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Systematic study of the focal shift effect in planar plasmonic slit lenses

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Abstract
In this paper, we systematically studied the focal shift effect in planar plasmonic slit lenses. Through theoretical derivations and numerical simulations, we found that there is a focal length shift between the traditional design model and the finite-difference time-domain simulations. The shift is not only dependent on the Fresnel number (FN) of the lens, like traditional dielectric lenses, determined by the lens width and the designed focal length, but also on the surface plasmon polariton (SPPs) interaction on the lens surfaces, dependent on the slit numbers. We also found that the FN-induced focal shift is predominant when FN < 1. However, the SPP interaction-induced focal shift plays a major role when FN > 1. An approximated theoretical model is presented to estimate the focal shift of plasmonic slit lens with FN < 1. (Some figures may appear in colour only in the online journal)

1. Introduction
Plasmonic devices have been rapidly developed in recent years due to their enormous potentials in miniaturized photonic circuits, data storage and solar cells [1–3]. Due to the confining properties of surface plasmon polaritons (SPPs), plasmonic metallic devices could break the diffraction limit and manipulate light in the near-field, as compared to traditional dielectric optical elements [4–9]. One important plasmonic device is the plasmonic lens for beam engineering. Among various plasmonic lenses, planar plasmonic slit lenses (PPSLs) are often used to realize integrated optical collimators [10–16]. The PPSL consists of a metallic slab, perforated with several well-designed nanoslits with various widths, thicknesses and material compositions. When a light wave propagates through these slits, it has different phase retardations [17]. Therefore, the focus of light can be realized by adjusting the materials and geometric parameters of the slits through the phase control. Compared to other plasmonic lenses, PPSLs have relatively simple structures, which can be easily fabricated. In addition, it has a better design flexibility because traditional diffraction optics theory, e.g. some iteration algorithms [18], can be applied to obtain the desired slit parameters.

However, the designed focal length obtained from the traditional in-phase model [19] is usually away, sometimes significantly away, from the actual focal length obtained from the rigorous vector simulations and experiments in PPSLs [12, 20]. For example, in the experimental work of Verslegers et al [12], the designed focal length is 20 µm, while the actual focal length obtained is only 5.3 µm, only a quarter of the designed focal length. This large focal shift could become a serious obstacle in the application of PPSLs. Therefore, it is important to understand the origin of this focal shift and the parameters affecting it.

In this paper, we present a comprehensive study on the parameters influencing the focal shift effect of PPSLs. By comparing the theoretical and simulated results, we found that the focal shift of a PPSL is mainly determined by the Fresnel number (FN), dependent on lens width and the designed focal length, when FN is small (<1). When FN is large (>1), the effect of the SPP interaction on the lens surfaces, dependent
on the number of slits, becomes obvious, causing deviation of the simulated actual phase from the one achieved by the traditional design model. In the end, we also present an approximated theoretical model to evaluate the focal shift of a designed PPSL with \( FN < 1 \).

2. Structure and basic principles

The design principle for a PPSL is based on the phase modulation of one-dimensional slits [12–16]. The schematic structure of the lens is shown in figure 1. A metal film laid on the \( x\)–y plane is perforated with \( 2N - 1 \) slits. The thickness and the dielectric constant of the metal film are denoted by \( h \) and \( \varepsilon_m \), respectively. A transverse magnetic (TM)-polarized (with magnetic field parallel to the \( y \) axis) plane wave with a wavelength of \( \lambda \), is propagating along the \( -z \) direction. The width of the \( n \)th slit is denoted by \( w_i \) from the middle of the slit lens to the two sides and the phase retardation of the plane wave propagating through the \( n \)th slit is denoted by \( \varphi_i \). The width of the lens, which is defined as the distance between the two outermost slits, is denoted by \( d \), see figure 1. Therefore, in order to ensure all the transmitted waves are in phase at a designed focal length of \( f_d \), the phase retardation of the \( n \)th slit should satisfy [13]

\[
\varphi_n = 2\pi \left( n + \frac{f_d - \sqrt{f_d^2 + x_i^2}}{\lambda} \right)
\]

(1)

where \( n \) is an integer and \( x_i \) is the position of the \( n \)th slit. The actual focal length is denoted by \( f_a \). We note that, based on this design principle, there is always a focal shift between the designed focal length \( f_d \) and the actual focal length \( f_a \) [12]. We express this focal shift as \( \Delta f = f_a - f_d \). In order to study the focal shift effect, we define a focal shift coefficient as

\[
\eta = \frac{\Delta f}{f_d} = \frac{|f_a - f_d|}{f_d}.
\]

(2)

Then we need to decide how to choose the slit widths to obtain the phase distribution in (1). First, we consider a single slit with a width \( w \) and a length \( h \), respectively. If we assume \( h \sim \infty \), a metal-clad waveguide structure [19] (also called a plasmon slot waveguide [21] or metal–insulator–metal structure [22]) is formed and the dispersion relation in this waveguide is expressed as

\[
\sqrt{\varepsilon_d k_0^2 - \beta^2} - 2 \arctan \frac{\sqrt{\beta^2 - \varepsilon_m k_0^2 \varepsilon_d}}{\sqrt{\beta^2 - \varepsilon_d k_0^2 \varepsilon_m}} = m \pi
\]

(3)

for TM polarization, where \( \varepsilon_d \) and \( \varepsilon_m \) denote the dielectric constants of the dielectric in the slit and the metal, respectively. \( m \) is the mode number, \( k_0 \) is the wavevector in vacuum and \( \beta \) is the complex waveguide propagation constant. If \( \beta = 0 \), we can obtain the cutoff width of each mode from (3) as

\[
w_c^{TM} = \frac{\lambda}{2 \sqrt{\varepsilon_d}} \left[ \frac{m}{2} + \frac{1}{\pi} \arctan \left( \sqrt{\frac{\varepsilon_d}{\varepsilon_m}} \right) \right]
\]

(4)

where \( w_c^{TM} \) denotes the mode cutoff width of TM-polarized waves. From (3), one can find that all the modes have a cutoff width except the fundamental mode \( TM_0 \) because, when \( m = 0 \), the calculated \( w_c^{TM} \) is less than zero.

According to (3), the dispersion relation of \( TM_0 \) can be written as

\[
\tanh \left( \frac{\sqrt{\beta^2 - \varepsilon_m k_0^2 \varepsilon_d}}{2} \right) = \frac{-\sqrt{\beta^2 - \varepsilon_m k_0^2 \varepsilon_d}}{\sqrt{\beta^2 - \varepsilon_d k_0^2 \varepsilon_m}}
\]

(5)

which corresponds to the symmetric surface plasmon modes [22, 23]. Therefore, when the slit width is narrower than the cutoff width of the \( TM_1 \) mode (also called the antisymmetric mode), calculated by \( w_c^{TM_1} = \lambda/\sqrt{\varepsilon_d} [1/2 - \arctan [\sqrt{\varepsilon_d(\varepsilon_d + \varepsilon_m)}] / \pi] \), only the symmetric mode can propagate in the slit. In this situation, for a slit with a finite length \( h \), the slit can be simply considered as a single-mode Fabry–Perot (FP) cavity, with the refractive index of the \( N \) slits.

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The entrance and transmitted magnetic field at the exit, respectively.

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respectively. The metal is chosen as silver and the dielectric constant is applied the same as \([13]\) that \(\varepsilon_{\text{Ag}} = -17.36 + 0.715i\).

3. Effect of the phase deviation caused by diffraction and SPP interaction

It is well known that, when a plane wave passes through a PPSL, complicated diffractions and SPP coupling exist at both the entrance and the exit surfaces, which may cause deviation of the actual phases from the traditionally designed phases. However, all these effects are not considered in (5) and (6). In this section, we will study the contribution of this phase deviation on the focus. We first consider a single slit structure. In figure 2, we give the comparison of phase retardations as a function of slit width of a single slit, calculated by finite-difference time-domain (FDTD) method \([24]\) (red line with dots) and the FP model (blue line) from (5) and (6). The calculated range of the slit width is from 10 to 100 nm, which is narrower than \(w_{\text{TM}}\) to ensure single-mode propagation in the slit. The results show that the phases obtained from both FDTD and FP models decrease about \(2\pi\) in the slit. The incident wavelength is 650 nm. The thickness of the film is \(h = 500\) nm. The inset shows the single-slit structure.

We then consider a double-slit structure. For simplicity, the two slits have the same widths. In this situation, SPPs can be generated at the exit of one of the slits, propagates along the surface and partially transmitted into the other slit (as indicated in the inset of figure 3), which may have a contribution to the phase retardation of the other slit. In order to study the effect of an adjacent slit on the phase retardation, we define this effect as \(\Delta\phi = \phi' - \phi\), where \(\phi'\) is the phase retardation of the double-slit structure and \(\phi\) is the phase of a single slit, calculated in figure 2. The results of \(\Delta\phi\) as a function of the two slit distances are plotted in figure 3. Without loss of generality, the two slit widths are set as 25 nm. It is found from the FDTD results that \(\Delta\phi\) oscillates with an amplitude of \(0.1\pi\) when the distance increases. The good prediction of the ASP model of this oscillation indicates that the SPP interaction and coupling at the exit surface can change the phase retardation of a slit. However, this effect is also not considered in the FP model.

Since both of the SPP diffractions and SPP coupling of adjacent slits have an influence on the slit phase retardation, it can be inferred that, in a more complicated PPSL system, the obtained phase retardation will not coincide strictly with the designed one. We then design a lens with a designed focal length of \(f_\text{d} = 10\lambda\) from (1), (5) and (6). The lens width is \(d = 4\ \mu\)m. The slits are separated equidistantly and the slit number is 41, i.e. \(N = 21\). The corresponding slit positions and widths are depicted in figure 4(a). Then we extract the phase information of the slits at the exit surface from the FDTD result, which is shown as the red line in figure 4(b). It shows clearly that the actual phase distribution does not agree well with the objective phase distribution (blue line), due to the scattering of the slits and the strong coupling of the SPPs excited by the slits \([26]\). The largest deviation reaches \(0.6\pi\). In figure 4(c), we present the transmitted field distribution. Here the normalized Poynting vector of the \(z\) direction \(|S_z|/|S_{\text{o}}|\) is used to express the field intensity, where \(S_{\text{o}}\) is the \(z\)-direction Poynting vector of the incident plane wave. As indicated in figure 4(c), although the designed focal length is \(10\lambda\), the actual focal length is only \(6.2\lambda\). The focal shift coefficient is \(\eta = 38\%\). Then one may ask: is this large focal shift entirely caused by the phase mismatch in figure 4 (b)? In order to answer this question, we need to have the field distribution.
Figure 4. Effect of deviation of the actual phase from the objective phase on the focal shift. The parameters are set as $\lambda = 650$ nm, $h = 500$ nm, $d = 4 \mu$m and $f_d = 10\lambda$, respectively. (a) The slit positions and corresponding slit widths. (b) Objective (blue) and actual (red) phase distribution of the slits at the exit surface of the metallic slab. (c) Distribution of normalized Poynting vector in the $z$ direction $|S_z|/|S_{z0}|$ calculated by FDTD with actual phase. The actual focal length is 6.2$\lambda$. (d) Distribution of normalized Poynting vector in the $z$ direction $|S_z|/|S_{z0}|$ calculated by 2DGF with objective phase.

4. Effect of the structure parameters on the focal shift

In this section, in order to find the causes of the focal shift, we will study the effect of the structural parameters on the focal shift. We first present the simulation results of lenses with different slit numbers in figure 5. In the simulations, the lens width and designed focal length are set as $d = 4 \mu$m and $f_d = 5\lambda$, respectively. The slit number is increased from 11 to 41, i.e. $N$ is increased from 6 to 21, and the slits are separated equally. Therefore, the distance between two adjacent slits becomes smaller with $N$ increasing. Figure 5(a) shows the $|S_z|$ distribution of the transmitted light for $N = 11$, 16 and 21. For comparing the focus intensity, we let the maximal $|S_z|$ for $N = 21$ be unity, which is the strongest of the three. An obvious feature of the picture is that the focus intensity is increased with the slit number. This feature can also be seen in figures 5(b) and (c), which show the intensity distribution of more lenses in the actual focal plane $z = -f_a$ and the middle line $x = 0$, respectively. The Poynting vector in these two pictures is normalized with that of the incident plane wave $S_{z0}$. Another feature that is indicated in Figure 5(c) is that the focal shift is increased with the slit number. In table 1, we give the detailed main parameters of the focus of each lens, including focal shift, full width at half-maximum (FWHM) in both $x$ and $z$ directions, and the maximum normalized Poynting vector. It shows that the focal shift increase from 0.3$\lambda$ to 1.08$\lambda$ when the slit number increases from 13 to 42. The FWHM in $x$ and $z$ directions are not changed much. However, the slit number has a remarkable effect on the intensity of the focus. When the total slit number increases three times (from 13 to 42), the intensity increases by about 7.5 times (from 0.37 to 3.03). This is because more energy is transmitted through the lens with more slits.

We then change the designed focal length from $1\lambda$ to $15\lambda$, and set the lens width and slit number at $d = 4 \mu$m and $N = 21$, respectively. The results are depicted in figure 6. It can be seen that, with the increase of the designed focal length, the focal spot is larger and the intensity is weakened. Another important feature is that the focal shift is much increased with $f_d$. When the designed focal length is only one wavelength, the simulated result gives good agreement. However, when $f_d = 15\lambda$, $f_a$ is less than 8$\lambda$. In table 2, we give $\Delta f$, $\eta$, FWHM in the $x$ and $z$ directions, and the maximal intensity of the focus. It can be found that the focal shift coefficient increases...
to compare the focal pattern, the calculated Poynting vectors for large lens width is very weak, in order to decrease from 6.3 to 2.86.

In addition, the lateral and vertical FWHM are broadened from 25% when the designed focal length is larger than 15λ. It can be inferred that the focal coefficient will larger than 1% to 48% when f = 2d.

We now study the effect of lens width on the focus. Like the analysis above, the other two parameters are set as f = 5λ and N = 21. The Poynting vector distributions for d = 2, 4, 6, 8 and 10 μm are plotted in figure 7. Because the focus intensity for large lens width is very weak, in order to compare the focal pattern, the calculated Poynting vectors of a lens are normalized to the maximum |Sz| of the focus. The result shows that the focal spot becomes smaller with the increase of the lens width. The focal shift is very large for lenses with small width. In table 3, it can be found that the shift coefficient η reaches 59.2% for d = 2 μm and reduces to 0.6% for d = 10 μm. Both the FWHM in the x direction and z direction are decreased.

5. Discussions about the focal shift

In this section we will discuss the reason for the dependence on the lens parameters of the focal shift. From tables 2 and

Table 1. Dependence of the focus on slit number.

| N     | Slit distance (nm) | Δf (%) | η (%) | x-FWHM | z-FWHM | |Sz| Max/|Sz| 0 |
|-------|-------------------|--------|-------|--------|--------|-------------------|--------|
| 6     | 400               | 0.38λ  | 6     | 0.67λ  | 4.33λ  | 0.37              |        |
| 8     | 286               | 0.24λ  | 4.8   | 0.63λ  | 5.44λ  | 0.59              |        |
| 11    | 200               | 0.56λ  | 11.2  | 0.62λ  | 4.44λ  | 1.32              |        |
| 13    | 167               | 0.58λ  | 11.6  | 0.62λ  | 4.82λ  | 1.24              |        |
| 16    | 133               | 0.85λ  | 17.0  | 0.62λ  | 4.27λ  | 1.98              |        |
| 18    | 118               | 0.96λ  | 19.2  | 0.60λ  | 4.05λ  | 2.32              |        |
| 21    | 100               | 1.08λ  | 21.6  | 0.60λ  | 3.88λ  | 3.03              |        |

Table 2. Dependence of the focus on designed focal length.

| N     | Slit distance (nm) | Δf (%) | η (%) | x-FWHM | z-FWHM | |Sz| Max/|Sz| 0 |
|-------|-------------------|--------|-------|--------|--------|-------------------|--------|
| 12λ   | 0.01λ             | 1      | 0.4λ  | 1.05λ  | 6.3    |                   |        |
| 13λ   | 0.03λ             | 1      | 0.6λ  | 2.44λ  | 4.1    |                   |        |
| 15λ   | 1.08λ             | 21.6   | 0.6λ  | 3.88λ  | 3.03   |                   |        |
| 18λ   | 2.3λ              | 28.8   | 0.91λ | 8.05λ  | 3.85   |                   |        |
| 19λ   | 3.6λ              | 38     | 0.96λ | 9.32λ  | 3.23   |                   |        |
| 22λ   | 5.25λ             | 43.8   | 1.03λ | 12.87λ | 2.84   |                   |        |
| 24λ   | 7.2 λ             | 48     | 1.18λ | 14.53λ | 2.86   |                   |        |

Figure 5. Effect of the slit number on the focus of the lens. The lens width and designed focal length are set as d = 4 μm and f = 5λ, respectively. (a) Distribution of Poynting vector in the z direction |Sz| for N = 11, N = 16 and N = 21, respectively. Here we let the maximum |Sz| of the distribution for N = 21 be 1 as a reference. (b) Normalized |Sz| of the calculated focal plane z = −f. Sz is the Poynting vector in the z direction of the incident plane wave. (c) Normalized z direction Poynting vector at x = 0 of the transmitted region.
Figure 6. Effect of the designed focal length on the focus of the lens. The lens width and the slit number length are set as $d = 4 \mu m$ and $N = 21$, respectively. (a) Distribution of $|S_z|$ for $f_d = 1\lambda$, $f_d = 8\lambda$, and $f_d = 15\lambda$. We let the maximum $|S_z|$ of the distribution for $f_d = 1\lambda$ be unity as a reference. (b) Normalized $|S_z|$ of the calculated focal plane. (c) Normalized $|S_z|$ at $x = 0$ of the transmitted region.

Table 3. Dependence of the focus on lens width.

| $d$ (\(\mu m\)) | $\Delta f$ | $\eta$ (%) | $x$-FWHM | $z$-FWHM | $|S_z|_{\text{Max}}/|S_{\text{std}}|$ |
|------------------|-----------|------------|--------|--------|-----------------|
| 2                | 2.97\(\lambda\) | 59.2       | 0.73\(\lambda\) | 4.77\(\lambda\) | 2.77            |
| 4                | 1.08\(\lambda\) | 39.2       | 0.63\(\lambda\) | 3.94\(\lambda\) | 3.04            |
| 6                | 0.14\(\lambda\) | 2.8        | 0.51\(\lambda\) | 2.77\(\lambda\) | 0.8             |
| 8                | 0.08\(\lambda\) | 1.6        | 0.46\(\lambda\) | 2.05\(\lambda\) | 0.49            |
| 10               | 0.03\(\lambda\) | 0.6        | 0.41\(\lambda\) | 1.66\(\lambda\) | 0.49            |

one can find that the focal shift becomes larger when the designed focal length increases and the lens width decreases. This phenomenon is also found in the traditional dielectric lenses, in which the focal shift effect becomes obvious when the $FN$ is small ($FN$ is expressed as $FN = d^2/(4\lambda f_d)$) [28–30].

The basic physical mechanism behind the focal shift is due to the competition of the interference effect and damping effect of the transmitted light. According to the Huygens–Fresnel principle, although the Huygens wavelets from all the slits are in phase at the designed focus, the amplitude of the transmitted light wave decreases with the distance from the lens [28]. Therefore, the position of the largest intensity is closer to the lens than the designed focal length. When the designed focal length is increased or the lens width is decreased, the damping effect is increased. As a consequence, the focal shift is increased.

On the other hand, it is also found from table 1 that even if the structures have the same $FN$, the focal shift increases with the slit number. When $N$ increases from 6 to 21, the distance between the two adjacent slits decreases from 400 to 100 nm (see table 1). Considering the skin depth of silver is only 24 nm in our structure [1], which is much smaller than the slit distance, mode coupling between two slits should not be the reason for this focal shift. However, the propagating distance of SPPs is 40 $\mu m$ [1]. Based on the results in figure 3, it can be inferred that the focal shift is caused by the SPP interaction on the exit surface of the lens. When the slit number is increased and the slits become nearer and nearer, the SPP coupling effect from other slits is more and more obvious, which causes the phase distribution not to agree with the designed one like figure 4(b). Because this effect is not included in the results calculated by ‘perfect’ phases, we can also draw a conclusion from figure 4 that the focal shift from the designed focal length (10$\lambda$) to the focal length of ‘perfect’ phases (7.1$\lambda$) is caused by $FN$, while the focal shift from 7.1$\lambda$ to the actual focal length (6.2$\lambda$) is induced by the SPP interaction.

It should be noted that the SPP-induced focal shifts are all less than 1$\lambda$ in all of our simulations. However, when
Figure 7. Effect of the lens width on the focus. The designed focal length and the slit number are set as \( f_d = 5 \lambda \) and \( N = 21 \), respectively. (a) Distribution of \( |S_z| \) for \( d = 2, 6 \) and 10 \( \mu m \). (b) Normalized \( |S_z| \) of the focal plane. (c) Normalized \( |S_z| \) at \( x = 0 \) of the transmitted region.

Table 4. Comparison of \( FN \)-induced and SPP-induced focal shifts as a function of \( FN \).

<table>
<thead>
<tr>
<th>( FN )</th>
<th>9.47</th>
<th>3.16</th>
<th>1.98</th>
<th>1.18</th>
<th>0.95</th>
<th>0.79</th>
<th>0.63</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta f_{FN}/\lambda )</td>
<td>0.019</td>
<td>0.165</td>
<td>0.651</td>
<td>3.128</td>
<td>4.422</td>
<td>6.615</td>
<td></td>
</tr>
<tr>
<td>( \Delta f_{SPP}/\lambda )</td>
<td>0.0737</td>
<td>0.144</td>
<td>0.426</td>
<td>0.366</td>
<td>0.674</td>
<td>0.825</td>
<td>0.584</td>
</tr>
</tbody>
</table>

FN is less than 1, the \( FN \)-induced focal shifts are usually much larger than the SPP-induced ones. In table 4, we give the comparison of \( FN \)-induced focal shift (\( \Delta f_{FN} \)) and SPP-induced focal shift (\( \Delta f_{SPP} \)) for different \( FN \) according to table 2. It shows that, when \( FN \) is 9.47, \( \Delta f_{SPP} \) is larger than \( \Delta f_{FN} \), which means the SPP-induced focal shift plays a major role. However, for \( FN = 0.63 \), \( \Delta f_{FN} \) is predominant in the total focal shift, which is ten times larger than \( \Delta f_{SPP} \). The same phenomenon can also be found for table 3.

Because the focal shift is inevitable and very large when \( FN \) is small due to the \( FN \)-caused focal shift, a fast calculating estimating method is needed to predict this effect. Although approximated formulae were presented for estimating the focal shift in traditional lenses [28, 29], these approximations are not applicable for a PPSL, in which the far-field and paraxial approximations are not valid because the focus is much closer to the plasmonic lenses than that of the traditional dielectric lenses, and the lens width is comparable with the wavelength. Therefore, we apply the Huygens–Fresnel principle to estimate the focal shift of a PPSL. Then the magnetic field component of the transmitted light on the middle plane \( x = 0 \) of figure 1 is expressed as [27]

\[
H(x = 0, z) = \int_{-d/2}^{d/2} A_T e^{j k z} \frac{e^{j k_0 \sqrt{z^2 + x_i^2}}}{(\sqrt{z^2 + x_i^2})^{1/2}} \, dx_i
\]  

(7)

where \( \Gamma \) is the exit surface of the lens, and \( A_T \) and \( \varphi_T \) are the amplitude and phase of the magnetic field component of the points on \( \Gamma \). If we only consider the electromagnetic field of the slits and assume that the phase at each slit exit is planar and fulfill equations (1) and (7) is changed to

\[
H(z) = \sum_{i=-N}^{N} e^{j k_0 (z - \sqrt{z^2 + x_i^2})} \frac{e^{j k_0 \sqrt{z^2 + x_i^2}}}{(\sqrt{z^2 + x_i^2})^{1/2}} w_i
\]  

(8)

where \( i \) presents the \( i \)th slit. The negative sign represents the slits of the left side. For simplicity, we let \( A_T = 1 \). The position of maximal intensity satisfies

\[
\frac{\partial I}{\partial z} = \frac{\partial [H(z) \cdot H^*(z)]}{\partial z} = 0
\]  

(9)
Therefore, according to (8) and (9), the actual focal length $f_a$ can be calculated by

$$
N \sum_{i=-N}^{N} \sum_{m=-N}^{N} \frac{e^{i k_0 (\alpha_i - \beta_j - a_m + b_m)}}{\sqrt{a_i \mu_m}} \times \left[ \frac{1}{a_i} - \frac{1}{a_m} \right] = \frac{1}{2} \left( \frac{1}{a_i^2} + \frac{1}{a_m^2} \right) w_i w_m = 0 \quad (10)
$$

where $a_{i,m} = \sqrt{a_i^2 + x_{i,m}^2}$, $b_{i,m} = \sqrt{b_i^2 + x_{i,m}^2}$. Equation (10) does not have an analytical solution; however, it can be easily solved numerically. In figure 8, we present the comparison of the focal shift coefficient calculated by FDTD and (10). It can be seen that the estimating method gives a good prediction in figures 8(c) and (b) when $F_D < 5 \lambda$. The deviation of about 7% of $\eta$ in (a) and (b) for $f_d < 5 \lambda$ is caused by the SPP-induced focal shift, because this effect is not considered in the estimating method and $FN$ in these structures are all larger than 1. However, considering that $FN$ is usually very small in a PPSL design, this evaluation method is acceptable. For example, in [14], the $FN$ of the PPSL is 0.18 and the evaluation method gives the prediction of the focal length is 4.7 $\mu$m, which is very close to the experimental result 5.3 $\mu$m. In addition, the calculating time of the evaluation method is much shorter than the FDTD method. It only takes about one second for calculating 20 PPSLs; however, for the FDTD method it takes more than 10 h. With the consideration of the fast calculation speed of the estimating method, it is recommended in predicting the focal shift of a PPSL with $FN$ smaller than 1.

### 6. Conclusion

In summary, we give a comprehensive study on the focal length shift effect of a PPSL. By studying the dependence of the focal shift on lens parameters including the slit number, designed focal length and lens width, we found that the focal shift is caused by both the small Fresnel number and SPP interaction effects. When the $FN$ of the PPSL is larger than 1, the SPP-induced focal shift is larger than the $FN$-induced one. However, when $FN$ is less than 1, which often happens in a PPSL, the $FN$-induced focal shift plays a major role. Furthermore, we give an estimation method to efficiently predict the focal shift. The result shows that it gives a good estimation of the focal shift when $FN$ is less than 1 with a fast calculating speed. The work presented here provides a systemic study of PPSLs that is broadly applicable to the design of other plasmonic devices.

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