INTEREST-RATE PARITY AND DIVERGENCE OF VIEWS ON EXCHANGE-RATE CHANGES: AN UPWARD-SLOPING SUPPLY CURVE OF FUNDS EVEN FOR A SMALL COUNTRY*

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I. INTRODUCTION

It is generally accepted by economists, given reasonable assumptions on capital mobility, that international asset arbitrage maintains interest parity, and that a ‘small’ country faces a horizontal supply curve of loanable funds (capital). The empirical counterpart of ‘smallness’ is less precise than the analytical abstraction. Many, if not most, countries fall into this category in view of the overwhelmingly large size of the international capital market compared to any particular national financial market, and the relatively small transaction costs of international financial transactions.

In this paper, we argue that, while maintaining all conventional assumptions on capital mobility and risk neutrality, relaxation of the strong assumption that different investors always hold identical views with respect to expected exchange rate changes is sufficient to invalidate interest-rate parity (except for marginal suppliers of funds) and to make the supply curve of funds upward-sloping for any but ‘super-small’ countries. This result helps to explain a number of empirical phenomena, some of which are inconsistent with existing theories.

Section II motivates reservations about the validity of typical implementations of the small country assumption and provides the rationale for rational diversity in exchange rate expectations. Section III analyses the central result of the paper. Section IV shows that this result is not affected by the introduction of transaction costs and is in fact strengthened by the presence of risk aversion; it also touches on a ‘super-small’ country. A formal derivation of the supply curve of funds for an individual transactor with risk aversion is provided in the Appendix.

II. EXPECTATIONS

Uncovered interest parity constitutes a strategic building block in asset models of exchange rate determination.¹ As a pure arbitrage condition it does not address issues of causality, nor does it determine the level of the exchange rate. Specification of some minimal structure

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¹ This class of models is typified by Dornbusch’s (1976) seminal contribution. It constitutes a prominent variant of the asset approach which encompasses ‘a bewildering complexity’ (Frankel, 1983, p.84) of particular specifications, including the assumption of imperfect asset substitutability.
characteristically includes the assumptions of monetary neutrality, purchasing power parity (PPP), and rational expectations. These rather exacting requirements fix the international yield differential but not the absolute levels of interest rates. For small countries, this indeterminacy is eliminated by postulating the exogeneity of the foreign interest rate. International investors are modelled as if they are prepared to supply any amount of funds to the small country at the given foreign interest rate, irrespective of exchange rate expectations.

The widely acknowledged conspicuous failure of this type of model to predict or explain exchange rate behaviour (Meese and Rogoff, 1983; Koromzay et al., 1987) may be attributable to misspecification. Typically, small countries do not have access to a perfectly elastic supply of funds in international financial markets. Furthermore, secular growth has made the small country assumption increasingly anachronistic: ‘In the real world of the 1970’s there are probably few countries which are either completely “closed” or “small” in the Mundellian sense’ (Flanders, 1972, p.310). Intrinsically plausible and rational scenarios are preempted by the assumption of risk neutrality.

The presumption of uniform exchange rate expectations follows directly from the postulate of rational expectations in conjunction with the assumption of market efficiency. ‘This jointness has become so accepted that it has taken efficient market tests out of the realm of arbitrage and ... [diluted] the arbitrage intuition in many applications of the economic theory of information to problems of modelling financial markets’ (Ross, 1987, pp.32-3). The alternative scenario of divergent expectations is fully compatible with rational behaviour on the part of differently situated transactors in so far as efficiency considerations prevent at least some agents from fully exploiting all relevant extant information or that they use different models of the economy.

In general, it is immaterial whether heterogeneity derives from differences in expectations formation per se or in the utilisation of information and in attitudes to risk. The strategic factor is the existence of differences among transactors in their subjective perceptions of the opportunity costs of holding assets denominated in a particular currency. Except for occasions of most extraordinary regime instability that elicit convergence of these perceptions and monolithic portfolio readjustments, some agents characteristically hold long positions while others hold short positions in any denomination at any particular point in time.

Non-uniform expectations cannot be dismissed a priori nor on the basis of empirical evidence. The impossibility of direct observation and the inconclusiveness of joint hypothesis testing compromise the a priori validity of any particular expectations hypothesis, including the presumption that expectations are necessarily uniform. This ambivalence imparts legitimacy to the analytical exploration of alternative working assumptions.

The implications of non-uniform expectations are investigated in the following section within the standard analytical framework of rational behaviour. Retaining the conventional assumptions of perfect capital mobility and asset substitutability as well as risk neutrality, it is demonstrated that relaxation of the assumption of homogeneous exchange rate expectations is sufficient to make the domestic interest rate in the small open economy endogenous. The conventional exogeneity result is confined to the atypical case of ‘super-small’ countries.

2 For instance, ‘hedgers’ and ‘speculators’ in McKinnon (1979, ch.7) or ‘fundamentalists’ and ‘chartists’ in Frankel and Froot (1986).
The endogeneity of the interest rate may persist even though, formally, it may be regarded a short period result. Divergent expectations across transactors are, in principle, incompatible with long-run full stock-flow equilibrium. However, the length of duration of the short period is indefinite a priori. It is determined by the rapidity with which agents revise their expectations. High adjustment speeds in asset markets need not entail comparably fast revisions of expectations. These are entirely distinct phenomena, the former a consequence of the institutional environment (communication and transaction costs, depth of market, etc.), whereas the latter a behavioural phenomenon. Agents may dismiss current information as white noise or they may follow a slow adaptive process. They may interpret market signals differentially according to their preferred economic model (Frankel, 1979). Differences in the expected path of the real exchange rate may validate short-run violations of uncovered interest parity.

Thus, expectational 'mistakes' are feasible, and they need not be corrected instantaneously. Ultimate convergence towards full equilibrium is compatible with persistence of divergent expectations for protracted periods of real time.

III. Analysis

Uncovered IRP stipulates equality between the international nominal interest differential and the expected rate of change, $e$, of the exchange rate (domestic price of foreign exchange).

$$i = i^* + e$$

With the possible exception of a very small number of dominant countries (the US and Japan), domestic events are assumed to exert negligible effects on the world interest rate, $i^*$. The conventional 'small' country implementation of uncovered IRP, applicable to all but the few dominant countries, postulates exogeneity of the world rate of interest. Hence, for any given expected rate of depreciation the country is faced with a perfectly elastic supply of funds. Any shift in the domestic excess demand curve for funds leaves the domestic rate of interest unchanged unless it changes the expected rate of depreciation. In that event the perfectly elastic supply curve shifts up or down without any alteration in its slope.

The preceding analysis is based on a uniformly expected rate of depreciation. Suppose instead, more plausibly, that transactors are not fully agreed on the prospective behaviour of the exchange rate. They may hold divergent views concerning the extent of the impending depreciation and also concerning the time path of exchange rate adjustment. Then the domestic interest rate at which it is expected to be profitable to place funds in this particular country, given the world interest rate, will differ among transactors according to their particular expectation of depreciation. Any agent $j$ has a specific subjective probability distribution of the expected rate of depreciation, $e_j$. To start with, assume risk neutrality and the absence of transaction costs (both

3 By way of illustration, suppose that real interest parity holds continuously for perfectly substitutable assets. Then uncovered interest differentials can be sustained by deviations from ex ante purchasing power parity. Concretely, the expectation of a real appreciation validates an uncovered interest differential in favour of the home country:

$$r - r^* = (i - i^* - e) + (e + \pi^* - \pi) = 0$$

where $r$ and $\pi$ are the real interest rate and the expected rate of inflation, respectively. The first term in brackets represents the uncovered interest differential, and the second ex ante PPP.
assumptions will be relaxed later), transactor \( j \) will find it profitable to invest in this country if the domestic interest rate is at least as high as \( i^* + e^a_j \), where \( e^a_j \) is the mean expectation of depreciation.

Due to the diversity of expectations, the \( e^a_j \) differ among market participants. Let the distribution of transactors (weighted by the amount of funds at their disposal) over \( e^a_j \) (and hence over \( i^* + e^a_j \) which is measured along the vertical axis in Figure 1) be \( F \) (where the density is measured horizontally from the vertical axis). For example, point \( A \) refers to transactors with stationary expectations, \( e^a_j = 0 \), and the distance \( i^*A \) represents the (fund-weighted) frequency of that expectation.4

Let \( e^a \) be the weighted (by \( F \)) average of \( e^a_j \). Then, at a domestic interest rate equal to \( i^* + e^a \), those transactors with \( e^a_j \) below \( e^a \) will supply funds, and those with \( e^a_j \) above \( e^a \) will demand funds from the country concerned. By the definition of \( e^a \), supply equals demand at \( i^* + e^a \). Hence, the net international supply of funds to the home country at \( i = i^* + e^a \) is zero by construction.

For \( i > i^* + e^a \) the net international supply of funds is positive. Note that this supply is not netted out against domestic demand. It is the excess supply of funds to the domestic economy or, equivalently, the excess demand for domestic bonds on the world financial market. For example, at point \( B \), the net supply is represented by the area \( BCDE \), where the dotted area and the dashed area are equal by construction. (If \( F \) is not symmetrical around \( i^* + e^a \), the widths of these two

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4 'Fund-weighted' frequency is the frequency of participants weighted by the amount of funds each participant is supplying. On the determination of the amount of funds an individual wishes to supply, see the Appendix. It is true that if investors at the margin have an unlimited amount of funds, the supply curve will become horizontal. However, this is clearly an extremely unrealistic assumption.
shaded areas may be slightly different.) This is so because at \( i = B \), the supply of funds is measured by the area bounded by \( F \) below \( BC \), and the demand is the area above \( BC \), noting that the area below \( ED \) is equal to the area above \( BC \).

By varying the domestic interest rate, the net supply curve of funds from world financial markets to an individual country can be derived. If \( F \) is a normal distribution, the corresponding supply curve is obviously an inverted letter ‘S’ as illustrated in Figure 2. This is so because at the point \( i^* + e^a \), the supply curve intersects the vertical axis. At a slightly higher (lower) rate of interest, the net supply increases (decreases) substantially as the distance of the curve \( F \) to the vertical axis in Figure 1 is large. At a much higher (lower) interest rate, that distance is much smaller. Of course, \( F \) may not be a perfectly normal distribution. But if it has a somewhat normal distribution (i.e. more people holding medium than extreme views), the supply curve retains an inverted ‘S’ shape similar to that in Figure 2. It would take a rather peculiar distribution to change this general inverted-S shape of the supply curve.

![Figure 2](image-url)

Next, draw in the domestic excess demand curve for funds, \( D \).\(^5\) It is clear that, depending on whether the domestic excess demand is positive/negative at \( i = i^* + e^a \), (i) the equilibrium domestic rate of interest \( i \) will be above/below \( i^* + e^a \), (ii) the amount of net capital inflow will be positive/negative. It can thus be seen that \( IRP \) does not hold except for the marginal suppliers of funds and, in particular, it does not hold with respect to the mean expectation of depreciation except when the demand curve happens to intersect the horizontal axis at \( i^* + e^a \).

\(^5\) The shape and position of this demand curve may also be affected by the distribution of the expected rate of depreciation. However, going through a similar analysis as in the derivation of the supply curve would not change the result. Hence, we may take the demand curve as drawn as already reflecting the given distribution of the expected rate of depreciation. Alternatively, instead of distinguishing the (net) supply and (net) demand curves according to the location of the participants (overseas or domestic), we may distinguish between the speculative (where expectation regarding the exchange rate is important) and the non-speculative wealth owners. Yet a third way is to lump all participants together, yielding only a single (net) supply curve. In either case, our result is unaffected except for pedagogical elucidation.
If we assume that the expected rates of depreciation prove to be correct on average, then $e^a$ equals the actual rate of depreciation. The analysis above then suggests that the equilibrium domestic interest rate should be higher/lower than the world rate plus the rate of depreciation if there is net capital inflow/outflow.

Due to the shape of the supply curve in Figure 2, we may also expect that there will be little effect on the domestic interest rate for relatively small shifts in demand around the point $i^* + e^a$ (unless the $S$ curve also shifts as a result), but proportionately larger effects on $i$ for bigger shifts in demand (even in the absence of supply shifts). This seems to be consistent with the empirical evidence reported in Hoelscher (1986).

IV. SOME EXTENSIONS

Transaction costs

Transaction costs can be introduced into the analysis in terms of a bid-ask spread around the domestic interest rate. For simplicity suppose that international investors supply funds to the country only if the domestic interest rate exceeds the expected-depreciation-augmented world interest rate by a premium commensurate with transaction costs. Analogously, international demand for funds (borrowing) from the country emerges only at a corresponding negative premium in the domestic interest rate. Consequently, for any given domestic interest rate, those international investors/borrowers whose expected return falls within the spread will neither supply nor demand funds.

In terms of Figure 3, for a domestic interest rate $i = i^* + e^a$, not all transactors with a mean expectation of depreciation $e_j^a$ less than the average expectation $e^a$ will supply funds. Investment in that country is profitable only for those transactors whose supply price $(i^* + e_j^a)$ does not exceed the domestic interest rate net of transaction costs. Suppose the transaction cost margin is given by the distance between $i = i^* + e^a$ and point $G$. Then, only transactors with a supply price...
not exceeding \( G \) will supply funds to this country. Similarly, only those transactors whose mean expectation of depreciation exceeds the average expectation by a margin that places them above \( I \) will demand funds. The distance \( IG \) represents the bid-ask spread, and those transactors whose expectations fall within that range remain inactive.

The existence of uncovered \( IRP \), based on average expectations, preempts incentives for net international capital flows and signifies stock equilibrium. Irrespective of transaction costs (but assumed symmetrical for borrowing and lending), the net supply schedule of international funds intersects the vertical axis at \( i = i^* + e^a \). The effect of transaction costs is a reduction in the elasticity of the schedule and a decline in the absolute magnitude of the net supply of international funds for any domestic interest rate other than the parity rate, as shown below.

Let the domestic rate of interest be given by point \( K \) (Figure 3). In the absence of transaction costs, the net supply of capital is the area \( KLJZHNM \), as discussed above. (The area bounded by \( F \) and the vertical axis above the point \( K \) is equal to that below the point \( M \) by construction). This area \( KLJZHNM \) is represented by point \( L \) on supply schedule \( S \) (no transaction costs) in Figure 4. With transaction costs, given by the distance \( KI = KP \), the net supply becomes the area \( LIZHNR \). (The area bounded by \( F \) and the vertical axis above the point \( P \) is equal to that below the point \( R \) by construction. Transactors with \( i^* + e^a \) within the range \( PI \) are inactive.) This area \( LIZHNR \) is somewhat smaller than \( KLJZHNM \) and is represented by point \( J \) on schedule \( S' \) (with transaction costs). The reduction in the absolute amount of negative net supply for \( i < i^* + e^a \) can be shown similarly. This reduction in the absolute magnitude of net capital inflow/outflow is not surprising (though not obvious). Since the presence of transaction costs prevents some transactors from participation, the balance of the rest at any \( i \) is smaller in absolute value except at \( i = i^* + e^a \) when net supply equals zero.

As the domestic interest rate is changed (holding \( i^* \) and \( F \) constant), the bid-ask spread and, thus, the band representing the set of inactive transactors, moves up or down along the vertical axis in Figure 3. (In the absence of transaction costs, all transactors are active, and changes in \( i \)
move a line, rather than the bid-ask band, along the ordinate.) This manipulation traces the net supply schedule \( S' \) in Figure 4. Its shape, resembling the inverted letter ‘\( S \)’, is similar to that of the zero transaction costs schedule, \( S \). The effect of transaction costs is to twist the supply curve in an anti-clockwise direction around the fixed-point \( i = i^* + e^a \). The shape of the curve is also changed slightly since \( S' \) should be close to \( S \) for levels of \( i \) close to and significantly different from the parity value, \( i = i^* + e^a \), while in the intermediate range \( S' \) should diverge more from \( S \). This follows from the shape of the distribution \( F \). For example, at very high \( i \), when \( F \) becomes very small and rather flat, \( S' \) cannot differ from \( S \) by very much.

If transaction costs are not symmetrical for borrowing and lending or if the distribution function \( F \) is not symmetrical, then the net supply curve \( S' \) need not necessarily intersect the vertical axis at \( i^* + e^a \). However, since transaction costs are not very large in international capital flows, and since the distribution function \( F \), even if not normal, is unlikely to be drastically different from a generally single-peaked function, \( S'' \) will intersect the vertical axis in the vicinity of \( i^* + e^a \) and assume a general inverted-‘\( S \)’ shape like \( S \) and \( S' \) in Figure 4.

**Risk aversion**

It is not very realistic to assume that a transactor supplies all the funds that can be mustered once \( i \) exceeds a critical level, and that as much is borrowed as possible as soon as it is below a critical level (same level in the absence of transaction costs). The introduction of a concave utility function of income destroys this unrealistic all-or-nothing result. As shown in the Appendix, the supply of funds of a risk averse transactor is an increasing function of the domestic interest rate. Moreover, it is likely to be (must be in the case of a quadratic utility function) of inverted-‘\( S \)’ shape. The horizontal summation of all the upward sloping inverted-‘\( S \)’ individual supply curves that intersect the horizontal axis at different levels distributed by a roughly normal \( F \) still gives us an inverted-‘\( S \)’ market supply curve. However, since the individual curves are themselves upward-sloping (e.g. \( S' \) in Figure A in the Appendix) instead of horizontal with upper and lower limits (\( S \) in Figure A), the market supply curve \( S' \) will also be steeper. Thus, both the introduction of transaction costs and the relaxation of risk neutrality tend to make the supply curve steeper, reinforcing our arguments without changing any of our previous conclusions.

![FIGURE A](image_url)
Super-small countries

There may be certain countries which are so small that the supply of funds from a subset of international investors holding similar views regarding their currencies is enough to flood their markets. The market supply curves for such countries may be taken as virtually horizontal at $i^* + e^*$, even in the presence of diversity of expectations, transaction costs, and risk aversion. Such countries may be designated ‘super-small’ countries as distinct from what we normally regard as small countries. Conventionally, a ‘small’ country is distinguished by the condition that its domestic actions exert negligible effects on world variables such as the world interest rate. For a small country in this usual sense, we cannot assume the existence of a horizontal supply curve of international funds even under the assumptions of risk neutrality and no transaction costs. As shown above, diversity of views with respect to its currency depreciation is sufficient to make the supply curve upward-sloping.

V. Conclusion

The foregoing argument challenges the validity of exchange rate models that invoke uncovered IRP to postulate the exogeneity of domestic interest rates. It demonstrates that interest rates can vary, even in ‘small’ countries, in response to shifts in the domestic excess demand for funds, even in the absence of changes in the world interest rate and in the expected rate of depreciation.

This unconventional result follows directly from the non-deterministic nature of expectations about the future path of the exchange rate. Expectations are not formed in a vacuum, but are influenced by transactors’ particular circumstances and by their knowledge and understanding of the relevant economic structure. Consequently, exchange rate expectations are likely to differ across transactors. The assumption of heterogeneity of expectations is compatible with rational expectations formation as long as the differences are not systematic. The present analysis demonstrates that this property is reflected in a finite elasticity of the supply of funds.

If expectations turn out to be correct on average – the rational expectations presumption – then the domestic interest rate will conform to the uncovered IRP condition only if there are no net international capital movements. Conversely, if expectations are correct, then net movements of funds occur only if the domestic interest rate differs from the expectations-augmented exogenous world interest rate. These conclusions are reinforced by relaxation of the assumption of risk neutrality, as well as by recognition of transaction costs. The argument does not apply, and the conventional presumption of exogeneity of the domestic interest rate continues to hold, in the case of trivially small or ‘super-small’ countries where domestic capital markets are dominated by the investment decisions of a small subset of international investors.

The present analysis provides a potential explanation for the intractability of uncovered IRP in empirical work. It vindicates the rationality of the ostensibly paradoxical phenomenon of two-directional capital movements, i.e. of simultaneous inflows and outflows. And it confirms the continued existence, under floating exchange rates, of scope for discretionary interest rate management for all but the ‘super-small’ countries.
Supply of funds by an individual transactor with risk aversion

With risk neutrality, a transactor places all funds available for disposal in this country if \( i > i^* + e^p_j \). The introduction of risk aversion in the form of a utility function concave in income destroys this all-or-nothing (horizontal supply curve) result. It can be shown that the supply curve is upward-sloping, i.e. supply is increasing in \( i \).

Assume that incomes of the transactor from other sources are unaffected by the investments in the currency of this country. We may thus write

\[
y = \tilde{y} + sE
\]  

where \( y \) = total income, \( \tilde{y} \) = income from other sources, \( s \) = amount of funds invested in this country, \( E = i - i^* - (e^p_j + \mu) \) is the expected per dollar earning from foreign investment, and \( \mu \) is a stochastic variable with mean zero, and probability density distribution function \( f(\mu) \). The transactor maximises

\[
V = \int -U(y + sE)f'(\mu)d\mu
\]

The following first-order condition is obtained by setting \( \partial V/\partial s \) equal to zero,

\[
\int EU_jf'(\mu)d\mu = 0
\]  

where \( U_j = \partial u/\partial y \). Note that \( U_j \) depends on \( y \) and hence on \( \mu \).

Since a change in \( s \) does not affect any other terms on the R.H.S. of (1), we have

\[
dy/ds = E
\]  

Differentiating (3), holding \( i^*, e^p_j, \mu, \) and \( f'(\mu) \) constant, we have

\[
\int \left( U_{yy}dy + EU_{yy}dy \right)f'(\mu)d\mu = 0
\]  

where \( U_{yy} = \partial^2 U_j/\partial y^2 \).

Dividing (5) through by \( ds \) and substituting in \( dy/ds \) from (4), we obtain,

\[
\int \left( E^2U_{yy} + U_{yy}/\partial i \right)f'(\mu)d\mu = 0
\]  

Since \( U_{yy} > 0, U_{yy} < 0 \) (concavity of \( U \), and \( f(\mu) \) is a non-negative probability distribution function, \( ds/di \) must be positive for (6) to hold. In other words, given the world interest rate \( i^* \) and a risk-averse transactor’s subjective estimate of depreciation of the currency of this country, the expected utility-maximising supply of funds to this country is increasing in the interest rate of this country.
From (6), it is also clear that, with risk neutrality ($U_y = 0$), the supply curve is infinitely elastic, i.e. $ds/di = \infty$. Some upper and lower bounds on $s$ must then be introduced to reflect the non-infinite access to funds in reality.

Differentiating (6), dividing through by $di$ and substituting in $dy/di = s$ we have,

$$\int \left( sE^2U_y + 2EU_{yy} + \frac{sU_y}{ds/di} - \frac{U_y}{(ds/di)^2} \right) f'(\mu)d\mu = 0 \quad (7)$$

If we take the case where the utility function in the relevant range can be approximated by a quadratic function as is commonly assumed for simplicity, we have $U_{yy} = 0$. Since $U_{yy} < 0$, $ds/di > 0$, and since $E$ and $s$ have the same sign, the second and third terms in big brackets in (7) are negative/positive if $s$ is positive/negative. From the fourth term, this means that $d^2s/di^2$ must be negative/positive if $s$ is positive/negative. This in turn means that the individual supply curve is also an inverted ‘S’ as the market supply curve $S$ in Figure 2.

With $U_{yy} \neq 0$, $d^2s/di^2$ in general depends also on $f(\mu)$. However, just as it is reasonable to impose upper and lower bounds for $s$ in the all-or-nothing case in recognition of fund/credit limitations, similar limitations would ensure that, even if the individual supply curve is not of a perfectly inverted-‘S’ shape, it has to be in its general shape. In other words, the relaxation of risk neutrality transforms the individual supply curve from the extreme $S$ curve in Figure A into one of a general shape like $S'$.

\[ ^6 \text{This is obvious from the fact that } \delta s > \delta i \text{ if } 0 \text{ and from (1), given } U_i > 0. \]
REFERENCES

Dornbusch, Rudiger (1976), ‘Expectations and Exchange Rate Dynamics’, *Journal of Political Economy*, vol. 84.


