion

Outline

NC Gravity from gauging Bargmann

The Schrödinger Method

NC Gravity with Torsion

Future Directions

The Relativistic Conformal Method

Conformal = Poincare + D (dilatations) + K_μ (special conf. transf.)

conformal gravity $\equiv$ gauging of conformal algebra

$$\delta b_\mu = \Lambda_K^a(x) e_\mu{}^a\,, \qquad f_\mu{}^a = f_\mu{}^a(e, \omega, b)$$

Poincare invariant $\Leftrightarrow$ CFT of real scalar

An example

$$\mathrm{P}: \quad e^{-1}\mathcal{L} = \frac{1}{\kappa^2} R$$

STEP 1

$$(e_\mu{}^{\mathsf{A}})^{\mathsf{P}} = \kappa^{\frac{2}{D-2}} \varphi \, (e_\mu{}^{\mathsf{A}})^{\mathsf{C}} \qquad \text{with}$$

$$\delta\varphi = -\Lambda_{\mathsf{D}}\varphi\,, \qquad \delta(e_\mu{}^{\mathsf{A}})^{\mathsf{C}} = \Lambda_{\mathsf{D}}(e_\mu{}^{\mathsf{A}})^{\mathsf{C}}$$

STEP 2

$$(e_\mu{}^{\mathsf{A}})^{\mathsf{C}} = \delta_\mu{}^{\mathsf{A}} \quad \Rightarrow \quad \partial_\mu\xi^\nu + \Lambda^{\nu\mu} + \Lambda_{\mathsf{D}}\delta_\mu{}^\nu = 0$$

make redefinition $\varphi = \phi^{\frac{2}{D-2}}\,, \quad D > 2 \quad \Rightarrow$

$$\mathrm{CFT}: \quad \mathcal{L} = 4\,\tfrac{D-1}{D-2}\,\phi\Box\phi \quad \text{with} \quad \delta\phi = \xi^\mu\partial_\mu\,\phi - \tfrac{1}{2}(D-2)\Lambda_{\mathsf{D}}\phi$$

from CFT back to P

$$\mathrm{CFT}: \quad \mathcal{L} \sim \phi\Box\phi \qquad \delta\phi = \xi^\mu\partial_\mu \phi + w\Lambda_{\mathrm{D}}\phi$$

STEP 1 replace derivatives by conformal-covariant derivatives $\Rightarrow$

$$e^{-1}\mathcal{L} = 4\,\tfrac{D-1}{D-2}\,\phi\Box^{\mathrm{C}}\phi$$

STEP 2 gauge-fix dilatations by imposing $\phi = \frac{1}{\kappa}$ $\Rightarrow$

$$\mathrm{P}: \quad e^{-1}\mathcal{L} = \frac{1}{\kappa^2}\,R$$

Three Different Invariants

1. **Kinetic terms** Example: $\mathcal{L} \sim \phi\Box\phi \quad \Leftrightarrow \quad e^{-1}\mathcal{L} = R$

   includes all CFT's with time derivatives

2. **Potential terms** Example: cosmological constant $(\kappa = 1)$

$$e^{-1}\mathcal{L} = \Lambda \qquad \Leftrightarrow \qquad \mathcal{L} = \Lambda\,\phi^2\,, \quad w = -\frac{D}{2}$$

3. **Curvature terms** Example: Weyl tensor squared

$$e^{-1}\mathcal{L} \sim \phi^{2\frac{D-4}{D-2}}\left(C_{\mu\nu}{}^{AB}\right)^2 \qquad D \geq 4$$

The Schrödinger Method

The contraction of the conformal Algebra is the Galilean Conformal Algebra (GCA) which has no central extension!

$z = 2$ Schrödinger $=$ Bargmann $+$ D (dilatations) $+$ K (special conf.)

$$[H, D] = zH\,, \qquad [P_a, D] = P_a$$

$z = 1$: conformal algebra, $z \neq 2$: no special conf. transf.

Schrödinger Gravity

Hartong, Rosseel + E.B. (2014)

Gauging the $z = 2$ Schrödinger algebra we find that the independent gauge fields $\{\tau_\mu, e_\mu{}^a, m_\mu\}$ transform as follows:

$$\delta\tau_\mu = 2\Lambda_D\tau_\mu \,,$$
$$\delta e_\mu{}^a = \Lambda^a{}_b\, e_\mu{}^b + \Lambda^a\tau_\mu + \Lambda_D e_\mu{}^a \,,$$
$$\delta m_\mu = \partial_\mu\sigma + \Lambda_a\, e_\mu{}^a$$

The time projection $\tau^\mu b_\mu$ of b_μ transforms under K as a a shift while the spatial projection $b_a \equiv e_a{}^\mu b_\mu$ is dependent $\Rightarrow$

$b_a\,(e,\tau)$ represents (twistless) torsion!

SFT's versus Galilean Invariants

the Schrödinger action for a complex scalar Ψ with weights (w, M)

$$\text{SFT}: \quad S = \int dtd^dx\, \Psi^\star \Big(i\partial_0 - \tfrac{1}{2M}\partial_a\partial_a\Big)\Psi$$

is invariant under the rigid Schrödinger transformations

$$\delta\Psi = \big(b - 2\lambda_D t + \lambda_K t^2\big)\partial_0\Psi + \big(b^a - \lambda^{ab}x_b - \lambda^a t - \lambda_D x^a + \lambda_K t x^a\big)\partial_a\Psi$$
$$+ w\big(\lambda_D - \lambda_K t\big)\Psi + iM\big(\sigma - \lambda^a x_a + \frac{1}{2}\lambda_K x^2\big)\Psi \qquad \text{for} \quad w(\Psi) = -d/2$$

The corresponding Galilean invariant G has inconsistent E.O.M.'s

Outline

Case 1: zero torsion: $b_a = 0$

Schrödinger method also works at level of E.O.M.'s

$$\text{foliation constraint}: \quad \partial_\mu(\tau_\nu)^{\mathrm{G}} - \partial_\nu(\tau_\mu)^{\mathrm{G}} = 0\,,$$

$$\text{Gal E.O.M.}: \quad (\tau^\mu)^{\mathrm{G}}(e^\nu{}_a)^{\mathrm{G}}\mathcal{R}_{\mu\nu}{}^a(G) = 0\,,$$

$$(e^\nu{}_a)^{\mathrm{G}}\mathcal{R}_{\mu\nu}{}^{ab}(J) = 0\,.$$

Schrödinger method leads to ($\Psi = \varphi e^{i\chi}$)

$$\text{SFT}: \quad \partial_0\partial_0\varphi = 0 \quad \text{and} \quad \partial_a\varphi = 0 \quad \text{with} \quad w = 1$$

Case 2: twistless torsion: $b_a \neq 0$

foliation constraint is conformal invariant $\rightarrow$ use the second compensating scalar χ to restore Schrödinger invariance:

$$\partial_0\partial_0\varphi - \frac{2}{M}(\partial_0\partial_a\varphi)\partial_a\chi + \frac{1}{M^2}(\partial_a\partial_b\varphi)\partial_a\chi\partial_b\chi = 0$$

$$\Downarrow$$

$$\boxed{-\triangle\Phi + \hat{\tau}^\mu\partial_\mu K + K^{ab}K_{ab} - 8\,\Phi\, b\cdot b - 2\,\Phi\,\mathcal{D}\cdot b - 6\, b^a\mathcal{D}_a\Phi = 0}$$

plus

$$\boxed{e^\nu{}_a R_{\mu\nu}{}^{ab}(J) = 0}$$

Afshar, Mehra, Parekh, Rollier + E.B. (2015)

Outline

NC Gravity from gauging Bargmann

The Schrödinger Method

NC Gravity with Torsion

Future Directions

New developments and Extensions

- relation to Hořava-Lifshitz gravity

Hartong and Obers (2015)

Afshar, Mehra, Parekh, Rollier + E.B. (2015)

- extension to $z \neq 2$ and Galilean conformal symmetries

- matter-coupled NC gravity

- non-relativistic supergravity $\rightarrow$ localization techniques

Andringa, Rosseel, Sezgin + E.B. (2013)

Knodel, Lisbao, Liu (2015)