Waveguide devices with homogeneous complementary media

Yueke Wang, Dao Hua Zhang,* Jun Wang, Xuefeng Yang, Dongdong Li, and Zhengji Xu
School of Electrical and Electronic Engineering, Nanyang Technological University, 639798, Singapore
*Corresponding author: edhzhang@ntu.edu.sg

Received June 23, 2011; revised August 24, 2011; accepted September 5, 2011; posted September 8, 2011 (Doc. ID 149724); published September 26, 2011

We report the design of microwave waveguide devices based on complementary media. Various kinds of waveguide devices that possess nearly 100% transmission efficiency are proposed, such as waveguide bends, splitters, connectors, and shifters. Compared with previous work on waveguide devices of low reflection and minimized distortion based on transformation optics, our transform media are homogeneous metamaterials. Electromagnetic simulations by a finite-element method on detailed examples have been performed to validate the designs and these functionalities can be close to the practical. © 2011 Optical Society of America


Since 2006, transformation optics has been widely studied because it provides many possibilities to control the propagation of an electromagnetic wave along an expected path [1,2]. This potent method is applied in various fields, including cloaks, concentrators [3], antennas [4], and so on [5]. Among the above applications, invisibility cloaks attract the most attention. Although the research on metamaterials has made great progress [6], practical realization of invisibility cloaks remains a challenge.

Recently, the concepts of complementary media were applied to design an invisible cloak [7]. A perfect lens could also be explained and designed by the complementary media [8]. In addition, complementary media were proposed as a coating around an object to enlarge the electromagnetic wave scattering cross section [9].

For conventional microwave waveguide devices, reflection and distortion are inevitable. The development of transformation optics provides an option to solve this problem. So far, transformation optics has been applied to design other waveguide devices, such as waveguide bends [10,11], connectors [12,13], and splitters [14]. However, the previous designs have demanded extreme material parameters, and such material is too complex to be easily realized. Recently, Han et al. [15,16] proposed a method for adaptive waveguide bends using homogeneous, nonmagnetic, and isotropic materials. The proposed method simplified the parameters of the bends. In this Letter, we propose a convenient method for designing waveguide devices by using complementary media. In this method, the designed material parameters are homogeneous, relatively relaxed, and need not be so stringent. Our design is, therefore, closer to practical realization for current nanofabrication technology.

Figure 1(a) shows the scheme of designing waveguide bends with no reflection or distortion by filling homogeneous complementary media into the bends. The bending area is divided into regions 1 and 2. The material parameters are \((\varepsilon_1, \mu_1)\) and \((\varepsilon_2, \mu_2)\), and an arbitrary point in the two regions can be expressed as \((x_1, y_1, z_1)\) or \((x_2, y_2, z_2)\), respectively. To eliminate reflection and distortion, the optical phases at boundaries AC and AC' must be exactly the same. The complementary medium \((\varepsilon_2, \mu_2)\) is formed by a coordinate transformation of folding region 1 into region 2. The transformation equations can be expressed as

\[
x_2 = a_1 x_1 + b_1 y_1 + c_{11},
\]

\[
y_2 = a_{12} x_1 + b_{12} y_1 + c_{12},
\]

\[
z_2 = z_1.
\]

The permittivity and permeability tensors of the complementary medium in region 2 can be obtained from

\[
\varepsilon_2 = \frac{\Lambda_1 \varepsilon_1 \Lambda_1^T}{\det(\Lambda_1)}, \quad \mu_2 = \frac{\Lambda_1 \mu_1 \Lambda_1^T}{\det(\Lambda_1)},
\]

where \(\Lambda_1\) are the Jacobian transformation tensors corresponding to the coordinate transformations of folding region 1 into region 2. So when the material parameters satisfy the above condition, the waveguide mode can perfectly tunnel through the bends without reflection or distortion, and is independent of the surroundings.

Now we take several examples of waveguide bends to verify the above design principle. Full-wave simulations based on the finite-element method are performed, and the TE0 mode is excited from port 1. All boundaries around the simulation regions are selected as perfect electric conductors and the working frequency is 10 GHz. First, let us consider a 90° bend \((\alpha = 90°)\) designed by the proposed method and that connects the two uniform waveguides, as shown in Fig. 1(a). The widths of the two waveguides are chosen to be 5 cm. If we choose air as the material in the two waveguide arms and region 1 (the following is the same), then \(\varepsilon_2 = \mu_2 = -1\) for this method. Figure 1(b) shows the snapshots of the total electric fields. In the bending area, the fields around the interface between the two regions are strong due to the excitation of the surface waves induced by the multiple scattering of light. This indicates that the perfect tunnel effect is a steady-state phenomenon.

We now look at where the bending regions are unsymmetrical, as shown in Figs. 1(c) and 1(d), in which the lengths of the BC boundaries are 4 and 12 cm, respectively, while all the other parameters are the same as in Fig. 1(a). The perfect tunnel effect still happens when
we choose suitable complementary media in region 2:
\[ \varepsilon_2 = -0.5, \quad \mu_2 = \begin{bmatrix} -1 & 1 \\ 1 & -2 \end{bmatrix} \]
for Fig. 1(c) and
\[ \varepsilon_2 = -1.5, \quad \mu_2 = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \]
for Fig. 1(d). For the cases when the bending regions are symmetrical but the bending angles are different, the total electric fields of the 45° (\( \alpha = 45° \) and BC = 8√2 cm) and 60° (\( \alpha = 60° \) and BC = 12 cm) bending are shown in Figs. 1(e) and 1(f), respectively. The transmission efficiencies are nearly 100% and no distortion is observed where \( \varepsilon_2 = \mu_2 = -1 \).

Figure 2(a) shows the schematic of the designed waveguide splitters by filling homogeneous complementary media into the splitting regions. For simplicity, the medium in regions 1 and 2 is air; and the complementary medium in regions 3 and 4 is obtained according to Eq. (2). All the widths of the waveguide arms are the same as in Fig. 1(a), and AC = DC = 4 cm. Assume that the TE\(_1\) mode is excited from port 1 and the bending angle between the original arm and the output arms is \( \alpha \). The snapshots of the total electric fields for \( \alpha = 90° \) are shown in Fig. 2(b). The propagation modes in ports 2 and 3 are TE\(_0\) mode and the sum of the transmitted power in the two arms is equal to the TE\(_1\) mode from port 1. So the splitter can perfectly split the guided wave into two uniform divisions.

Next we design waveguide splitters with different bending angles of 45°, 60°, and 120°. The snapshots of the total electric fields are shown in Figs. 2(c)–2(e). It is clearly shown that the TE\(_1\) mode excited from port 1 is split into two uniform divisions with the suitable complementary media in the splitting region. It is believed that the method can be used to design arbitrary angle splitters.

Then we propose a three-step-fold transformation method to design the waveguide shifters and connectors. To transfer the TE\(_0\) mode from port 1 to port 2, as shown in Fig. 3, the phase of the AC boundary must be identical to that of the FD boundary. The complementary media of region 2 are obtained based on Eq. (2). Region 3, denoted by the triangle ECD, is folded from the triangle BCD, and the complementary medium \((\varepsilon_3, \mu_3)\) in region 3 can be expressed by \((\varepsilon_2, \mu_2)\) based on Eq. (2). Region 4, denoted by triangle FED, is folded from the triangle ECD, and the complementary medium \((\varepsilon_4, \mu_4)\) in region 4 can also be expressed by \((\varepsilon_3, \mu_3)\) based on Eq. (2).

Now we design waveguide shifters and connectors to verify the above three-step-fold design principle.
Fig. 3. (Color online) (a) Schematic illustration of the designed waveguide shifters or connectors with the complementary media. Snapshots of the total electric fields: (b) and (c) for waveguide shifters and (d) and (e) for waveguide connectors with different geometries.

Figures 3(b) and 3(c) show the electric fields of the waveguide shifters. In Fig. 3(b), the boundaries AB, AC, EF, DF, and CD are all equal to 8 cm, $\varepsilon_2 = \mu_2 = \varepsilon_4 = \mu_4 = -1$, $\varepsilon_3 = \mu_3 = 1$; while in Fig. 3(c), the boundary CD is 8 cm and AB, AC, EF, and DF are 4 cm.

$$
\begin{align*}
\varepsilon_2 &= -0.5, \\
\mu_2 &= \begin{bmatrix} -2.5 & -0.5 \\ -0.5 & -0.5 \end{bmatrix}.
\end{align*}
$$

$$
\begin{align*}
\varepsilon_3 &= 0.5, \\
\mu_3 &= \begin{bmatrix} 2.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}.
\end{align*}
$$

$$
\begin{align*}
\varepsilon_4 &= \mu_4 = -1.
\end{align*}
$$

The waveguide mode in the original waveguide shifts to another waveguide with no distortion or reflection. Next, we design the waveguide connectors that connect two waveguides with different widths. Figures 3(d) and 3(e) show the electric fields of the waveguide connectors with different geometries. The width of port 1 is 4 cm and that of port 2 is 8 cm. In Fig. 3(d), AB = EF = 4 cm and CD = $4\sqrt{5}$ cm.

$$
\begin{align*}
\varepsilon_2 &= -\frac{1}{3}, \\
\mu_2 &= \begin{bmatrix} -\frac{5}{3} & -\frac{4}{3} \\ -\frac{4}{3} & -\frac{5}{3} \end{bmatrix}.
\end{align*}
$$

$$
\begin{align*}
\varepsilon_3 &= -0.2, \\
\mu_3 &= \begin{bmatrix} -1 & -1 \\ -1 & -2 \end{bmatrix}.
\end{align*}
$$

$$
\begin{align*}
\varepsilon_4 &= \frac{1}{2}, \\
\mu_4 &= \begin{bmatrix} 0.5 & 0 \\ 0 & 2 \end{bmatrix}.
\end{align*}
$$

In Fig. 3(e), AD = 8 cm and AB = 4 cm.

$$
\begin{align*}
\varepsilon_2 &= -1, \\
\mu_2 &= \begin{bmatrix} -5 & 2 \\ 2 & -1 \end{bmatrix}.
\end{align*}
$$

$$
\begin{align*}
\varepsilon_3 &= -0.5, \\
\mu_3 &= \begin{bmatrix} -2.5 & 0.75 \\ 0.75 & -0.625 \end{bmatrix}.
\end{align*}
$$

$$
\begin{align*}
\varepsilon_4 &= \frac{1}{2}, \\
\mu_4 &= \begin{bmatrix} 0.5 & 0.25 \\ 0.25 & 2.125 \end{bmatrix}.
\end{align*}
$$

No matter whether the waveguide mode is excited from port 1 or port 2, it can propagate through the connector perfectly.

In conclusion, based on fold transformation, various waveguide devices are proposed and designed theoretically in this Letter. The general expressions of constitutive tensors of the bending regions are derived. Compared with the previous work on adaptive waveguide bends based on transformation optics [15, 16], we use the complementary media theory to design homogeneous metamaterials that can cancel reflection or distortion, and this method expands to the design of waveguide bends, splitters, connectors, and shifters.

The project is supported by the National Research Foundation (NRF-G-CRP 2007-01) and A*Star (092154009), Singapore.

References