UTILITY AND PROFIT MAXIMIZATION BY AN OWNER-MANAGER: TOWARDS A GENERAL ANALYSIS*

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I. INTRODUCTION

The traditional and still widely accepted theory of the firm is based on the assumption of profit maximization. This assumption has, however, been extensively criticized. In particular, it is argued that the businessman will maximize his utility but not his profits. This criticism may have some plausibility in the case of big corporations where ownership is separated from control. For the case of an owner-managed firm, I wish to show that utility maximization leads to profit maximization, provided 'profit' is appropriately defined.

By the appropriate definition of profit, I mean, in particular, that 'profit' is not only net of the outlays on hired or purchased inputs (including interest on capital), but also net of the imputed wages to the owner-manager qua manager. It is obvious that the maximization of profit gross of this imputed wages makes no sense as it implies that, as long as the marginal return is positive, the owner-manager will work 24 hours a day and 365 days a year. The definition of profit as being net of the owner-manager's imputed wages was in fact used by Scitovsky (1943) when he attempted to show that a utility-maximizing owner-manager would maximize profit only if he has a peculiar indifference map between income and leisure. I (Ng, 1969) have already shown that profit maximization does not imply this peculiar indifference map. However, both Scitovsky and I did not take account of the possibility of hiring or selling managerial services by the owner-manager. In this paper, I wish to generalize my argument to include such possibilities and also to supplement the argument by a general mathematical treatment.1

II. FREEDOM TO BUY AND SELL MANAGERIAL SERVICES

For simplicity, assume that the utility of the owner-manager is a function of

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1 It must be mentioned that Koplin (1963) also has a verbal argument that utility maximization by an owner-manager implies profit maximization for his firm. On the other hand, using the Scitovsky framework, Ladd (1969) argues that utility maximization is sufficient for short- and long-run competitive survival and hence rejects profit maximization as a necessary condition for long-run competitive equilibrium. If my argument that utility maximization leads to profit maximization is correct, this rejection is meaningless. Recently, Olsen (1973) also argues that, where competition has wiped out any excess profit, utility maximization and profit maximization occur simultaneously. But for the case where excess profits exist, Olsen reaffirms Scitovsky's conclusion that profit maximization occurs only as a special case, though he queries Scitovsky's methodology.
his income $M$ and the amount of his leisure $L$. Consider Figure 1 where the point $N$ denotes zero amount of entrepreneurial labour. The amounts of both labour and leisure are measured on the horizontal axis in opposite direction. The slope of $MN$ measures the market-determined managerial wage rate that the owner-manager could earn if he decided to work as a hired manager. His possible income for any amount of managerial services he wishes to sell is shown by the height of $MN$ at the respective level of $L$.

![Figure 1](image-url)

If he wishes to operate his own firm, the amounts he can earn at different levels of $L$ are represented by the income curve $A$. This income consists of his imputed wages plus the profit (positive or negative) of the firm, and is net of outlays on other inputs. One may regard the difference between the

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2 This assumption seems to rule out the possibility of positive or negative marginal utility of work. But this need not be so as any non-zero marginal utility of work can be included as a component of the marginal utility of leisure. For example, if the marginal utility of work is positive, then the marginal utility of leisure time will be the marginal utility of leisure activity as such minus the marginal utility of work. This is so despite the argument of Evans (1972, p. 3, n. 5). It is, however, true that any possible difference in preferences between different types of work has been assumed away. This can be taken into the analysis at a cost of some complication without affecting the central issue here.

3 This figure is similar to Olsen's Figure 1.
income curve $A$ and the market wage curve $MN$ as the amount of profit $\pi$. This profit curve has its maximum at the level of leisure (and also labour) $L_2$. If the shape of the indifference curves is such that the highest one is touched by the income curve $A$ at $L_2$, as is the case depicted in Figure 1, then utility maximization and profit maximization will occur at the same point. But it is clear that the indifference curves need not necessarily be of this particular shape. Hence it is tempting for one to conclude that, where positive profits are being made, simultaneous profit and utility maximization will be the exception rather than the rule. $^4$

The above conclusion can be queried on two different grounds. First, the argument is based on the implicit assumption that the owner-manager is faced with only two alternatives: (i) working solely for himself without employing other managers; and (ii) working solely for someone else, though the hours of work can be varied in each instance. Under the assumption that the owner-manager can freely choose the hours of work he is prepared to do for other firms, it seems more logical to assume that, even after working for his own firm, he should still be able to work for other firms if he so desires. Hence, if his indifference curves are as represented by I and II in Figure 2, he can increase his utility after reaching the tangency point $G$ by selling his labour at the market price to reach the point $H$. Similarly, under the assumption of divisibility, it is logical to assume that the owner-manager can also hire any amount of managerial services to help him manage his own firm. Thus, if his indifference curves are $I'$ and $II'$ in Figure 2, he can increase his utility by hiring managerial services, moving from $G'$ to $H'$.

$^4$ This is actually the conclusion reached by Olsen (1973, p. 394).
Neither $H$ nor $H'$, however, indicates a point of maximum utility. As the owner-manager can freely buy or sell managerial services at the market price his feasibility set is given by $OM'N'$ in Figure 3, where $M'N'$ is parallel to $MN$ and touches the $A$ curve at $G$. The point of utility maximization can be either to the left of $G$ such as $U$ or to the right of $G$ such as $U'$, depending on the shape of his indifference curves. But in all cases, he operates his firm till the point $G$ which gives the maximum amount of profit. This operation may be assisted by hired managers, so he moves from $G$ to $U'$. Alternatively, he may be the sole manager of his firm as well as working for other firms and he moves from $G$ to $U$. Of course he may stay at $G$ if the $A$ curve touches his highest indifference curve at this point. The main point is that, even though the owner-manager is maximizing utility and hence may not choose a leisure-income mix at the point $G$, he will still operate his firm at this point $G$ which maximizes profit. It can thus be concluded that, if the owner-manager is free to buy and sell managerial services, utility maximization also leads to profit maximization. This conclusion is, of course, based on the implicit assumption of non-satiety which is weak enough.

Another query that can be made with respect to the argument of the second paragraph of this section may now be raised. If the owner-manager is faced only with the mutually exclusive alternatives of either working for himself or for another firm, his wages are not, in general, represented by the $MN$ curve. To see this, suppose he decides to work for his own firm. What is his imputed wages? The accountant and the businessman himself may answer that it is measured by the line $MN$. But as economists, we must figure in terms of opportunity costs measured by the best alternative foregone. It is
true that $MN$ is one of the foregone alternatives. But another foregone alternative is leisure. It is quite possible that the opportunity cost in terms of foregone leisure may well exceed that of the market price of managerial services. In this case, his imputed wages has to be measured by the cost of foregone leisure. The correct way to measure this cost of foregone leisure is discussed in the following section.

III. LIMITED MARKET FOR MANAGERIAL SERVICES

The condition that the owner-manager can sell or buy any amount of managerial services may not be satisfied in many cases. Different degrees of indivisibility may exist in different cases. A contrasting case to the one discussed in the preceding section is where the owner-manager cannot sell or buy managerial services at all, but is free to choose his level of work for his own firm.

Consider Figure 4 which is similar to the preceding three figures except that the owner-manager is not faced with the market opportunity line $MN$. Without such a market, how do we (or the owner-manager) determine the imputed wages? Since he can neither sell nor buy managerial services, the opportunity cost of working (for his own firm) is the leisure foregone. The problem is how to place a monetary value on the foregone leisure. One possible method is to take indifference curve I that passes through the point of zero amount of work $N$ as the measure of his imputed wages. This is based on the reasoning that, if his work is not paid sufficiently high as to bring him
above this indifference curve, it is better for him to stay at $N$. With this measure of managerial wages, the profit curve $\pi$ can then be derived by taking the difference between the income curve $A$ and the indifference curve $I$. It is then obvious that the point of maximum profit $P$ and the point of maximum utility $U$ occur, in general, at different levels of $L$, as shown in Figure 4.

The above argument is acceptable only if the indifference curve $I$ can be regarded as an appropriate measure of the imputed wages. For example, at $NS$ amount of work, $RS$ is wages and $UR$ profits. In taking this measure, one is comparing the points $N$ and $S$. If working the amount $NS$ does not bring at least $RS$, it is better to stay at $N$. If the owner-manager's choice is exclusively between $NS$ amount of work and no work at all, then $RS$ is the appropriate measure of his wages, the minimum amount of payment necessary to induce him to work. In this case of an all-or-nothing choice, the income curve dissolves into the points $N$ and $U$. If $U$ is higher (lower) than $R$, $U$ ($N$) will be the point where maximum profit and maximum utility coincide. No divergence between profit and utility maximization can arise.

In general, the owner-manager is free to choose different levels of work. In this case, the indifference curve $I$ is no longer, in general, the appropriate measure of his wages. The reason is that, though this indifference curve represents the locus of points which are indifferent to the point of zero amount of work, $N$, a point on it (e.g., $R$, with $NS$ amount of work and $RS$ amount of income) is not necessarily indifferent to doing some amount of work between $NS$. If working an intermediate amount of work brings an amount of income ($wages + profits$) different from the height of indifferent
curve I, then the marginal opportunity cost of work (or the marginal value of leisure) is no longer measured (by the slopes) along indifference curve I. This point is more clearly seen by referring to Figure 5. It is clear that the owner-manager will not choose any level of work to the right of B and to the left of N since this will make him worse off. FB therefore represents his wages for the amount of work NB. In considering whether to increase his work from B to C (disregarding the intermediate levels, i.e., assuming semi-divisibility), he is still consulting the indifference curve I. By working at B, his income is FB which brings him on the indifference curve I. His wages for BC amount of work is JH (the valuation of the lost leisure CB), leaving JK as profits. But once he is to the left of B, say at C, he is no longer on the indifference curve I. Rather, he is now on the point K and the indifference curve I'. The amount of wages for the amount of work CD has therefore to be ZY.

According to the analysis above, it can easily be shown that U is the point of simultaneous profit and utility maximization. To the right of U, the indifference curves passing through the income curve necessarily lie below U. Movement to U will then bring additional income greater than that necessary to compensate for the increase in the amount of work. Hence, positive profits are being earned for each successive movement from F until U is reached. On the other hand, it is not profitable to work to the right of U as the income curve lies below the indifference II. Hence, U, the point of maximum utility, is also the point of maximum profit.

If managerial activity can be varied continuously, we can draw a curve (see Figure 6) E from the point F with slope at each point equal to the slope...
of the corresponding indifference curve passing through the income curve between $U$ and $F$. For example, the slope of the curve $E$ at $H$ is equal to the slope of the indifference curve $I'$ at $J$. To the left of the point $U$, the curve $E$ is the vertical displacement of the indifference curve $II$. The reason is simple, to induce the owner-manager to increase his activity level after he has attained the point $U$, additional income (wages) must be sufficient to keep him at least at the satisfaction level represented by the indifference curve $II$.

![Figure 7](image)

The curve $E$ measures the sum of the minimum marginal increments in money income necessary to induce the owner-manager to supply his services continuously up to the respective level, given the income curve. If the owner-manager is free to vary his activity level as assumed, this $E$ curve gives the appropriate measure of his imputed wages. The vertical distance between the $E$ curve and the $A$ curve is then the residual profit that the owner-manager seeks to maximize.

It is now obvious that $U$ is the point of maximum profit as well, the slopes of $E$ curve and $A$ curve are equal at this point. To the right of this point, the slope (in absolute terms) of $E$ curve (for example, the slope at $H$) is smaller than that of the $A$ curve, because the slope of the indifference curve ($I'$) at the respective point ($J$) on $A$ curve, is smaller than that of $A$ curve. To the left of $U$, the slope of $E$ curve is clearly greater than that of $A$ curve, since it is the vertical displacement of indifference curve $II$.

Now consider Figure 7 where the $E$ curve is derived as above. If we introduce the possibility of working for another firm, the line $MN$ becomes
relevant. If $MN$ is above the curve $E$ in the relevant range, it seems that the point of maximum profit is no longer $U$ but $G$, where the income curve is parallel to $MN$. In this case, the derivation of the $E$ curve has to be slightly revised. Instead of starting from the point $F$, we start at $F'$, where $MN$ cuts the income curve, provided it is higher than $F$. (If the reverse is true, we start at $F$ as before.) We start at $F'$ because this represents the minimum income (wages) necessary to attract him to work for his own firm. From $F'$ onward (leftward), the derivation of the new $E$ curve ($E'$) depends on whether we assume (i) that the owner-manager can sell his services after committing to his own firm; or (ii) that he cannot. In the latter case, $E'$ is derived after $F'$ similarly to the derivation of $E$ curves above. In the former case, $E'$ is drawn in such a way that its slope (in absolute values) is equal to the higher of: (a) the slope of $MN$; (b) the slope of the indifference curve passing through the income curve, as above. This has to be the case because opportunity cost is measured by the best alternative foregone. In either case, profit maximization and utility maximization will still occur simultaneously.\(^5\)

We can also show simultaneous utility and profit maximization for the case where the owner-manager can work for his firm himself as well as hiring managerial services to help him. I refrain from doing this to avoid repetition. The case where he can both hire and be hired has already been analysed in Section II above. We have thus established that, whichever assumption we adopt with respect to the possibility of the owner-manager to sell and/or buy managerial services, profit maximization and utility maximization always occur simultaneously. We have to use so many figures because of the limitation of geometry; each figure can usually depict a particular situation. To analyse the various cases together, I use below a mathematical formulation.\(^6\)

IV. MATHEMATICAL FORMULATION

It is convenient to list the notations used.

**Notations**: $U =$ the utility function of the owner-manager.

$M =$ his money income.

$L =$ the amount of his leisure time.

$T =$ total given amount of his time.

\(^5\) At $U$, if his indifference curve is steeper than $MN$, then $E'$ is similar to $E$ and profit maximization and utility maximization coincide on the same point $U$. If his indifference curve is less steep than $MN$, then it is the case of indifference curve III in Figure 3. Profit and utility maximization still occur simultaneously, but at different points.

\(^6\) The formulation is similar to the one used by Olsen (1973) in some aspects, but differs in the following points. First, in our formulation, the imputed wages of the owner-manager is not necessarily the market price for managerial services. Secondly, Olsen does not allow for the possibility of the selling or buying of managerial services. Thirdly, Olsen (1973, p. 390) writes the amount of each input as an indirect function of the amount of managerial services. This is not only unnecessary but also misleading. For example, the last summation term on his equation (15) (p. 391) should not be there. (Compare our condition (5) below.) In addition, I have also imposed the non-negativity constraints and hence do not rule out corner solutions.
\[ P = \text{the parametrically given price of his product.} \]
\[ F = \text{the output of his product.} \]
\[ p^i = \text{similar price of his } i\text{-th input.} \]
\[ x^i = \text{quantity of the } i\text{-th input used.} \]
\[ \gamma = \text{amount of managerial service used in his firm.} \]
\[ \gamma^s = \text{amount of managerial services supplied by himself to his firm.} \]
\[ \gamma^h = \text{amount of managerial services hired.} \]
\[ \gamma^s = \text{amount of managerial services hired.} \]
\[ w = \text{his imputed wage-rate for his services.} \]
\[ w^m = \text{the market wage-rate for managerial services.} \]

The owner-manager maximizes

\[ U(M,L) \]

where

\[ M = PF(x^1, \ldots, x^n, y) - \sum_{i=1}^n p^i x^i + w^m (\gamma - \gamma^h) \]
\[ L = T - \gamma^s - \gamma^h \]
\[ \gamma = \gamma^s + \gamma^h \]

and all variables must be non-negative.

The following necessary conditions can then be derived.

\[ PF_i - p^i \leq 0, = 0 \text{ or } x^i = 0 \ (i = 1, \ldots, n) \]
\[ PF_y - w^m \leq 0, = 0 \text{ or } \gamma^h = 0 \]
\[ U_M PF_y - U_L \leq 0, = 0 \text{ or } \gamma^s = 0 \]
\[ U_M w^m - U_L \leq 0, = 0 \text{ or } z = 0 \]

where a subscript denotes partial differentiation, e.g.,

\[ F_i = \partial F / \partial x^i \text{, etc.} \]

Condition (5) requires that, for every input employed, its physical marginal product equals the ratio of input to output prices. Condition (6) requires the same thing with respect to managerial services employed.

If \( \gamma^s \neq 0 \), and either \( \gamma^h \neq 0 \) or \( z \neq 0 \), we can derive the following two equations either from the equality parts of (6) and (7), or (7) and (8),

\[ U_L / U_M = w^m \]
\[ PF_y = w^m \]

This means that the (absolute) slope of the income curve (A curve in the figures above), which is \( PF_y \), must be equal to the slope of \( MN \), which is \( w^m \). Moreover, the slope of the indifference curve \( (U_L / U_M) \) is also equated with the slope of \( MN \). This corresponds with the analysis of Section II and the second half of Section III above.

On the other hand, if the possibility of buying and selling managerial
services is excluded, conditions (6) and (8) become irrelevant, then we have, from (7), provided $y^s \neq 0$

\[(11) \quad U_L / U_M = PF_y\]

or the direct tangency of the indifference curve with the income curve, and corresponds to the case in the first half of Section III above.

Now turn to the issue of profit maximization. The profit of the owner-manager (or rather his firm) is given by

\[(12) \quad PF(x^1, \ldots, x^n, y) = \sum \hat{p}^i x^i - w^m y^h - \int w \, dy^s\]

where $w$ is his imputed wage rate which, in general, is not constant with respect to $y^s$. Hence the integral. Maximization of (12) requires the following conditions.

\[(13) \quad PF_i - \hat{p}^i \leq 0, \quad = 0 \text{ or } x^i = 0\]
\[(14) \quad PF_y - w^m \leq 0, \quad = 0 \text{ or } y^h = 0\]
\[(15) \quad PF_y - w \leq 0, \quad = 0 \text{ or } y^s = 0\]

We can now compare the above conditions for profit maximization with those for utility maximization. Conditions (13) and (14) exactly duplicate (5) and (6). So we may just concentrate on (15). First consider the case where the owner-manager is free to buy and sell managerial services at a constant market price $w^m$. In this case, his imputed wages $w$ equals $w^m$. If $y^s \neq 0$, we have, from (15), $PF_y = w^m$ which is exactly equation (10). Secondly, consider the case where no market for managerial services exists. In this case, as argued above, his imputed wages $w$ equals the slope of his indifference curve $U_L / U_M$. Hence, (15) gives $PF_y = U_L / U_M$ which is exactly equation (11). We may thus conclude that, one way or the other, the necessary conditions for profit maximization by the owner-manager’s firm is exactly the same as those for his personal utility maximization. Assuming the satisfaction of the second-order conditions, these necessary conditions also constitute the sufficient conditions. Hence, profit maximization must occur simultaneously with utility maximization. The acceptance of our argument means that the traditional theory of the firm is on a strong ground at least for firms where ownership and control are closely related.7

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REFERENCES


7 Complications arising from income effects are not likely to be very important since substitution effects usually predominate.


