The Possibility of a Paretian Liberal: Impossibility Theorems and Cardinal Utility

Y. K. Ng

University of New England, Australia

Since the publication of Arrow's celebrated works (Arrow 1950, 1963) on the paradox of social choice, dozens of publications have appeared attempting to reject or bypass the General Impossibility Theorem. Sen's (1970) recent paper is interesting in that, unlike the mainstream, he actually presents another impossibility theorem, namely, that the Pareto principle is inconsistent with liberalism. It is the intention of the present paper to argue that, since impossibility theorems hold only if the relative intensity of preference is not taken into account, the Pareto principle and liberalism are not inherently contradictory after all. This paper also attempts to refute an argument of Arrow which has not been called into question.

I

The gist of Sen's (1970) theorem is that there is no social decision function which satisfies the following conditions.

CONDITION U (Unrestricted Domain): Every logically possible set of individual orderings is [admissible]. . . .

CONDITION P [Pareto principle]: If every individual prefers any alternative \( x \) to another alternative \( y \), then society must prefer \( x \) to \( y \). . . .

CONDITION L (Liberalism): For each individual \( i \), there is at least one pair of alternatives, say \( (x, y) \), such that if this individual prefers \( x \) to \( y \), then society should prefer \( x \) to \( y \), and if this individual prefers \( y \) to \( x \), then the society should prefer \( y \) to \( x \). [In other words, each individual is decisive over at least one pair of alternatives.]\(^3\) [P. 153]

---

1 In an earlier paper, Sen (1969) shows that the Impossibility Theorem does not apply to social choice function where only the best alternative is indicated. But he also revives the theorem in relation to social choice function by some additional rationality requirement.

2 This is defined to be a rule which specifies the best or chosen alternative according to individual orderings of the relevant alternatives.

3 Sen (1970) actually shows that a weaker version of Condition L is sufficient for his proof.
In others words, given the Condition of Unrestricted Domain, Pareto principle and liberalism are incompatible.

The rationale of Condition L is that there are some choices which should be the "private affairs" of some individuals, and, hence, society should let the individual concerned have decisive power: "Whether you should sleep on your back or on your belly is a matter in which the society should permit you absolute freedom, even if a majority of the community is nosey enough to feel that you must sleep on your back" (Sen 1970, p. 152).

In the present stage of the discussion on the problem of social choice, it should be common knowledge that the General Impossibility Theorem holds because only the ordinal preferences is or can be taken into account. If the intensity of preference or cardinal utility can be known or is reflected in social choice, the paradox of social choice can be solved (cf. Kemp and Asimakopulos 1952; Coleman 1966; Mueller 1967). I hasten to add that cardinal utility as used here subsumes some sort of interpersonal comparison. The problem of social choice is to formulate a social preference or choice function from those of the preferences of various individuals. Thus, if there is no way to compare the preference intensities among the individuals, the knowledge of these intensities alone gives little help. However, it is difficult to know and compare the preference intensities of different individuals. Liberalism may be regarded as a partial solution to the problem of social choice in view of these difficulties. Thus, whether you sleep on your back or belly is a matter in which you

---

4 Economists nowadays are very hostile to cardinal utility. This is rather unfortunate. In those cases where the assumption of ordinal utility will do the job, it is to be preferred on grounds of Ockham's razor. However, there are certainly some areas of economics where we need the help of cardinal utility, for example, involving risk, optimum population, etc. The belief that utility is not cardinally measurable in nature (not referring to the practical difficulty of measurement) is clearly unfounded. If utility or preference is not cardinal, then one is unable to compare the difference in utility between having an apple and an orange and that between an orange and a house; one is also unable to compare the difference in disutility between a bite of an ant and a bite of a bee and that between a bite of a bee and being thrown bodily into fire. It is also widely believed that cardinal utility is not assumed in an indifference map. This belief can also be shown to be erroneous. In fig. 1, it can clearly be seen that the utility level of indifference II is twice that of I, if X and Y are independent goods, and more (less) than twice if they are Edgeworth-complementary (substitutary) goods. Thus, if X and Y are independent (complementary, substitutary) goods, the utility implied by indifference curve II, for example at P, equals (is more than, is less than) the utility at R plus the utility at S, while R and S are both on indifference curve I. But we cannot speak of "twice," "more than twice," or "less than twice" if we have a strictly ordinal utility index.

5 The required interpersonal comparison need not necessarily be scientific and objective. Some sort of ethical, common-sense, or political comparison—such as that used in Kemp and Asimakopulos (1952), where the same weight is given to the first preference of each individual—will ensure a transitive social choice, although it may not be optimal from the viewpoint of the utilitarians.
may have a strong preference but does not usually affect others in any significant way. If this matter is decided by majority voting, the outcome may be very undesirable. Suppose you strongly prefer sleeping on your back, but two individuals mildly prefer your sleeping on your belly, and all others are indifferent. Majority voting would dictate that you should sleep on your belly, which is clearly undesirable. Liberalism may thus be seen as an alternative to majority rule in this and similar cases. Whenever the choice is likely to affect some particular individual significantly but is very unlikely to affect others in any significant way, society agrees that this choice should be left entirely to that individual.

![Diagram](image)

**Fig. 1**

In terms of Sen’s conditions, this means that whenever Condition L is applicable, Condition U is generally not applicable. Where individual 1 is given decisive power over x and y, other individuals are not likely to prefer strongly y over x (or vice versa). Thus, the preference pattern \((y > z > w > x)\) of individual 2 in Sen’s proof is not likely to happen, given that individual 1 is decisive over x and y.

We are not arguing that Sen’s proof is incorrect. Given his interpretation of liberalism which requires only a decisive power over a pair of alternatives by each individual without restricting the preference pattern of other individuals on this pair—that is, given his Condition L and Condition U—Pareto principle cannot be sustained generally. What we argue is that the spirit of liberalism is not adequately represented by Condition L, and, hence, this spirit of liberalism is not inherently inconsistent
with Pareto principle. Thus, while Sen's theorem is logically valid as it stands, it is not very significant in practice.

However, it is not argued that the practice of liberalism will not be inconsistent with the Pareto principle at all times. In Sen's example of the reading of Lady Chatterley's Lover, the dictates of liberalism contradict Pareto principle, because one individual's reading externally affects the feeling of another so much that the liberal presumption that reading is solely the individual's own business is violated. However, this does not mean that the spirit of liberalism and the Pareto principle are inherently inconsistent. The extraordinary important external effect may be a special case. The enforcement of liberalism would lead to a social improvement in nine out of ten cases, but the reverse may be true in some special cases, such as Sen's example. However, if this is persistent and important, the liberals may well agree to restrict the scope (but not the spirit) of liberalism. For example, it is now the belief of most liberals that one should have absolute freedom in deciding whether to paint one's house white or green. However, if scientists discover that the color white has a devastating effect on eyesight, most liberals may well agree to the prohibition of or a special tax on white houses. Liberalism does not mean that one may do whatever he likes; one is permitted to do anything he likes only if he does not significantly affect others. The remaining possible Pareto inefficiency due to the practice of liberalism may be regarded as a cost of the enforcement of a general rule. The rule that motorists must stop at red traffic lights surely saves a lot of accidents and congestion, but it also creates some unnecessary waiting time. Similarly, liberalism also leads to many social improvements. But we cannot know exactly every case (for example, white house) where liberalism should well be abandoned. Hence, the enforcement of liberalism in the generally agreed scope must necessarily result in some "inefficiency." This is due to the requirement of practical convenience (in enforcing a general rule, rather than to examine each particular case) rather than to the inherent inconsistency between Pareto principle and the spirit of liberalism.

II

We turn now to the criticism of an argument of Arrow. Having proved his Impossibility Theorem, Arrow goes on to say that "similarly, the market mechanism does not create a rational social choice" (Arrow 1963, p. 59). Since the word "similarly" (that is, similar to the Impossibility

6 However, liberals are not easily persuaded to restrict the freedom of reading on the ground that reading may have strong external effects, as in Sen's example. This is because the restriction of the freedom of reading would have some profound undesirable effects in the long run. Hence, the solution to the paradox in Sen's example may consist in disregarding the external effects in the short run and in "developing individual values that respect each other's personal choices" in the long run.
Theorem) is used, we understand that the quoted sentence does not refer to the market solution of the provision of public goods but rather to the social state—that is, production and distribution of goods—resulting from the functioning of the market. One may also understand this from his earlier statements, for example, "The methods of voting and the market . . . are methods of amalgamating the tastes of many individuals in the making of social choices" (Arrow 1963, p. 2).

The deduction of Arrow seems to be questionable. I leave open whether the market solution is "rational" (in what sense?), but the logic of Arrow is faulty. The solution of the market and the problem of social choice posited by Arrow are distinct in one important sense (see below), so that one cannot say that the market solution is irrational by analogy to the Impossibility Theorem.

As mentioned above, the Impossibility Theorem holds because only ordinal preference is taken into account. However, the market mechanism takes cardinal preference into account. A consumer does not only prefer oranges to apples, but he has to pay exactly five cents for each orange. And this five cents reflects the marginal utility of an orange to him. Thus, if a rock melon costs him ten cents, the marginal utility of a rock melon is exactly (disregarding indivisibility and irrational choice) twice that of an orange. Moreover, the preference intensities of different individuals are also interpersonally "compared" in the market by giving each dollar one vote; that is, the preference of each individual is weighted by the amount of his income. It is this characteristic which leads the market to achieve Pareto optimum under "classical" environments, that is, perfect competition and the absence of externality, indivisibility, and increasing returns. On the other hand, no such cardinal significance is possible in Arrow's postulate of the individual orderings of social states. Arrow's extension of his Impossibility Theorem to market mechanism is therefore invalid.

References

腹部


7 We must, therefore, qualify Arrow's assertion that "market mechanism however, takes into account only the ordering according to tastes" (Arrow 1963, p. 18, italics added).

8 Buchanan (1954a) also criticizes Arrow's deduction on market mechanism, but his criticism is not based on the fact that the market takes cardinal preference into account. This important difference between market and voting is also not mentioned in Buchanan (1954b).


