A Note on “Some Cambridge Controversies in the Theory of Capital”

G. C. Harcourt’s brilliant review essay [3, 1969] contains a misleading statement in section 3.14.¹ He points out that the distribution of per capita income can be written as

\[ y = rk + w \]

whence

\[ k = \frac{y - w}{r} \]

while, when

\[ r = \frac{dy}{dk}, \]

\[ k = -\frac{dw}{dr} \]

He then claims that “\( k \) cannot equal both \( (y-w)/r \) and \( -dw/dr \) unless the factor price frontier of each method [of production] is a straight line” [3, Harcourt, 1969, pp. 392-393]. This is shown in Figure 1 where \( \tan \Theta = \tan \Psi \) measures \( (y-w)/r \) and \( \tan \Psi = -dw/dr \), and \( \tan \Theta \neq \tan \Psi \). E. J. Nell [4, 1970] points out a case where \( k \) may equal both \( (y-w)/r \) and \( -dw/dr \). However, Nell’s case takes the rate of growth into consideration and Nell himself says that Harcourt’s claim is “fair enough” for the switching problem where the rate of accumulation of capital is assumed to be zero. Moreover, referring to Figure 1, Nell also states that “clearly these (the two tangents) will only be equal when the \( w-r \) relationship is a straight line.” [4, 1970, p. 41].

I shall offer a counter example where the two tangents are equal. My example is within the confines of the switching problem; i.e., problem of growth is not considered.

Consider Figure 2 where the factor-price frontier is not a straight line. But clearly the two tangents are equal for the point \( P \). This means that, contrary to the contention of Harcourt, nonstraight line \( w-r \) relation-

¹ Harcourt was reporting the work of P. Garegnani [2, 1970] which became available to me after I wrote this note. Of course my criticism applies to Garegnani’s paper and to Amit Bhaduri’s [1, 1969, pp. 536-37] reproduction of Garegnani’s argument.

**FIGURE 1.**

**FIGURE 2.**
ships may not be inconsistent with the neo-classical parable that \( r = \frac{dy}{dk} \). This latter equation of the neoclassicals is supposed to hold only at the equilibrium point and not for all points on the frontier. And we have shown that such a point is possible for some non-straight line frontiers.

However it is also clear that such a point is not possible if the frontier is wholly concave or wholly convex. But for the neoclassicals this may also raise doubts about the possibility of such curves, rather than the possibility of \( r = \frac{dy}{dk} \). This seems to be an interesting problem to be pursued by the participants on both sides of the controversy.

For the simple example given by Nell,

\[
w = \frac{1 - ar}{B + r(ab - aB)},
\]

\[
\frac{dw}{dr} = \frac{-ab}{[B + r(ab - aB)]^2}
\]

\[
\frac{d^2w}{dr^2} = \frac{2ab(ab - aB)}{[B + r(ab - aB)]^3}
\]

If \((ab-aB)\) is negative, \(d^2w/dr^2\) may change signs as \( r \) increases. This could be consistent with Figure 2. But if \((ab-aB)\) is positive, the \( w=r \) frontier must be convex to the origin. This result does not seem to be pleasing to the neoclassicals. But the latter may still argue that \( r \) may still equal \( \frac{dy}{dk} \) if growth is taken into account.

Another problem which may be pursued further is the Solow-Nell controversy. Solow proves that the social rate of return to investment always equals the switching rate of profits [6, 1967]. Nell [4, 1970] claims to have shown this to be false by using Spaventa's diagram. However, Solow's proof appears to be mathematically elegant and his opponent admits that the result, though tautological, is necessarily true in the accounting sense [5, Pasinetti, 1969, pp. 525-26]. This casts doubts upon the validity of Spaventa-Nell diagram—at least as applying to Solow's proof.

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REFERENCES


G. C. Harcourt's Reply to Ng

Mr. Ng's counter example is certainly right—I should have confined my statement to entirely concave and convex \( w-r \) relationships as he says. When I say "I," I do wish people would acknowledge that I was reporting the work of Bhaduri and Garegnani in this context. His criticism is also, in my opinion, minor. For while it is quite true that at that point on the counter example \( w-r \) relationship, curvature is not inconsistent with \( \frac{dy}{dk} \) being treated "as if" it were equal to \( r \), it is only true of that point.